Essays in Modeling the First Bid in Retail Secondary Market Online Auctions: A Bayesian Approach

by Babak Zafari

B.Sc. in Applied Mathematics, January 2006, Sharif University of Technology
M.S. in Operations Research/Computer Science, August 2009, Bowling Green State University

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Refik Soyer
Professor of Decision Sciences

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Babak Zafari

Dissertation Research Committee:

Refik Soyer, Professor of Decision Sciences, Dissertation Director

Srinivas Y. Prasad, Associate Professor of Decision Sciences, Committee Member

Thomas A. Mazzuchi, Professor of Operations Research & Engineering Management, Committee Member

Nicholas Polson, Professor of Econometrics & Statistics, The University of Chicago, Committee Member

Wenjing Duan, Associate Professor of Information Systems & Technology Management, Committee Member

Dedication

I dedicate this dissertation to my first teachers, my beloved parents Ahmad Zafari and Shahin Ghobadi. All I have and will accomplish is only possible due to their unconditional love and their endless support. I also dedicate this dissertation to my uncle Amir Zafari for his continued support even when I was away. Finally, I would like to thank my sister Golbarg and my brother-in-law Mehdi who have been always there for me during my hard moments.

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Abstract of Dissertation

Essays in Modeling the First Bid in Retail Secondary Market Online Auctions: A Bayesian Approach

The online commerce has greatly changed the trading markets for both businesses and consumers. A big part of this environment is the way online auctions are conducted. In this dissertation, I develop a series of models to study some of the elements relevant to the formation of first bids in the secondary market online auctions.

In the first essay, I propose models to study bidders' participation in auctions and their first bidding behavior in their participated auctions. Through these models I predict two major elements of given auctions: who will bid and who will place the first bid. I apply these models to different sets of auctions and compare their fit measures and predictive performances. In developing the models, in addition to some auction-specific characteristics, I consider some time-varying bidder's characteristics as well which show how bidders change their strategies by learning from their past activities. In addition, I show how the information from other overlapping auctions may influence bidders' decisions in participating in the focal auction and the time they place their first bid.

In the second essay, I focus on the time that auctions receive their first bid and study the effect of auctions features on arrival of the first bid. I propose a model of finite mixture of beta distributions to model the time to the first bid. The developed model helps in studying the auctions that receive their first bids very early or very late and how this may impact their dynamics. I also discuss its predictive aspects and its managerial implication.

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1 Introduction

The online commerce has greatly changed the trading markets for both businesses and consumers. A big part of this environment is the way online auctions are conducted. In addition to the changes in the Business-to-Consumer (B2C) and Business-to-Business (B2B) auctions, internet has also expanded this market by giving the opportunity to a user to sell his merchandise to other users (Consumer-to-Consumer or C2C auctions). In the current literature, problems like optimal design of an auction for a seller, optimal bidding behavior for a consumer and role of major factors in these processes have been extensively studied. In this work, however, we study the formation of the first bids in online auctions and we address the question of what factors might influence bidder's decision in participating and placing the first bid in a focal auction. We also try to see whether people learn and adopt different strategies over the time and whether we can benefit from studying the history of their bidding and in predicting their bidding behavior in the future auctions.

The first bid has an important role in an auction. Bajari and Hortacsu (2003) show how the first bid influences the number of bidders and consequently final price of an auction. In a similar work, Ariely and Simonson (2003) study the role of the first bid on the herding which eventually will affect the recovery rate (final price divided by retail value of the item) of the auction. Li et al. (2009) also discuss how early bids signal information about the value of an item to other bidders and how this may increase the final price and subsequently the recovery rate of an auction. However, while most of these studies are in the context of Business-to-Customer (B2C) or Customer-to-Customer (C2C) auction platform such as eBay, the focus of the current study is on the formation of (first) bids and their arrival time in retail secondary market online Business-to-Business auctions platform where there is overall more uncertainty involved regarding auctioned items and characteristic of the auctions are different. In Pilehvar et al. (2013), authors investigate the impact of bidders' internal and external reference price on the first bid and how it is moderated by bidders heterogeneity which is closely related to their experience and their level of participation in concurrent auctions. In their study, which is conducted over the same secondary market platform as our study, they also cluster bidders in order to distinct bidders that behave differently in the way they use the available information.

As empirical analyses show, the impact of the bidders' experience on their decision is significant. For example, in the context of some online auction marketplaces such as eBay, it has been claimed that experienced bidders tend to bid later than the less experienced bidders (Wilcox, 2000) and that less experienced bidders may over-value an auction and suffer from winner's curse problem and pay higher prices that the true value of the item (Bajari and Hortacsu, 2003). This can be interpreted as less experienced bidders are still learning about the characteristics of these auctions or that experienced bidders has adopted a strategy over time that has worked better for them.

The online auction platform that we study is the same as the one studied in Pilehvar et al. (2013). This platform deals with the resale of excess, returned and salvaged consumer electronic merchandise from a big-box retailer in North America. It differs from common online B2C platforms studied in the literature as the majority of the bidders are different re-sellers with different level of bidding activities and experience and as the there is a lot of uncertainty involved in the contents of the pallets. The retailer sends these pallets (of returned/excess items) to the liquidator warehouse in big-boxes and they are auctioned in 'as-is' condition where neither the retailer nor the seller (liquidator) takes responsibility for the condition of the contents. In other words, bidders are uncertain about the exact quality of the items in the pallet and the way a bidder evaluates these pallets is mainly based on their resale value which is determined by the condition and composition of the pallet. This may also depend on the fact that different retailers have different return policies and different inventory and packaging processes. However, as also discussed in Pilehvar et al. (2013), some of these factors may be learned by bidders over the time and after opening the pallets.

Another motivation in conducting this study is the lack of Bayesian inference in modeling online auctions and the potential insights one may get by using the Bayesian approach. In a paper by Park and Bradlow (2005), authors suggest a Bayesian framework to capture some key behavioral components of an internet auction such as who bids, when they bid and how much they bid at the auction level. As it is also discussed in their work, one of the the main advantages of using Bayesian methodology is that it allows for sharing of information over auctions; a fact that becomes even more important where there are auctions that have received only few bids. Li et al. (2009) have implemented a hierarchical Bayesian framework based on the developed model of Park and Bradlow (2005) to study how different features of an auction will change bidders' perception toward the uncertainty of the auction and how it affects their bidding behavior. They also suggest a need for studying the dynamics of bidding behavior over time to develop stronger models.

In this dissertation, our objective is to develop a series of models to study some of the elements relevant to the formation of first bids in the secondary market online auctions:

In the first essay, we propose models to study bidders' participation in auctions and their first bidding behavior in their participated auctions. Through these models we intend to predict two major elements of given auctions: who will bid and who will place the first bid. We apply our models to different sets of auctions and compare their fit measures and predictive performances. In developing the models, in addition to some auction-specific characteristics, we consider some time-varying bidder's characteristics as well which helps us to study how bidders change their strategies by learning from their past activities. In addition, we try to see how the information from other overlapping auctions may influence bidders' decisions in participating in the focal auction and the time they place their first bid.

In the second essay, we focus on the time that auctions receive their first bid and study the effect of auctions features on arrival of the first bid. We propose a model of finite mixture of beta distributions to model the time to the first bid. The developed model helps in studying the auctions that receive their first bids very early or very late and how this may impact their dynamics. We also discuss its predictive aspects and its managerial implication.

To the best of our knowledge, none of the models have been developed before. Our work is the first attempt in developing a dynamic Bayesian model in studying auctions over time to predict bidders' behavior in upcoming auctions. Also, none of our proposed models for modeling timing of the bids have been studied in the literature before. In both essays, we first introduce the basic approach and its implementation to develop an understanding of the potential advantages as well as possible limitations of the proposed approach. We then show how we will utilize the retail secondary market online auction dataset, which consists of bid information of 2000 unique bidders who have participated in over 11000 auctions over the course of five years (2003-2008), in developing and validating our models. Finally, we conclude each essay with a summary of the results, managerial implications and implementation issues that may arise and possible extensions.

2 Essay 1:

2.1 Summary

In this essay, motivated by a paper by Pilehvar et al. (2013) in which they study the effect of market price information on the bidding behavior of the first bidder, we first propose a Bayesian dynamic probit model to see who will bid an auction. Given participation, we then use a beta regression model to predict bidders' time to place their bid and consequently the bidder who will place the first bid earlier than other participants. In developing both of the models, we consider some auction-specific and some bidder-specific explanatory variables. The bidder-specific characteristics take into account both bidders' bidding behavior and their experience based on their previously participated auctions and their current activities at the time of the bid. The primary focus of the models will be on the predictive performance of the models, however, we will also study how different auction features and bidders heterogeneity will affect the dynamics of participation and bid timings.

2.2 Introduction

In this essay we focus on the use of a Bayesian dynamic probit framework and a beta regression model to study bidders' participation and the time they place their first bid in auctions. In the most general expression, our observation of a bidder's participation in an auction can be represented in the form of covariate structure and model links where Y is the binary outcome that gets the value of 1 if the potential bidder has placed a bid in the focal auction and 0 otherwise, β the vector of parameters and X the vector of explanatory variables. Then, given a set of participants for an auction, we can model the time of the first bid for each bidder and compare them to predict the first bidder of the auction; the bidder that has the minimum time of the first bid among others. In the literature, there are various approaches to model bidding behavior and the factor that influence this behavior.

There is a vast amount of literature on the effect of experience on the bidder's behavior in the auctions. Wilcox (2000) examines the effect of bidder's experience (defined as 'the number of previous auctions in which he/she has participated and won') on his/her bidding behavior. In this study which is done over bidding data of four product categories on eBay, he finds empirical support that in an auction, experienced bidders tend to bid in the final moments of an auction (i.e., snipe) and that they are more likely to place less bids compared to inexperienced bidders. The data also suggests that this learning leads to better bidding strategies and higher success rate. Similar results are obtained in works of Easley et al. (2010) and Livingston (2010) where they note that bidders learn from their experience and they find bidding strategies that increase their final payoffs. In addition, Borle et al. (2006) find that experienced eBay bidders tend to place their bids either closer to the beginning of the auction or closer to the end of the auction. Similar to some other works in the literature, they define experience based on eBay's feedback point system which for each bidder is the 'net of number of positive and negative feedbacks received over the time'. They also find that experienced bidders tend to be less involved in multiple biddings. On the other hand, Wang and Hu (2009) study the effect of two different types of experience (winning and losing) on the bidders' bidding behavior. They analyze the full 6-months participation history of the bidders and show that losing experience plays more important role in refining bidding strategies than do winning experience. In the market that we're studying, there are bidders with different levels of participation and experience both in terms of winning and losing; active bidders with high winning rate, active bidders with very low winning rate, bidders with low participation but high winning rate and so on. While in analyzing their behavior we find some similar results to the literature (like late bidding of experienced bidders), we will also discuss how bidders may behave differently in this market.

One of the less studied topics in the auction literature is the formation of the first bid and role of bidders' past and present activities in this process. While the strong influence of the first bid on the recovery rate has been discussed in works of Ku et al. (2006) and Simonsohn and Ariely (2008), lesser attention has been paid to the formation of the first bid. Ku et al. (2006) show how lower starting prices lower the entry barriers to attract more bidders and that early bidders tend to bid more in an auction which will make the final price higher. Simonsohn and Ariely (2008) discuss how the first bid (its value and its time) can impact the number of bids in an auction. They call early bidding as a "necessary condition for herding". They also discuss how some sellers lower the starting price of the auctions to trigger this behavior. In Elhadary (2012), the author examines the first bid effect on the dynamics of eBay auctions of Montblanc pens. The study shows the high correlation between the time of the first bid and both the number of bidders and number of bids. In Pilehvar et al. (2013), however, authors analyze the formation of the first bid in more details. For each first bid, they construct a set of latent bidders, of which one becomes the actual first bidder. In doing so, they take into account the bidder's experience and his current level of activity on the platform. They show bidders who have won or bid on a set of just finished auctions are less likely to bid first. In contrast, bidders that are participating in similar open auctions, tend more to place the first bid. In addition, by clustering the bidders they show that more experienced bidders are more likely to be the first bidder while inexperienced bidders wait to gather more information by observing the placed bids. In the next sections, we will show how in our dataset, which is the same used by Pilehvar et al. (2013), how first bid and its arrival time affects the dynamics of an auction.

In the framework of Bayesian inference, Park and Bradlow (2005) model bidding behavior in Internet auctions by deriving a probability model which is based on a latent construct called willingness-to-bid. Using this construct, they develop an integrated framework to model four modules of an Internet auction among the potential bidders (whether, who, when and how much to bid) over the course of the auction duration. In an extension to this work, Bradlow and Park (2007) consider a latent competing set of bidders and use Bayesian data augmentation method to develop a Bayesian record breaking model for bidders' changing valuations of the auction from bid to bid. In developing their model, they use some auction design variables, sellers' ratings and product features to learn about different parameters. Using these variables, the number of latent bidders at each bid is modeled using a truncated Poission distribution where the parameters are modeled through a covariate structure. Having modeled a set of latent bidders at each bid, a truncated normal distribution (truncated at 0) is used to model bid increments. Finally, in order to to find the arrival time of the next bid, bid timings of the pool of latent and already-participated bidders are modeled. Authors model these timings using exponential distribution and then model arrival time of the next bid as the minimum of set of these exponential distributions (which itself follows an exponential distribution). In a similar approach, Li et al. (2009) study the effect of consumers' perception of seller credibility and product quality across different product categories on reducing some of their uncertainties toward an auction. They develop a model under the hierarchical Bayesian framework by adapting the integrated modeling framework of Park and Bradlow (2005). They also differentiate the impact of the bidder's willingness-to-bid at different stages of the auction which is based on the work of Ariely and Simonson (2003) in showing that bidder's valuations and decisions dynamics in entry stage is different from during the auction. In none of these models, however, bidding history of participants have been incorporated in the models and no attention has been paid to specific bids like first or the last bid.

In this essay, we aim to extend the use of Bayesian dynamic model and beta regression models to understand bidders' behavior and to make predictions about their behavior in their future auctions. More specifically, we answer questions like how available information across auctions and bidders' past activities may influence their strategy and how we can use this information to make predictions about future auctions. In so doing, we also observe the change in the significance of these metrics to see whether it changes over the time. The organization of the rest of the essay is as follows. We start by explaining the secondary market auctions platform fully discussing our dataset and its variables of interest. We then introduce the notations and the basic theory behind Bayesian dynamic probit models and explain how we use it to model bidders' participation using this approach. We will explore the core algorithm behind the implementation of our proposed model and will provide the corresponding results. In the second part of the essay, we explain beta regression models and we show how we utilize them to model bid arrival times. We then give details of our methodology in using the developed model to predict first bidders and provide the results. We conclude the essay with potential managerial implications of the models, limitations of our proposed approach and possible extensions.

2.3 The Auctions Platform

In this section, we explain business-to-business secondary market auctions hosted on an online platform. We also explain our dataset and the steps in creating the models core explanatory variables.

2.3.1 Retail Secondary Market Online Auctions

According to National Retail Federation's annual report, total merchandise returns account for almost $284 billion (8.89% of total sales) in lost sales for US retailers in 2014. Traditionally retailers used to sell some of the their excess inventories and their return through manual liquidation sales channels with insufficient capacity and very low recovery rate. But in the last decade online auction marketplaces have replaced the traditional methods and have increased retailers' profits through their reverse supply chain process. While some of the retailers sell their store returns to small group of brokers, some have started their own online marketplaces to liquidate their returns or use websites such as eBay. In addition, there are retailers that rely on wholesalers logistics companies to liquidate their products. These companies receive retailers return and sell them in pallets or truckloads either in fixed-price format or in online auctions. In what follows, we discuss the details of the studied marketplace which is a major logistics company specialized in wholesale returns and liquidation.

As mentioned earlier, this study is conducted on the online auction marketplace which is studied in Pilehvar et al. (2013). In this market, auctioned products are (uninspected) returns, open box, excess and salvage consumer electronics of big electronic retailers in North America and most of the customers (bidders) are themselves re-sellers such as off-price retailers and eBay power sellers. The process starts when retailers send these items to the liquidator's warehouse where they arrive in pallets. The information regarding the pallets are then posted on the liquidator's online platform. However, these postings provide little information regarding the contents of the pallets, mainly the overall retail value of the pallet and the quantity of the items. The retail value of each pallet is estimated based on an overall quality of the items and the current market price. The overall quality of the items somehow depends on the return policy of the main seller of the items and the way it handles its return and excess inventory (at the business-to-customer level). Because of the nature of these products, there is no major return policy defined for the items and they are said sold 'as-is'. This causes a lot of uncertainty about the condition of the items in the box as they might be some damaged items with low retail value or some open-box returned items with much higher retail value. In addition, unlike business-to-customer platforms like eBay, there is no seller feedback available to help reducing some of the uncertainties. This lack of information and the unique characteristics of the secondary B2B auction platform, highlights the role of the first bid in this environment which we will discuss in details later.

In addition to the limited information provided by the auctioneer, observing and participating in similar overlapping auctions can be very informative to help the bidders reducing some of their uncertainties regarding the pallet. The seller (i.e., auctioneer) usually receives multiple pallets from the retailers and auctions many of them at the same time. Bidders can see the auctioned pallets in the online platform along with the limited provided information regarding the contents of the pallets including retailer's name, items condition (i.e., return, excess or salvage) and the warehouse where it is stored. On the other hand, even though they cannot see the history of the bids and bidders' information, they can see the value of the current highest bid in the similar open auctions and the final price of the recently closed similar auctions. These price signals are influential in bidders' decision toward bidding in similar auctions that have not received any bid yet. When buyers are uncertain about their valuations of an item, they are likely to rely on other bidders' behavior (Bajari and Hortacsu (2003), Li et al. (2009)).

Additionally, bidders' own experience is an important factor in moderating their bidding behavior. The experience is even more valuable in the secondary market platform auction because not much is known about the value and the contents of pallets. For example, a bidder with a purchase history from the same seller in the past might have learned about the average quality and value of pallets sold by that seller which might help him/her in better valuing a currently posted pallet from the same seller. At the same time, experience influences bidders' bidding strategy. As we mentioned in the introduction, multiple works shows that experienced bidders are aware of the winner's curse problem and they avoid early bidding which might attract more bidders toward the auction and possibly increasing the final price. They may also shade their bids (bidding lower than their actual valuation of the auction) to keep the final price lower. In the current dataset, however, even though we see a similar relation between bidders' experience and the time they place their first bid, the relation is not very strong. Figure 2.1 shows bidders' experience versus the time of their first bids (Top chart: Experience defined as the number of participations in the last three months/Bottom chart: Experience defined as the number of wins in the last three months).

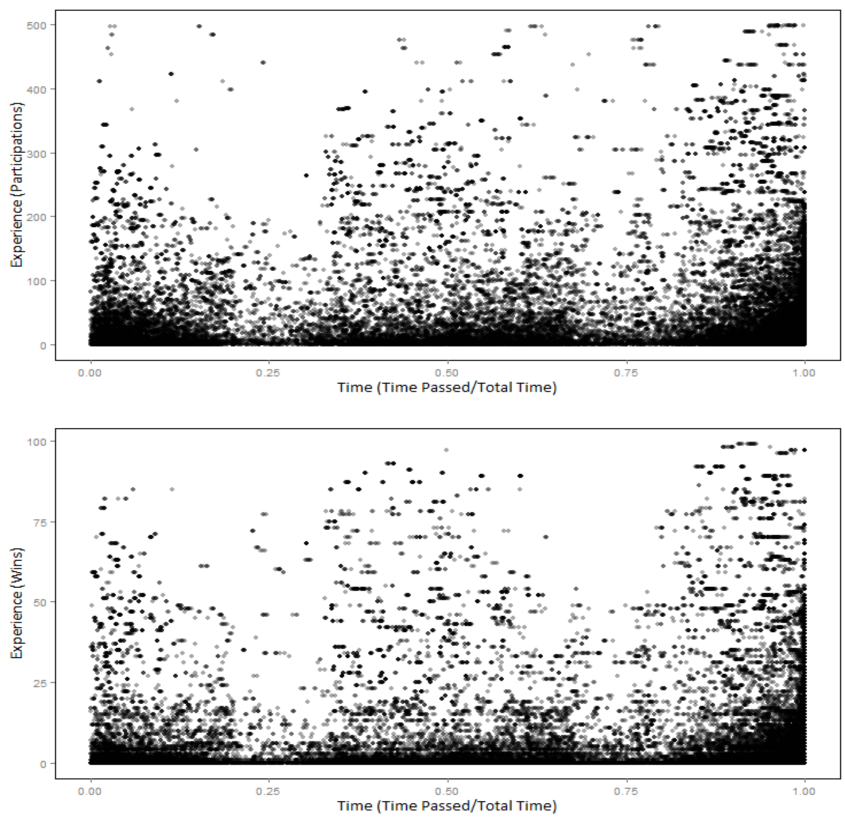

Figure 2.1: Bidders' Experience and Time of Their First Bid - (Top: Number of Participations - Bottom: Number of Wins)

This data is collected from an online secondary electronics market auction platform over the course of five years from 2003-2008. The observations are at the bid level and the dataset has bid information of 2000 unique bidders who have participated in over 11000 auctions. The average of number of bidders per auction is 5.18 and average number of bids 8.02 (Figure 2.2). As it is seen in Figure 2.3, the busiest season of this market is Winter (January, February and March) and this is due to the returns of holiday shopping season which starts in late November and lasts till end of December. The rich panel dataset enables us to track bidders' activity over the time. In other words, we can measure bidders' overall experience which can be defined in terms of number of auctions that he/she has participated, the number of auctions he/she has won and/or the number of losses. Also, we can see bidders' cross-bidding activities in similar open or recently closed auctions at the time of the bid and we can study the effect of these activities in his upcoming bidding decisions.

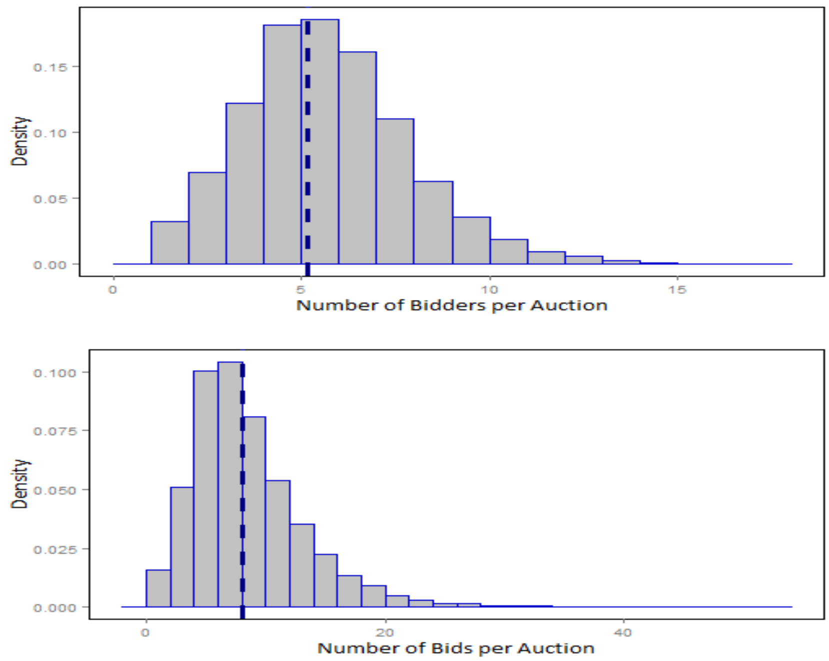

Figure 2.2: Average Number of Bidders (top) and Average Number of Bids (bottom) per Auction

On other hand, getting to know the dynamics of secondary market auctions has important managerial implications. In general, auctioneer is selling many of these items under a consignment agreement with the retailer, meaning that it will pay the retailer only if the items are sold. So the liquidator needs to sell the items as fast as possible to avoid inventory problems and to be able to pay retailers. In addition, secondary market auctions on average have a low recovery rate (0.261 in our dataset) where recovery rate is defined as the final price divided by the pallet's retail value. So its of auctioneer's main interest to minimize the number of unsuccessful auctions and by selling the items as soon as possible with higher final prices.

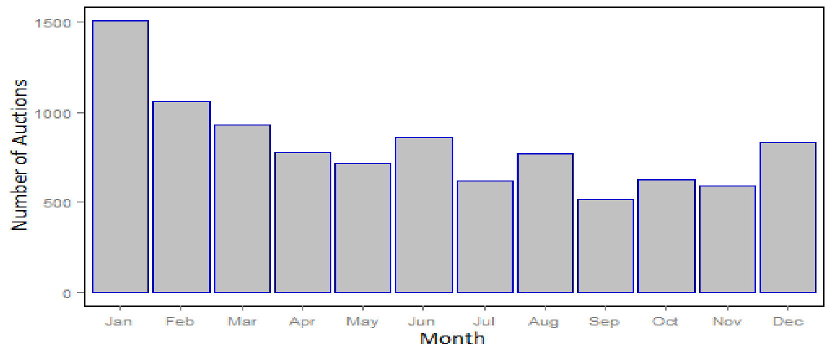

Figure 2.3: Number of Auctions Per Month

2.3.2 Dataset and Variables

In this section, we aim to discuss some aspects of the dataset which relate to the first bid formation and describe how we plan to extract required information from our dataset. In general, we consider two types of variables, some related to the auctions itself and some related to the bidders including their past and current activities.

In considering the auction-specific characteristics, as it was discussed in the previous section, we should note that compared to customer-to-customer auctions platforms such as eBay, buyers in the business-to-business secondary market auctions have access to limited information regarding the posted items. The major pieces of information are the quantity of the items in the pallet and the pallet's declared retail value. So as auction-specific variable, we use quantity and a normalized version of the retail value (per unit price of the pallet defined as the pallet's retail divided by its quantity). We also consider the duration of the auction which for most of the auctions is either two or three days depending on the day they are posted. The other auction-specific variables are item conditions (return or salvage), day and time of the day the auction starts and its retailer name. We should note that these auctions have no reserve price and the starting price is almost the same for all of them.

We can divide the bidder-specific constructs into two sub-categories. The first category is related to bidder's past activities where we measure variables such as number of participation, number of wins/losses, the time between the focal auction and the last time he/she bid and etc. As mentioned earlier, bidder's experience, its definition and its influence on bidder's strategy has been studied in multiple works (Wilcox (2000), Wang and Hu (2009) and Borle et al. (2006)). However, most of the studies in the literature are about business-to-customer auctions like eBay where more information is available to the customers. The fact that in B2B auctions environment little information is known about the items makes bidder's experience an even more important variable. As for the definition, experience can be defined in terms of overall participation, number of wins or number of losses. It can also be defined using a moving-window approach (i.e., considering only a windows of most recent activities). In our study, we define experience as bidders' number of wins or number of losses in the the last three months when we believe is a better representative of bidders level of experience at the time of the auction. The moving-window approach is based on internal reference price literature in marketing and is also suggested by Pilehvar et al. (2013) to account for bidder inactivity over a long period of time where authors use a six-months window to measure experience.

The second group of bidder-specific variables are related to bidder activities around the time of focal auction. These variables, also called cross-bidding activity variables, measure several factors such as the number of similar overlapping auctions which is participating in or number of similar auctions which have just ended and the bidder has either bid or won the auction. In order to define overlapping and similar auctions, similar to work of Chan et al. (2007), we first pool all the auctions that are running at the time the focal auction opens and calculate the mean quantity and retail value of these auctions. Next, we select auctions that are within one standard deviation of the focal auction quantity and retail value. We define these auction as the 'similar auctions' superset. The market price information that the bidder is receiving from these similar auctions that he has outstanding bids in, is influential. At one hand, the price and the dynamics of those auctions can help the bidder having a better idea about a similar auction that has not received a bid yet. Also, the loss or win experience in the just finished similar auctions might influence bidder participation in similar overlapping auctions.

In developing our model, we will the above-mentioned factors and few more variables (which will be discussed later) in understanding and predicting bidders behavior in future auctions.

2.4 Modeling Participation using Bayesian Dynamic Probit Model

Modeling categorical longitudinal data using time-varying coefficients have been studied by authors such as Carlin and Polson (1992) and Frühwirth-Schnatter (1994). While in most of the previous works logit-type models have been considered, in this section, we discuss using the Bayesian dynamic probit modeling paradigm and use of data augmentation approach and Gibbs sampling algorithm to simulate from the posterior distribution of the parameters based on the work of Soyer and Sung (2013). We then show how we plan to use this approach to model bidders' participation in auctions.

2.4.1 Participation in Auctions

Modeling bidder arrivals is important for studying auctions. The outcome of an auction cannot be predicted accurately unless we can build good models to predict the bidders and the frequency, arrival time and amount of their bids. In the auction literature, there are different models developed to study these processes. In Shmueli et al. (2007) authors model bidder arrivals through a Poisson process and they use this model to develop a more complex probabilistic model for bid arrivals through the auction stages according to a nonhomogenous process (i.e. time-varying intensity). They explain the relation and the differences between the two processes and discuss difficulties in modeling bidder arrivals since it is unobserved. On the other hand, some have focused on modeling who will bid rather than the number of bidders or the number of bids. As discussed earlier, Bradlow and Park (2007) have developed a Bayesian record breaking model for bidders' changing valuations of auction from bid to bid. Using this model, they construct a latent competing set of bidders at each bid and identify the bidder with the highest valuation to be the next bidder. In doing so they use a covariate structure model based on some auction features. While in most of the these studies simulated data or eBay datasets with no bidding history of the bidders have been used we take advantage of the rich dataset in hand to build a different model for bidders' participation in auctions.

As it was mentioned modeling bidder arrivals or the auction participants is not straightforward. First is that not all bidders' behavior is recorded by auctions website. For example, we do now know whether a bidder has been watching an auction or for how long he/she has been watching it before placing a bid. In fact, it is only the bid arrival which is recorded. In addition, bidder's decision to participate in an auction is an endogenous decision. Pilehvar et al. (2013) discuss how entry decision is derived from an underlying decision process and how the current marketplace structure (i.e. having multiple similar auctions running at the same time which may affect bidders' entry decisions) adds to the complexity of modeling it. They use a discrete choice model to study bidders' entry decision where the participant bidder will be identified among a set of latent bidders. They then follow Heckman sample selection procedure to model the first bidder on an auction. In the rest of this essay, we explain our approach to model entry into auctions and to identify the first bidder. We first develop a dynamic probit model to study bidders' participation in an auction. In doing so we try to track multiple bidders over a set of their auctions to see whether their decision process changes over the time or should be assumed constant. The fact that we are putting multiple bidders' data together enables us to build a stronger model that learns from more bidders' decision process. In addition, it is more robust and has better predicting performance since it will be tested against a set of heterogeneous bidders. Conditioning on the entry decision, we develop the second part of the model which identifies the first bidder among a set of participants. In the following section, we start by describing the Bayesian dynamic probit model and how it can be used to model bidders' entry.

2.4.2 Bayesian Dynamic Probit Model

In this section we review our proposed model for participation and its two variants which will be tested against each other. For simplicity we explain the model for the case of a single bidder and we later show how it will be updated to the multiple bidders' scenario. In its general form, participation of a bidder in an auction t can be modeled by the following static (i.e. no time-varying regression parameters) probit structure:

$$Pr\{Y_t = 1 \mid X_t\} = \pi_t \text{ with } \pi_t = \Phi(\beta X_t)$$

where the variable of interest $Y_t, t = 1, ..., T$ is a binary variable that gets value of 1 if the bidder participates in auction t and 0 otherwise and it is assumed to have a Bernoulli distribution $Y_t \mid \pi_t \sim Bern(\pi_t)$. Using a set of K explanatory variables to model participation, X_t is the $K \times 1$ vector of those covariates and β is the $1 \times K$ vector of regression parameters. In this equation Φ is the cumulative normal distribution function (linking function) which maps the values to the range of 0 to 1 and (i.e. generates π_ts). In considering the covariates, we choose both a set of bidder-specific and auction-specific variables where bidder-specific covariates will be a set of bidder's current and past activities.

In an extension to this model, we consider time-varying regression parameters to account for possible changes in the entry decision process (through change in the effect of different covariates). In the new form, β's are replaced by β_t's. Considering the observation equation of general dynamic linear model (DLM) form

$$Y_t = \beta_t X_t + v_t \text{ with } v_t \sim N(0, V_t)$$

the dynamic probit model for binary time series Y_t can then be written in the following form:

$$Pr\{Y_t = 1 \mid X_t\} = \pi_t \text{ with } \pi_t = \Phi(\beta_t X_t)$$

where as mentioned earlier X_t is the $K \times 1$ vector of covariates and β_t is the $1 \times K$ vector of regression parameters. The system equation (also known as state equation), which captures the dynamic nature of the problem, has the following form:

$$\beta_t = G\beta_{t-1} + w_t \text{ with } \omega_t \sim N(0, W_t)$$

where w_t's are uncorrelated multivariate normal error vectors with mean 0 and covariance matrix W_t which is the evolution matrix for β_t, and G is the known transition (transfer) matrix. It is common to assume that G is an identity matrix to imply a steady model. The general static regression model has the similar form, however, the evolution matrix (W_t) is equal to zero, so that β_t's are constant in time (i.e., $\beta_t = \beta$). Dynamic linear models are based on the assumption that the regression is only appropriate at the local level in time and that the regression parameters evolve according to a random walk. The changes in the parameters between times $t-1$ and t are described by the evolution error term ω_t. The deterministic part of the evolution is the transition from β_{t-1} to $G\beta_{t-1}$ through a linear transformation based on the transition matrix G. So overall we have a one-step Markov evolution because given β_{t-1} and the values of W_t and G, β_t is determined independently from data prior to time $t-1$.

In our model, we expect locally constant variation (in a few periods forward or backward) and we are assuming a local linear model for the underlying smooth evolution (i.e., steady model). In that regard, as also explained in West and Harrison (1999), we assume G as an identity matrix to have a steady model.

An important concept in understanding discrete-data types of regression is the use of latent (continuous) variables to express the discrete data. In Soyer and Sung (2013), authors introduce a method to use this latent structure in combination with data augmentation approach (Albert and Chib (1993), Tanner and Wong (1987)) and Gibbs sampling algorithm to simulate from the posterior distribution of β_ts which we will adapt for our calculations. The data augmentation approach of Albert and Chib (1993) in the context of dynamic probit models is through introducing the following latent variable representation of the model:

$$Z_t \sim N(\beta_t X_t, 1) \text{ such that } Y_t = \begin{cases} 1 & \text{if } Z_t > 0 \\ 0 & \text{otherwise} \end{cases}$$

which is the equivalent form of the above-mentioned probit model by using latent variables. In this equations Z_t's are the latent variables. Even though these variables are unobserved, they can be somehow interpreted. In our case, as it will be explained later, they can be seen as a continuous measure of willingness to participate where positive values mean entry into an auction and negative values mean otherwise.

In designing the Gibbs sampler, we need to obtain two full posterior conditional distribution of $p(\beta \mid D, Z^T)$ and $p(Z^T \mid D, \beta)$ where $\beta = (\beta_1, \beta_2, ..., \beta_T)$, $Z^T = (Z_1, Z_2, ..., Z_T)$ and $D = \{Y_t; t = 1, ..., T\}$ is the observed data. The posterior distribution of Z^T conditional on β has a simple form. By definition, the random variables $Z_1, Z_2, ..., Z_T$ are independent. Considering their independence and two possible values of Y_t (0 or 1), the posterior distribution of Z^T conditional on β can be written by the following two truncated normal distributions:

$$(Z_t \mid \beta_t, Y_t = 1) \sim N(\beta_t X_t, 1)\ I(Z_t > 0) \text{ truncated at left by 0}$$

$$(Z_t \mid \beta_t, Y_t = 0) \sim N(\beta_t X_t, 1)\ I(Z_t \leq 0) \text{ truncated at right by 0}$$

where I is the indicator function. In obtaining the posterior distribution of β conditional on Z^T, we can use sequential methods like the one used in West and Harrison (1999). We start by using the Markov structure of the model to re-express $p(\beta \mid Z^T)$ as:

$$p(\beta \mid Z^T) = p(\beta_1, \beta_2, ..., \beta_T \mid Z^T) = p(\beta_T \mid Z^T) p(\beta_{T-1} \mid \beta_T, Z^{T-1}) \ ... \ p(\beta_1 \mid \beta_2, Z^1)$$

where $Z^t = (Z^{t-1}, Z_t)$ for $t = 1, ..., T$ (i.e., data up to the time t). In this expression, $p(\beta_T \mid Z^T)$ can be calculated using the DLM posterior calculation. For some mean m_0 and variances C_0, V_t and W_t, we have:

$$(\beta_t \mid Z^t) \sim N(m_t, C_t)$$

where:

$$m_t = m_{t-1} + A_t e_t \qquad e_t = Z_t - f_t \qquad f_t = X_t' a_t$$

$$R_t = C_{t-1} + W_t \qquad Q_t = X_t' R_t X_t + V_t \qquad A_t = R_t X_t Q_t^{-1}$$

$$C_t = R_t - A_t A_t' Q_t$$

Here R_t^{-1} is the prior precision, C_t^{-1} is the posterior precision and the error variance V_t is assumed to be 1. Then, by sampling from β_T, we can sequentially sample from $\beta_{T-1}, ..., \beta_1$ using the above densities of the form $p(\beta_{t-1} \mid \beta_t, Z^{t-1})$ for $t = T, T-1, ..., 2$.

By using the filtering and the state equations of the DLM we can write:

$$p(\beta_{t-1} \mid \beta_t, Z^{t-1}) \propto p(\beta_t \mid \beta_{t-1}, Z^{t-1}) p(\beta_{t-1} \mid Z^{t-1})$$

where

$$p(\beta_t \mid \beta_{t-1}, Z^{t-1}) \sim N(\beta_{t-1}, W_t) \text{ (from the system equation)}$$

$$p(\beta_{t-1} \mid Z^{t-1}) \sim N(m_{t-1}, C_{t-1}) \text{ (from standard DLM setup)}$$

It then follows from the above that:

$$(\beta_{t-1} \mid \beta_t, Z^{t-1}) \sim N(h_{t-1}, H_{t-1})$$

where

$$h_{t-1} = m_{t-1} + C_{t-1} R_{t-1}^{-1} (\beta_t - m_{t-1}) \qquad H_{t-1} = C_{t-1} - C_{t-1} R_t^{-1} C_{t-1}$$

This recursive updating algorithm, also known as Forward Filtering Backward Sampling, was first proposed by Frühwirth-Schnatter (1994). In our case, we now have a scheme for sampling from the full conditional of $\beta = (\beta_1, \beta_2, ..., \beta_T)$ by sampling β_t from $N(m_t, C_t)$ by 'Froward Filtering' and sample β_t $(t = 1, 2..., T-1)$ from $N(h_{t-1}, H_{t-1})$ by 'Backward Sampling'.

2.4.3 Modeling Participation

In this section, we provide a detailed specification of the model and how we plan to run it over the dataset. This model is a probabilistic approach toward modeling bidders' entry into auctions. Having modeled bidders' entry into auctions, in the next section of the essay we discuss our proposed model for predicting the first bidder of an auction among a set of bidders.

In the first step, we create a set of participated and potential auctions for each bidder. For each bidder, a participated auction is an auction in which he/she has placed a bid ($Y_t = 1$). On the other hand, potential auctions are auction which we assume the bidder has seen and has shown interest in but hasn't entered it. We do so by searching the dataset during running time of that auction to see whether the bidder has placed a bid in a similar auction during that time. For participated auctions (in which the bidder has placed more than one bid), we consider his/her characteristics at the time of the first bid. For potential auctions (during which the bidder has placed multiple observed bids in other comparable auctions), we consider bidder's characteristics at the time of his/her last observed bid since this was the last he/she could potentially enter the current auction. We should note that by bidder's characteristic we are referring to some bidder's time-specific (i.e. time-varying) covariates such as number of open auctions he/she is participating, number of just finished auctions he/she has won which are specific to the time each bid is placed. The full list of model variables will be explained later.

The process is repeated for other bidders to construct a set of their participated and potential auctions. We then randomly select a set of n bidders and choose their T auctions. In other words, for each bidder, we are choosing T of his/her consecutive auctions activity which can either be a participation in an auction or being a potential bidder of an auction. The updated model will have the following form:

$$Pr(Y_{it} = 1 \mid X_{it}) = \phi(\beta_t X_{it})$$

where Y_{it} with $i = 1, ..., n$ and $t = 1, ..., T$ is a binary variable for the $\underline{i}^{\text{th}}$ bidder's participation/potential status in his/her $\underline{t}^{\text{th}}$ auction[1] and X_{it} is a set of auction-specific and bidder-specific covariates. Also, as it is shown we are using a common

[1]We should note that index t refers to a bidder's t^{th} auction and not necessarily one bidder's first auction is the same auction as another bidder's first auction and so on.

regression parameters for all bidders at each time (i.e. β_t with no $\underline{i}$ index) rather than bidder's specific parameter. We now have with $n \times T$ number of records. We then go through the following algorithm:

Step One: Divide data into two time frames (two data sets): main period (from time/auction 1 to time/auction s) and a prediction period (from auction $s+1$ to auction T) where T is the total number of auctions we have considered for each bidder.

Step Two: Run the model on the main period to calculate the parameters up to the time/auction s (i.e., β_t for $t = 1, ..., s$).

Step Three: Calculate the fit measures and make a one-step ahead prediction by estimating the probabilities of each bidder's participation in his/her upcoming auction using the following equations:

$$(\beta_{s+1} \mid D_s) \sim N(m_s, R_{s+1})$$

$$\pi_{i(s+1)} = Pr\{Y_{i(s+1)} = 1 \mid X_{i(s+1)}\} = \Phi(\beta_{s+1} X_{(is+1)})$$

Step Four: Calculate predictive performance measures and store π_{is+1}s as the posterior predictive values for the $\underline{i}^{\text{th}}$ bidder's participation/potential status in his/her $(s+1)^{\text{th}}$ auction.

Step Five: Update the main and prediction period datasets by moving the just-forecasted auctions from the prediction period to the main period. If the prediction dataset is empty, quit, otherwise go to **Step Two**.

For the choice of priors, we choose an inverse Wishart prior (with given scale matrix and degrees of freedom) for W and normal priors for β_t's with some specified values for m_0 and C_0. In order to test the performance of the dynamic model, we also run a static version of the probit model by setting $W_t = 0$ to have $\beta_t = \beta$ for all ts. The model selection approach will be assessed by both fit measures such as deviance and DIC and predictive measures such as predictive likelihood and log Bayes factor. Also, we use typical classification methods such as ROC curves to compare classification performance of static and dynamic models. In addition, the dynamic nature of the model and the time-varying parameters enables us to get more insight by observing possible trends in the value of the parameters over the time.

As mentioned in the model description, we estimate common regression parameters for all auction at each time rather than bidder's specific parameter which helps us learning from the behavior of more bidders. The advantage of the current setting is also in its applicability in being implemented in the online auction platform. In doing so, the predictive model can learn from the activities of all the bidders in the past to predict bidders' behavior in the upcoming auctions, observe the outcome of the predicted auctions and update the model accordingly. The predictive model can also be used along with an online recommendation system to attract more bidders by promoting running or upcoming auctions to specific group of bidder (for example to groups with interest but low probability of participation).

2.4.4 Results - Participation

In running and comparing the results of static and dynamic models of participation, we randomly select 50 ($n = 50$) bidders along with each bidder's 210 ($T = 210$) consecutive auction activities. We choose bidders' first 200 ($s = 200$) auctions as the main dataset and their next 10 auctions as the holdout set. Following the discussed algorithm, each model will be run 10 times where at each iteration we make a one-step ahead prediction, update the datasets and go to the next iteration. As for covariates (X_{it}s), we select two distinct categories. The first category specifies the current auction where we use the following variables: quantity of the items in the auction, retail value of items in auction (value per item), binary indicator to show whether the auction is posted over the weekend, binary indicator to show whether the auction is running in winter (Janauary, February, March) which is the busiest time of this market and finally the number of similar overlapping auctions running at the same time. The second category however explains bidders' characteristics. In this category we consider the followings: number of participation in the past three months, winning experience (ratio of wins in participated auctions of the past three months), number of open auctions in which bidder has placed a bid, number of wins in auctions that have finished after the start of the focal auction but before his/her bid placement, number of days from bidder's last bid (i.e. a measure of inactivity) and finally participation rate (ratio of participation in potential auctions of the past three months). In addition, we also consider the intercept term in the model.

In the dynamic probit model setting, the Wishart prior on W^{-1} has $r = K$ degrees of freedom with the scale matrix $R = diag(1, ..., 1)$ which has $K \times K$ dimension. Note that in our case $K = 13$. For this analysis we use R and WinBUGS on a personal computer with INTEL(R) i7-2600 CPU 3.40Ghz processor and 16GB RAM memory. In the model where we are jointly modeling mean and precision parameters, it takes 41 minutes for each run of the iterative process which includes simulations and calculation of fit and predictive measures. The inferences are made based 5000 posterior samples after burn-in sample of 10000 iterations and thinning by 5.

For model comparisons we use common Bayesian model comparison and fit measures. Deviance Information Criterion (DIC) of Spiegelhalter et al. (2002) is a model comparison criterion that takes into account both the fit and complexity of the model. Having deviance is defined as $D = -2log\mathcal{L}(\Theta)$ with Θ representing unknown parameters of the model, DIC in its general form it is defined as:

$$\text{DIC} = \overline{D} + p_D$$

where $\overline{D}$ is the posterior mean of the deviance and $p_D = \overline{D} - D(\hat{\Theta})$ where $D(\hat{\Theta})$ is a point estimate of the deviance obtained by substituting in the posterior means for Θ. In DIC formulation p_D is a penalty measure for the complexity of the model by its effective number of parameters. Table 2.1 shows the values of these measures for both class of models over 10 iterations. As it is seen dynamic model outperforms static model in all of the iteration by having smaller values for both $\overline{D}$ and DIC.

	Static			**Dynamic**		
Iter	**Dbar**	**pD**	**DIC**	**Dbar**	**pD**	**DIC**
1	10876.1	13.0	10889.0	10372.3	377.3	10749.6
2	10943.9	13.3	10957.2	10429.5	379.0	10808.6
3	11001.0	13.0	11014.0	10473.5	379.2	10852.7
4	11055.5	13.1	11068.6	10515	382.8	10897.8
5	11108.3	13.0	11121.4	10561.6	385.5	10947.2
6	11167.8	12.8	11180.6	10624.2	384.1	11008.4
7	11220.3	13.0	11233.3	10673	387.1	11060.1
8	11269.8	12.8	11282.7	10731.4	389.7	11121.1
9	11323.5	13.3	11336.8	10771	394.2	11165.2
10	11369.6	13.1	11382.7	10802.8	397.6	11200.4

Table 2.1: Static and Dynamic models fit comparison over main dataset in each 10 iterations

Besides the overall fit measures, we also evaluate model predictive performance in terms of bidders' future participations. As it was discussed in the algorithm, our one-step ahead predictions are on a rolling basis since we use observations up to time t to make a prediction for observation at time $t+1$, then the $(t+1)^{\text{th}}$ observation is included for the prediction of observation at $t+1$. In evaluating predictive quality of the models, we follow work of Geweke and Amisano (2010) by obtaining one-step ahead log predictive likelihoods and calculating the cumulative log predictive Bayes factors. Given two competing models A_1 (dynamic) and A_2 (static), the log Bayes factor may be decomposed to as:

$$log\left[\frac{p(Y_T \mid Y_s, A_1)}{p(Y_T \mid Y_s, A_2)}\right] = \sum_{t=s+1}^{T} log\left[\frac{PL_{A_1}(t)}{PL_{A_2}(t)}\right]$$

where $PL_A(t) = p(Y_t \mid Y_{t-1}, A)$ and PL_{A_1}/PL_{A_2} is the predictive Bayes factor in favor of A_1 over A_2 for observation t. This decomposition shows how individual observations contribute to the evidence in favor of one model over another. In our setup, at each time t we are making 50 one-step ahead predictions to account for all 50 bidders' next step prediction. In other words we have:

$$PL_A(t) = p(Y_t \mid Y_{t-1}, A) = \prod_{i=1}^{n} p(y_{it} \mid Y_{t-1}, A)$$

The evaluations are conducted according to Kass and Raftery (1995) scoring rules where log Bayes factor value between 0 and 1 'not worth more than a bare mention', values between 1 and 3 show positive evidence, values between 3 and 5 show strong evidence and values greater than 5 show very strong evidence in favor of dynamic model over static model. Table 2.2 shows the results of the one-step ahead predictions (i.e. classification) in each iteration. The first two columns are log of predictive likelihood of the models PL_As and the third columns is the log predictive Bayes factor in support of dynamic model over static model in that iteration. The cumulative log Bayes factor is $14.03 > 5$ which shows very strong support in favor of dynamic model. We can also check each iterations contribution to this support to see that even though in four iterations data supports the static model, this support is very low compared to the evidence in favor of dynamic model (in other iterations).

	Log Predicitve Likelihood		
Iter.	**Static**	**Dynamic**	**Log Predictive Bayes Factor**
1	-33.830	-30.490	3.340
2	-28.797	-23.614	5.184
3	-27.223	-23.712	3.511
4	-26.483	-23.649	2.834
5	-30.859	-31.575	-0.716
6	-27.177	-26.995	0.182
7	-26.937	-28.545	-1.609
8	-26.617	-26.933	-0.316
9	-23.149	-21.174	1.975
10	-24.062	-24.411	-0.349

Table 2.2: Log Predictive Likelihood and Log Bayes Factor Over 10 Iterations

In Figure 2.4 we can compare the estimated posterior probabilities of 50 bidders' participation for their 210th potential auction (i.e. $s = 209$ which is the last run of the model) versus their actual values. We can see that the both models are slightly underestimating participation probabilities but dynamic model predictions (bottom) are overall performing better than the static ones (top).

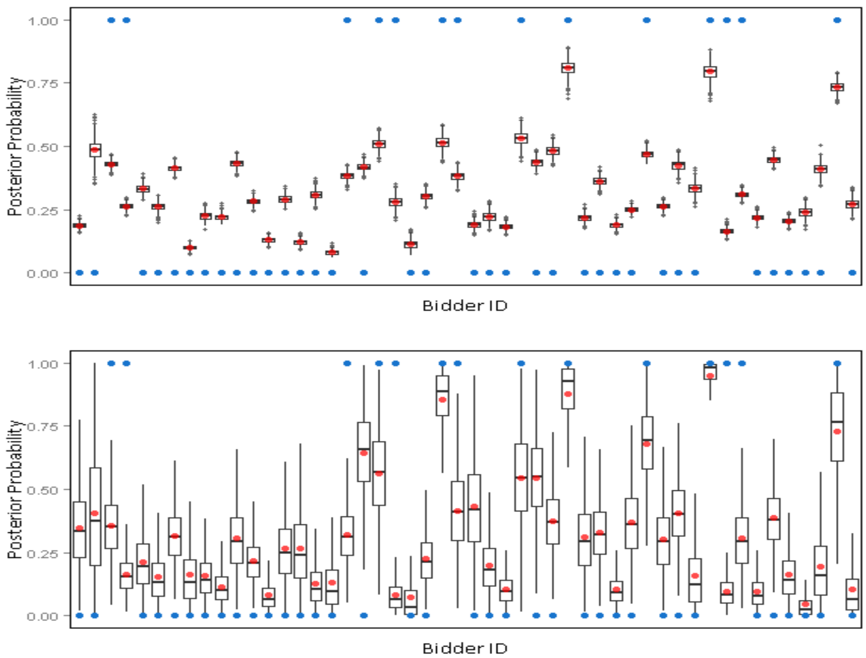

Figure 2.4: Posterior Probability of the 50 Bidders' Participation in their 210th Auction based on Static Model (top) vs. Dynamic Model (bottom) - (blue points are actual values and red points are the posterior means)

In comparing both models, we can also evaluate their predictive classification performances on as shown in Figure 2.5. The top plot compares the average (over iterations) accuracy of both models for different cutoff values. The plot at the bottom is a comparison of their ROC curves by plotting their average true positive rates versus false positive rates for different cutoff values.[2] The overall classification performance

[2]Each boxplot shows the spread of the estimated measure over 10 iterations of the dynamic model at that cutoff point.

curves and individual accuracy and ROC curves of each iteration of the prediction phase are available in the appendix (Figures 6.1, 6.2 and 6.3). In addition, we have also provided a cutoff sensitivity analysis for the performance of the dynamic model over the main dataset. If we are more interested in identifying participants of an auction, the results suggest that we can achieve reasonably high accuracy of around 70%, high true positive rate of around 66% and low false positive rate of around 25% by considering a cutoff value between 0.3 and 0.35. This finding is similar to what was shown in Figure 2.4 which was an overall underestimation of posterior probabilities. However, we may be more interested in identifying bidders with low probability of participation (so that they can be targeted to enter the auction). In that case, we may adjust the cutoff value to achieve a better classification of the bidders of interest. The full table of the cutoffs sensitivity analysis is provided in Table 6.1 in the Appendix.

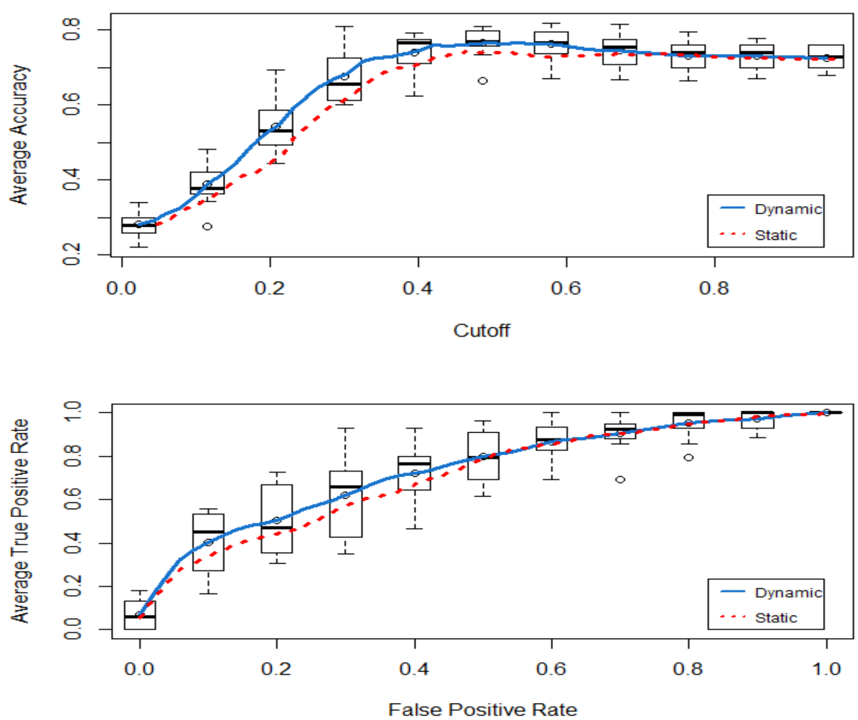

Figure 2.5: Average Accuracy and ROC Performance Curves Over Prediction Iterations

In addition to the performance analysis of the models, we can analyze regression parameters in terms of their significance and their possible changes over the study period. Figure 2.6 plots posterior mean of four of the regression parameters in the dynamic model. We can see the change in the parameters over the course of auctions which is another indication that a dynamic model can capture this behavior better. While some parameters such as 'number of past participations', 'quantity of item in the pallet', 'retail value per item' and 'days since last bid' show downward or upward trends, some other don't clearly show these patterns. The plots suggest that quantity has a negative effect on participation and this effect gets stronger over the time. On the other hand, retail value of auction items has a positive effect on bidders' entry decision and this effect gets stronger over the time. In other words, bidders are more willing to participate in auction with smaller pallet size (less items) and more valuable items and as they participate in more auctions, these features become more important to them. The same positive relation holds between the number of days passed since bidder's last activity and his/her participation probability. In other words, they have a higher chance of participation in a potential auction after a longer inactivity period.

Figure 2.7 shows estimated posterior means of another two parameters: number of open and similar auction is which the bidder has participated (a bidder-specific covariate) and number of similar overlapping auctions with the focal auction (an auction-specific covariate). The first one is an indicator of bidder's level of activity at the time and his/her interest in similar auctions and the second one is an indicator of the platform crowdedness at the time of the focal auction. The results suggest that bidders that are participating in more similar auctions are more willing to participate in the focal auction as well. On the other hand, participation rate decreases when auction platform is busy. This shows the negative effect of multiple auction postings (which is practiced in this marketplace) on the dynamics of the auctions through some key factors such as bid arrivals and their timings, bidders' participation and number of bids. As for the effect of other covariates, bidder's winning rate

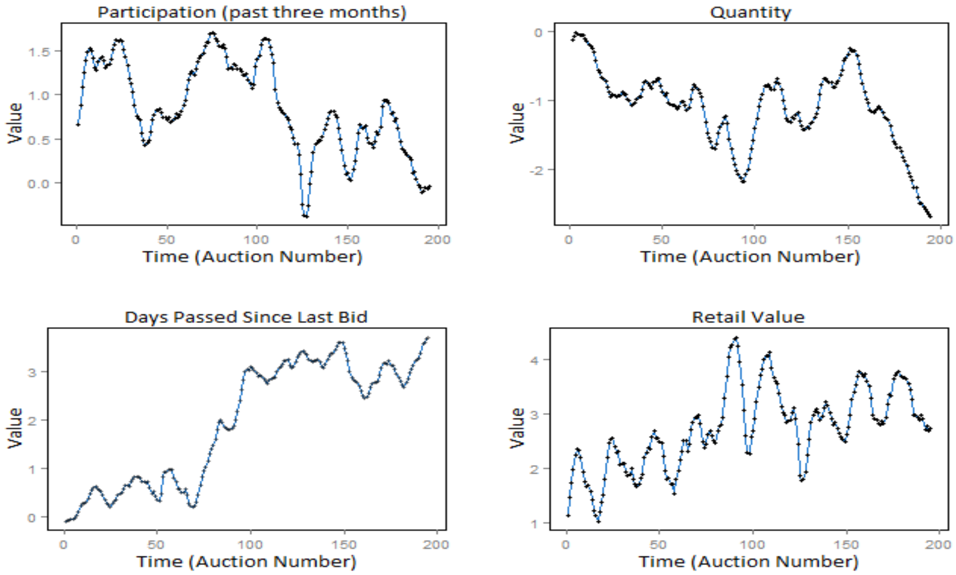

Figure 2.6: Posterior Mean of the Regression Parameters

in the past three months, auctions opening day and opening season do not appear significant while bidder's participation rate in the past three months (ratio of participation in potential auctions) has mostly a positive effect on bidder's entry decision.

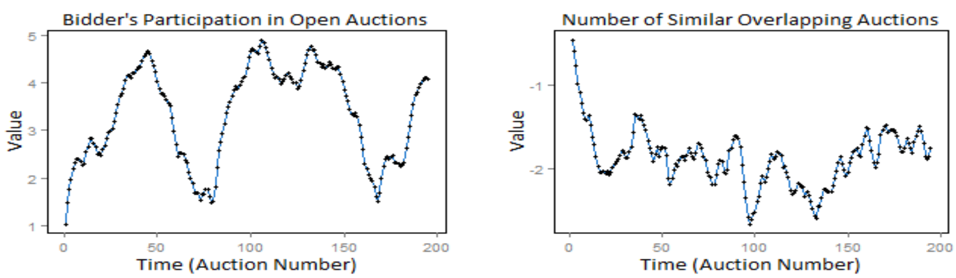

Figure 2.7: Posterior Mean of the Regression Parameters

In summary, in this section we discussed our development of a Bayesian dynamic probit model to study bidders' entry into auctions and the underlying decision process. The results, even though conducted over a period of 200 consecutive auctions, show how different factors play roles in this decision and how their significance may change over the course of auction (i.e. over time). In the next section, we move to the next step in the decision process. There we discuss details of our proposed model for predicting the first bidder among a given set of bidders. While throughout the upcoming section participation in the focal auction is assumed for the set of bidders, the already-developed model for participation can be used to jointly model a bidder decision in participating in an auction and placing the first bid of that auction.

2.5 Identifying the First Bidders using Bayesian Beta Regression Model

The importance of the first bid was fully discussed earlier in this essay. We explained how it's arrival time and it's value can change the final outcome of an auction. In Pilehvar et al. (2013) authors have studied the effect of this bid on the final outcome of the auction through its value in the studied marketplace. They demonstrate how higher first bid values lead to higher recovery rates through multiple models and numerical examples. In addition, they show that on average more experienced bidders place higher bids when they are first bidders. Given the major role that the first bid plays and the fact that it is influenced by bidders' heterogeneity, we propose to develop a model to predict first bidder of an auction. In doing so, for a given auction, we intend to estimate each (potential) bidder's time to their first bid and identify the first bidder by comparing bidders' time of first bid distribution.

As it was explained earlier, all the auctions in this marketplace have fixed duration of either two or three days which is set prior to the posting of the auction. In order to study the time that bids we need to define a time unit for bid placements. In our development, rather than using actual minutes/hours passed from the start of the auction, we define time as a ratio of the time passed over the total time of the auction [i.e, $\frac{TimePassed}{TotalTime}$] where "time passed" is the gap between auction start time and actual bid time and "total time" is the duration of the auction. Using this definition, bid-time has a range from 0 to 1 and as an example, in a two-day auction, a bid-time of 0.25 means the bid was placed $0.25 * 48 = 12$ hours after start of the auction. In modeling proportions, i.e. continuous data restricted to the interval of $(0, 1)$, beta distribution is a natural choice due to its flexibility and different density shapes it can take. This discussed transformation enables us to model the bid timings using beta distribution. In this section, we discuss Bayesian Beta regression models and how they can be used in modeling bid timings....

2.5.1 Bayesian Beta Regression Model

The two-parameter beta distribution is a family of continuous probability distributions which is defined on the range of $(0, 1)$ in the following form:

$$p(y \mid a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} y^{a-1}(1-y)^{b-1}, 0 < y < 1$$

where $a > 0$, $b > 0$ and $\Gamma(.)$ is the gamma function. The mean and the variance of y are, respectively,

$$E(y) = \frac{a}{a+b}$$

and

$$var(y) = \frac{ab}{(a+b)^2(a+b+1)}$$

Using beta distribution and its properties, a beta regression model can be developed in situations where the response variable can be modeled as a function of explanatory variables. However, since in regression analysis, it is more meaningful to model mean of the response and its precision, we use a re-parameterized form of the beta distribution that has mean and precision as its unknown parameters. In so doing, we let:

$$\mu = \frac{a}{a+b} \qquad and \qquad \phi = a + b$$

Using the new parameters, it follows that

$$a = \mu\phi \qquad and \qquad b = (1-\mu)\phi$$

$$E(y) = \mu \qquad and \qquad var(y) = \frac{\mu(1-\mu)}{1+\phi}$$

In other words, μ ($0 < \mu < 1$) can be interpreted as the mean and ϕ ($\phi > 0$) can be interpreted as the precision parameter, meaning that for fixed value of μ, the larger the value of ϕ, the smaller the variance of y. However, as it is shown, the variance is a function of both mean and precision parameter. So it should be noted that even though ϕ is the precision parameter and influences dispersion, it is not the only factor that controls dispersion.

By using the new parameters, beta distribution can be written in the following form:

$$p(y \mid \mu, \phi) = \frac{\Gamma(\phi)}{\Gamma(\mu\phi)\Gamma((1-\mu)\phi)} y^{\mu\phi-1}(1-y)^{(1-\mu)\phi-1}, 0 < y < 1$$

where $0 < \mu < 1$ and $\phi > 0$. Depending on the values of these two parameters, beta distribution can take many forms such as symmetric (for $\mu = 0.5$)) or left/right skewed and we will use this flexibility to model different bid-timing patterns across bidders and auctions.

As first proposed by Ferrari and Cribari-Neto (2004), if the response variable is related to some other variables, parameters of its distribution can be modeled using a regression structure. In that regard, the beta regression model can be developed in two forms: modeling μ as a function of explanatory variables and assuming a constant ϕ or modeling both μ and ϕ as functions of a vector of variables. In this study, we consider both of the classes and we compare their performances. Bayesian analysis of beta regression models was first developed by Branscum et al. (2007) which we will refer to for this study.

Let $y_1, y_2, ..., y_n$ be n independent random variables such that y_i follows a beta distribution with mean μ_i and unknown precision ϕ, i.e. $y_i \sim beta(\mu_i\phi, (1-\mu_i)\phi)$. The first form of the beta regression model can be obtained by modeling μ_i as

$$h(\mu_i) = \beta X_i$$

where β is a vector of unknown regression parameters and X_i is a vector on known covariates associated with the $\underline{i}^{\text{th}}$ observation and $h(.)$ is a link function that maps $(0, 1)$ into $\mathbb{R}$. The most commonly used link functions are *probit* with $h(\mu_i) = \Phi^{-1}(\mu_i)$ where $\Phi(.)$ is the cumulative distribution function (CDF) of the standard normal distribution and *logit* which is the inverse of the sigmoidal logistic function in the sense that $h(\mu_i) = \frac{\mu_i}{(1-\mu_i)}$. As a robustness check in development of our model, we will test both of the link functions and we compare their performance against each other.

Using the independence assumption of the observations, the likelihood function will have the following form:

$$\mathcal{L}(\beta, \phi) = \prod_{i=1}^{n} \frac{\Gamma(\phi)}{\Gamma(\mu\phi)\Gamma((1-\mu)\phi)} y_i^{\mu\phi-1}(1-y_i)^{(1-\mu)\phi-1}$$

We can use Gibbs sampling to generate MCMC samples from poster distribution of $p(\beta, \phi \mid Y)$ by repeatedly sampling from full conditional distribution of $p(\beta \mid \phi, Y) \propto L(\beta, \phi)p(\beta)$ and $p(\phi \mid \beta, Y) \propto L(\beta, \phi)p(\phi)$. In this development, we use normal priors for β's and gamma prior for ϕ.

As it was mentioned, the precision parameter ϕ may be assumed constant Ferrari and Cribari-Neto (2004) or may be modeled using a regression structure (Smithson and Verkuilen (2006), Simas et al. (2010), Figueroa-Zúñiga et al. (2013)). In addition to the joint modeling of the mean and the precision parameter, Cepeda-Cuervo (2015) have also considered modeling mean and variance of the distribution using a regression structure. However, in the development of our second model, we use a regression structure to model the mean and the precision of the distributions. In modeling ϕ_i's, the log link function is a natural choice as it makes sure the precision stays strictly positive. In other words, precision will be modeled as:

$$log(\phi_i) = \gamma F_i$$

where γ is a vector of unknown regression parameters and F_i is a vector of known covariates associated with the the $\underline{i}^{\text{th}}$ observation. It should be noted that that there is no restriction in terms of covariate structures X_i and F_i and they can consist of different covariates. In the next section, we will explain how this model will be used to model bidder's time of the first bid and subsequently predicting the first bidder of an auction.

2.5.2 Modeling Bidders' Time of The First Bid

Having discussed the details of beta regression model, we can now explain study bid timings using this model. As it was explained, a re-scaled form of bid timings is the key in helping us modeling them using beta regression structures where we define time of the bids as a ratio of $[\frac{timepassed}{totaltime}]$, where "time passed" is the gap between auction start time and actual bid time and "total time" is the duration of the auction. We use this definition for bidders' time of the first bid and we develop a model to estimate it for each bidder at a given auction. We then use this estimations to predict the first bidder in a given auction.

Let y_{ij_i} be the time to the first bid that bidder $\underline{i}$ places in his/her $\underline{j}^{\text{th}}$ auction. Assuming a beta regression model with constant precision, y_{ij_i} can be modeled as

$$y_{ij_i} \sim beta(a_{ij_i}, b_{ij_i})$$

where

$$a_{ij_i} = \phi * \mu_{ij_i}$$

$$b_{ij_i} = \phi * (1 - \mu_{ij_i})$$

and where

$$h(\mu_{ij_i}) = \beta X_{ij_i}$$

In the above equations $i = 1, ..., n$ is the bidder index with $\underline{n}$ being the total number of bidders, $j_i = 1, ..., N_i$ is the auction index with $\underline{N_i}$ being the total number of auctions that bidders $\underline{i}$ has participated, $\beta = (\beta_1, ..., \beta_k)$ is a vector of $\underline{k}$ regression parameters and $X_{ij_i} = (X_{1,ij_i}, ..., X_{k,ij_i})$ is a vector of $\underline{k}$ explanatory variables corresponding to bidder $\underline{i}$ in his/her $\underline{j}^{\text{th}}$ auction. In the second model, where ϕ is modeled using a regression structure, we have

$$y_{ij_i} \sim beta(a_{ij_i}, b_{ij_i})$$

where

$$a_{ij_i} = \phi_{ij_i} * \mu_{ij_i}$$

$$b_{ij_i} = \phi_{ij_i} * (1 - \mu_{ij_i})$$

where μ_{ij_i} is defined the same as the first model and

$$log(\phi_{ij_i}) = \gamma F_{ij_i}$$

and where $\gamma = (\gamma_1, ..., \gamma_k)$ is a vector of $\underline{k}$ regression parameters and similar to X_{ij_i}, $F_{ij_i} = (F_{1,ij_i}, ..., F_{k,ij_i})$ is a vector of $\underline{k}$ explanatory variables having information about bidder $\underline{i}$ in his/her $\underline{j}^{\text{th}}$ auction. In order to learn from all bidders' bidding behavior and to take advantage of information sharing across their bidding activities, we use common regression parameters β and γ to model unknown parameters of the distributions. Also, in the development of the models, covariates vectors X_{ij_i} and F_{ij_i} are assumed identical. In other words, we use the same explanatory variables in modeling mean and precision parameters. The covariates vectors consist of two groups of variables: one group provides information about the auction and the second group explains bidders' heterogeneity. The bidder-specific variables can be divided into two categories: variables related to bidders' bidding history (in the past three months) and variables explaining his activities around the time of the focal auction. While some of the variables are common with participation model which was developed in first part of this essay, some are specific to the current model as they focus on bidder's past first-bidding activities. In selecting the final set of covariates, we only include the instrumental variables to keep the model as parsimonious as possible. In the final setup we use the following set of explanatory variables:

Auction-Specific Covariates

Quantity: Number of items in the auctioned pallet(s)

Value: Average monetary value of each item calculated by dividing the pallet estimated retail value divided by its quantity

Weekend: A binary variable showing whether the auction is posted on Friday/Saturday

Winter: A binary variable showing whether the auction is posted on January/February/March

Simultaneous Auctions: Number of similar auctions that are posted at the same time with the focal auction

Bidder-Specific Covariates

Average TOFB: Average time of the first bid placement in the participated auctions in the last three months

First Bidder Rate: Proportion of bidder's participated auctions in the last three months in which the bidder has placed the first bid

Winning Rate: Proportion of bidder's participated auctions in the last three months in which the bidder has won the auction

Just Finished Auctions: Number of wins in similar auctions that ended after the start of the auction and before placement of the first bid

Due to the difference in the ranges and the scales of the variables, we normalize them to the range of 0 to 1. This step makes the interpretation and comparison of the the estimated parameters easier.

2.5.3 Predicting the First Bidder

The next step in our model development is to predict the first bidder in a given auction. In other words, the objective is to identify (within a given set of bidders for an auction) the bidder that places his/her bid before other bidders. In so doing, we use the developed beta regression model to estimate posterior distribution of time of the first bid of the given set of bidders. We then rank these distributions to find the bidder with the minimum bid arrival time which will be the predicted first bidder.

Let's assume we have a set of $\underline{n}$ potential bidders for a focal auction noting that in the current development, the set of potential bidders is the set of actual participants of the auction. In a different setup, this set can be identified by jointly using our previously-developed participation model and the current model (i.e. joint probability of participation and placing the first bid). However, in the current setup bidders' participation is assumed. In other words, we estimate each bidder's time to the first bid given that he/she participates in the auction.

Given the set of bidders for the focal auction and posterior distribution of the parameters β and ϕ (or β and γ if we are learning about the precision parameter using covariates) we obtain y_{ij_i}; the posterior distribution of time of the first bid of bidder $\underline{i}$ in the auction of interest which is his/her $\underline{j_i}^{\text{th}}$ auction. Having obtained all $\underline{\text{n}}$ potential bidders' distributions, we can calculate the probability of bidder $\underline{i}$ having the minimum time of the first bid as following:

$$P(y_{ij_i} < y_{kj_k}, \text{for all } k \neq i \mid Data)$$

In other words, this is the probability that bidder $\underline{\text{i}}$ places his/her first bid before everybody else, i.e. probability of being the first bidder. Also,

$$\sum_{i=1}^{n} P(y_{ij_i} < y_{kj_k}, \text{for all } k \neq i \mid Data) = 1$$

which means that the sum of these probabilities over all bidders of the auction is 1. In practice, the probability of being the first bidder for a specific bidder is calculated by going over each run of MCMC simulations and finding the number of times that the bidder has a minimum time among the set of bidders. In the next section, we review final results of the beta regression model. In so doing, we compare our two developed models both in terms of fit and predictive performances. In addition, we run a separate model using the full dataset to study the effect of each variable on this decision process.

2.5.4 Results - First Bidder

In this section, we explain data preparation, model development and results of running the beta regression model over the current dataset to study bidders' time of the first bid and identifying the first bidders of auctions.

In order to make the most of the dataset and make different runs of the model comparable, we take the following data preparation steps. Before sampling starts, we remove all auctions that have only received a single bid (i.e., they have only one participant). By removing these auctions we avoid a potential fake increase in the predictive performance of the model since in these cases the probability of the bidder being the first bidder is always 1. At each run of both models, the process starts then with sampling 550 auctions from the set of all remaining auctions noting that we only consider participant's first bids in these auction. The data is then re-arranged to have (for each bidder) the set of auctions that he/she has participated along with the time to the bidder's first bid in those auctions. We then split this sample into two sub samples of 500 and 50 auction. We use the larger sample to fit the model and test model's predictive performance on the smaller sample (also called holdout sample). We then separately run the two developed classes of models (model with constant precision and model with joint mean and precision) on the main sample to obtain the posterior distributions, assess the fit measures. The summary of number of bidders of each dataset is presented at Table 6.2 of the Appendix. Next, we test models against the out of sample set and calculate the predictive measures. We repeat this sampling and testing process 10 times. We should note that at each run, both models are tested against the same datasets so that we can compare their fit measures against each other.

In our analysis, we use proper but diffused priors for all parameters in the model. More specifically, for the model with constant precision, we specify gamma priors on ϕ with both shape and scale parameters of 0.01 and 0.01 and all covariate coefficients

β and γ in both models, are assumed to have independent normal priors with mean 0 and precision 0.01. For modeling mean parameters, both probit and logit link functions were tested. Given that their performances were very similar and that the we used probit in the first model of the essay, we choose the probit model in this model as well to stay consistent throughout the essay. For this analysis we use WinBUGS and R on a personal computer with INTEL(R) i7-2600 CPU 3.40Ghz processor and 16GB RAM memory. In the model with common precision it take 7 minutes for each run of the iterative process which includes simulations and calculation of fit and predictive measures and in the second model it each run takes 12 minutes to finish. All the results are based on running a Gibbs sampler with a burn-in sample of 10000 iterations and collecting 10000 posterior samples after thinning by 5. There were no convergence issues during the run of the models. In addition to the iterative sampling and processing of the models, two separate models using the full auction sets are conducted and their convergence diagnostics plots and tables are shown in the Appendix. The fact that the p-values that are associated with the Geweke diagnostic are large and Raftery and Lewis dependence factors are all smaller than 5 suggests that there are no convergence issues.

Table 2.3 compares fit measure of Model 1 (i.e. model with regression structure for mean parameters and constant precision) with fit measures of Model 2 (i.e. model with regression structure for both mean and precision parameters) after they are tested against 10 different datasets. Models are compared using Bayesian fit measures including $\overline{D}$ which is the posterior mean of the deviance wit deviance being defined as $D = -2log\mathcal{L}(\Theta)$, p_D which is a penalty measure for the complexity of the model by its effective number of parameters and Deviance Information Criterion (DIC) of Spiegelhalter et al. (2002) which is a model comparison criterion that takes into account both the fit and complexity of the model. In comparing $\overline{D}$ and DIC of two competing models, the model with smaller values is preferred. In addition to these measures we also compare them in terms of their accuracy where it is defined as the ratio of the auctions in the sample in which the first bidder was identified correctly by the model. As the results suggest, Model 2 is performing better in terms of fit measures over all 10 samples where it has considerably smaller values of $\overline{D}$ and DIC and higher accuracy in every iteration. In addition, p_D has higher values in Model 2 due to extra regression parameters used in modeling the precision parameters. The average accuracy for Model 1 over all runs is 0.36 and the average accuracy for Model 2 is 0.38. Given that the average number of bidders in these auctions is 5.4, the results suggest that both models are doing good in predicting the first bidder with Model 2 performing better.

	Model 1				**Model 2**			
Run.	**Dbar**	**pD**	**DIC**	**Accuracy**	**Dbar**	**pD**	**DIC**	**Accuracy**
1	-3783.11	9.96	-3773.14	0.361	-3876.15	17.84	-3858.31	0.372
2	-3720.05	9.92	-3710.13	0.369	-3842.5	17.86	-3824.64	0.380
3	-3599.04	10.03	-3589.01	0.368	-3748.29	17.77	-3730.51	0.382
4	-3438.22	10.04	-3428.17	0.370	-3577.3	17.88	-3559.42	0.385
5	-3673.28	10.08	-3663.2	0.372	-3799.12	17.76	-3781.36	0.387
6	-3739.25	10.04	-3729.21	0.372	-3860.23	17.86	-3842.37	0.387
7	-3917.33	10.02	-3907.31	0.356	-4023.83	17.82	-4006.01	0.370
8	-3738.02	10.00	-3728.02	0.383	-3887.92	17.79	-3870.13	0.399
9	-3693.43	9.98	-3683.45	0.364	-3803.5	17.81	-3785.68	0.374
10	-3747.83	9.97	-3737.85	0.361	-3847.68	17.87	-3829.8	0.374

Table 2.3: Model Comparisons - Fit Measures over Main Data Sets

Next two tables present comparison results of the models in terms of their performance over the holdout sets. Model 1 and Model 2 are compared in terms of their predictive log likelihood and accuracy as shown in Table 2.4 and log Bayes factor in favor of Model 2 as shown in Table 2.5. Consistent with fit measure results, Model 2 outperforms Model 1 with having higher predictive log likelihood values and higher accuracy. Using the likelihood values, log predictive Bayes factor values in favor of Model 2 are calculated and shown in Table 2.5 where all of them show evidence in support of Model 2. In terms of accuracy results are comparable to the results of Table 2.3 where higher accuracy is achieved by modeling both mean and precision parameters using explanatory variables. The average number of bidders in holdout datasets over the iterations is 5.5.

	Model 1		**Model 2**	
Run.	**Pred Log Likelihood**	**Accuracy**	**Pred Log Likelihood**	**Accuracy**
1	202.39	0.358	211.59	0.366
2	185.34	0.386	191.62	0.401
3	180.36	0.423	181.36	0.441
4	201.32	0.359	208.38	0.377
5	218.88	0.372	226.89	0.389
6	227.39	0.396	235.13	0.404
7	135.54	0.380	140.61	0.393
8	151.36	0.387	157.22	0.409
9	200.28	0.343	205.54	0.353
10	185.62	0.318	188.74	0.326

Table 2.4: Model Comparisons - Predictive Measures over Holdout Data Sets

Run.	**Log Bayes Factor**
1	9.20
2	6.28
3	1.00
4	7.06
5	8.01
6	7.74
7	5.07
8	5.86
9	5.26
10	3.12

Table 2.5: Log Predictive Bayes Factor in Favor of Model 2

In order to illustrate how the model works in terms of predicting the first bidder, we can take a closer look at a given auction and posterior distribution of time of the first bid of its two bidders. Figure 2.8 shows posterior distributions of bidders' time of the first bid. The plot on the right is generated by plugging posterior mean of estimated shape parameters. The posterior mean of shape parameters for Bidder 1 (shown in red) are $a = 1.19$ and $b = 0.51$ (i.e. $\mu = 0.69$ and $\phi = 1.71$) and for Bidder 2 are $a = 3.67$ and $b = 0.64$ (i.e. $\mu = 0.85$ and $\phi = 4.32$). As the results suggest and it is shown in the plots, Bidder 1 is more likely to place his/her first bid earlier than Bidder 2 ($\mu_1 = 0.69 < \mu_1 = 0.85$). Using the model, the probability of Bidder 1 placing his/her first bid before Before 2 is estimated to be 0.65. Looking at the dataset, the actual bid time of Bidder 1 is 0.88 and actual bid time of Bidder 2 is 0.96 which shows that Bidder 1 was in fact the first bidder.

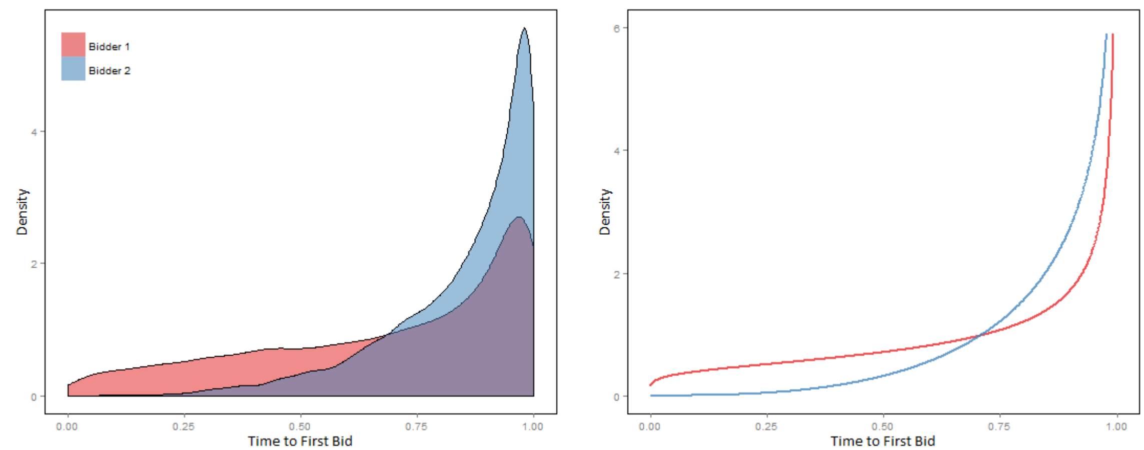

Figure 2.8: Posterior distribution of TOFB of the two participated bidders of a given auction and posterior predictive distribution of their TOFB (right)

In addition to multiple samplings for model comparisons, we run both models over the full set of auctions to fully capture the effect of different variables on this process. Table 2.6 shows posterior summaries of regression parameters of Model 2. Posterior summaries of Model 1 are presented in Table 6.3 of the Appendix. The result suggest that bidder's average time of first bid placements in his/her last three months participations and the number of wins in just finished auctions are two most important factor that positively affect first bid arrival time (i.e. delay it). The average time over bidding history suggest a bidding behavior that some bidders practice in their participated auctions. In other words, some tend to start late in auctions. This is consistent with prior findings and our discussions regarding bid placement patterns of different group (i.e. experience vs. inexperienced and more). On the other hand, bidder that have just won in some auctions show less interest in starting another auction right away. Another bidding behavior which is crucial is the extent each bidder has placed the first bid in his prior particiapted auctions. The more a bidder has placed the first bid, the earlier he tends to place his/her first bid in the focal auction. This is another bidding behavior that we are capturing in this marketplace where some bidders and it shows the effectiveness of studying bidders over the time. In addition, number of wins and number of participations in other open auctions do not seem to have significant effect of on the bid arrival time.

Parameter	Mean	StDev.	95% CCI
$\beta_{Ave.TOFB}$	1.035	0.016	(1.003,1.067)
$\beta_{FB.Rate}$	-0.623	0.029	(-0.681,-0.565)
β_{NJFA}	1.147	0.177	(0.800,1.498)
β_{NOA}	-0.158	0.067	(-0.291,0.025)
β_{Q}	-0.032	0.070	(-0.106,0.168)
$\beta_{Weekend}$	0.115	0.038	(-0.040,0.191)
β_{Win3}	0.053	0.039	(-0.024,0.130)
β_{Winter}	-0.062	0.012	(-0.087,-0.036)
$\beta_{RetailValue}$	0.205	0.056	(0.153,0.342)
$\gamma_{Ave.TOFB}$	0.392	0.026	(0.342,0.445)
$\gamma_{FB.Rate}$	-0.084	0.041	(-0.168,-0.006)
γ_{NJFA}	1.261	0.341	(0.571,1.921)
γ_{NOA}	0.050	0.115	(-0.173,0.273)
γ_{Q}	-0.790	0.105	(-0.995,-0.585)
$\gamma_{Weekend}$	-0.046	0.019	(-0.084,-0.007)
γ_{Win3}	-0.051	0.070	(-0.188,0.089)
γ_{Winter}	0.011	0.019	(-0.025,0.051)
$\gamma_{RetailValue}$	-0.604	0.084	(-0.774,-0.439)

Table 2.6: Posterior Summaries of Model 2

As for auction features, average retail value of items in the pallet seems to be the most important factor. Auctions with higher value items discourage bidders from entering the auctions earlier. This may be due to the fact that bidders are trying to avoid bidding wars in these auctions which may increase the final price and they may suffer from winners' curse. In addition, first bid arrival times are longer for the auctions which are posted over the weekend which may be due to increased inactivity rate during that period. On the other hand, bidders tend to enter auctions earlier during the busy season of the market (i.e. winter). Quantity of the items however doesn't seem to have a significant effect on this process. Table 2.6 show posterior summaries of the different γ parameters which are coefficients of the regression model that estimates the precision parameters. Similar to β parameters, average time of previous first bids and number of just finished auctions have the largest effect (i.e. they have positive values which leads to higher precision and lower variance in the distribution of time of the bid). On the other hand, quantity of items in the auction and their retail value adds more uncertainty to bidders' time of the first bid.

In what follows, we review the results of the models and discuss their managerial implication and how auction managers can benefit from this methodology and our findings.

2.6 Managerial Implications

There are a few managerial implications to take away from this study. First is that Bayesian dynamic probit and beta regression models can help the retail secondary marketplace managers to understand the underlying decision process of bidders in entering auctions and in the time they place their first bid. Also the proposed methods can assist auctioneers in devising operational strategies for efficient auction design. In addition, the developed models can support designing models that can predict bidders' decisions in given auctions.

Using the participation model, one can determine bidders with high or low probability of entering a given auction. While this be integrated with other models in estimating the number of bidders, it can also help in increasing the number of participations. The latter can be done by targeting bidders with low probability of participation or determining less attractive auctions and directing bidders toward those auction. To do so, interested bidders can be notified of the running auctions or coupons or other incentives may be given to lower their entry barriers. In addition, possible changes in the order and placement of the auctions on the website may increase their chance of attracting more bidders.

On the other hand, the second model can help auctioneers identifying bidders who have a higher chance of starting an auction. As it was discussed, bidders with different levels of experience and activity have different bidding patterns both in terms of the amount they bid and the time they place their bids. Given these differences and the importance of the first bid on the dynamics of the auction, it is valuable to know who will start the auction since it can be instrumental in changing the final outcome of the auction. Similar to the participation model, managers may use this model to target bidders. In addition, models which are designed to predict final outcomes of auctions can benefit from our developed model because knowing the first bidder and his/her history of bidding leads to a more accurate estimation of the value of the first bid and eventually the final outcomes.

Another key finding of this paper which may be useful to management is through the dynamic structure of the model. In the first part of the essay we showed how the effect of different factors on bidders' participation decision changes over the time. This can be important since auctioneers can learn how bidders' decision process changes over the time and can design more efficient marketplaces to adapt with these changes and benefit from them.

2.7 Concluding Remarks

In this essay we proposed two models to study bidders' entry into auctions and the time of their first bid. In the first part we proposed a Bayesian dynamic probit approach that can model bidders' participation in auction by adapting to some aspects of the changing environment of the retail secondary market online auctions. The proposed approach uses the Gibbs sampler presented in Soyer and Sung (2013) which is based on using data augmentation and sequential updating method of Forward Filtering Backward Sampling in the context of Bayesian dynamic models (West and Harrison (1999)). To the best of our knowledge, the proposed work is the first to use dynamic Bayesian model in studying auctions over time to predict bidders' behavior in upcoming auctions.

We discussed different auction-specific and bidder-specific variables and explained the final set of covariates which will be incorporated in the model to study participation. The overall significance and the effect of predictors were presented where we showed how some positively or negatively affects bidders' decision in participating in auctions and how their effect may increase or decrease as bidders participate in more auction (i.e. over the time). The dynamic model was also validated for both the fit and predictive performances and it was compared with a static version of the model, and it was shown that the performances were vastly improved by using the dynamic modeling approach.

In the second part, we developed a Bayesian beta regression model to study bidders' time to their first bid in participated auctions. We discussed two different setups of the proposed model and how they can be used in modeling bid timings. It was shown how they can be part of a predictive model to identify first bidders in given auctions. The results supported the model that uses auctions features and bidders characteristics in modeling both parameters of the bid placement time distribution. Further analysis showed the effect of bidders' heterogeneity on this distribution and how bidders with different bidding behavior in the past and at time of the auction tend to place their first bid in different times.

The proposed model can be used to evaluate and predict first-bidding activity of bidders ahead of time. In addition to the application of this model to the first bid, it can be also generalized to study different bids and their arrival times throughout the auction. In doing so, the model can predict upcoming bidders by comparing their bid arrival times. However, modeling the arrival time of the first bid stays instrumental and it needs to be modeled before moving to next bids since the arrival time of the next bids will all be truncated below by the time of the first one.

There are possible limitations to the study, which some were identified in the course of our analysis. The first limitation stems from the nature of dataset. The current dataset is at the bid level meaning that we only have information of the placed bids (including auction and bidder characteristics associated with that bid). A major part of this analysis is based on the definition of available bidders (i.e. their online status) at the time of the auction. In our work we have defined potential bidders as the ones who have placed a bid in similar auctions during the time of a given auction and the results are promising meaning that using the current definition we could capture bidders' status around the auction time. However, a more robust model can be developed if we are to be able to run our analysis on a dataset that provides bidders online status or whether they are watching an auction before they bid or whether they leave auctions after watching them and so on. Another limitation is one-step ahead forecasting and the updating procedure in the dynamic models. The iterative process of a prediction and re-running of the model can be impractical and computationally expensive for large dataset and large number of prediction. In order to address this issue and given that possible extension of our method for practical applications are crucial, we will explore further extensions of the model by using particle filtering as a more efficient way of updating the model in real-time rather than running it over all the past data which might be computationally expensive.

3 Essay 2:

3.1 Summary

In this essay, we propose to develop a model to estimate the distribution of the time of the first bid (TOFB) in secondary (retail) market online auctions. The proposed estimation is based on a Bayesian mixture model of finite beta distributions. Our main interest is to study this distribution from auctions heterogeneity point of view. In doing so, we try to incorporate some auction features in the estimation process and we analyze their effect on first bid arrival time. We test multiple competing models both in terms of fit and predictive performance and choose the best one to study underlying patterns of the auction market. We also discuss managerial implications of the study and suggest how auctioneers can benefit from both the explanatory and predictive aspect of the model.

3.2 Introduction

The effect of timing of the bid arrivals on the dynamics of auctions has been investigated by many researchers. We can divide the researches in this area into two main categories. In one category, researchers have focused on bidders' or auctions' heterogeneity and how these difference influence bid timings. For example they study how bidders with different levels of experience place their bid or study the effect of different auction features on bidders' behavior. On the other hand, some have studied the effect of different bid arrival patterns on dynamics and *Recovery Rate* [Final Price/Declared Retail Value] of auctions. For example, Dholakia and Soltysinski (2001) discuss herd behavior bias (i.e. the effect of early bids in attracting more bidders) and Ariely and Simonson (2003) and Li et al. (2009) discuss the effect of this behavior on recovery rate through attracting more bidders. Dholakia and Soltysinski (2001) discuss that the herd behavior bias is mainly due to bidders' "use of others' bidding behavior" as a signal and the "escalation of commitment" after they place their first bid. They argue that the first (psychological) mechanism is a reason why some auctions get overlooked and the second one a reason why some auctions become more popular even though they are comparable to other listings at the same time. In Pilehvar et al. (2013), authors study the importance first bid in secondary market online business-to-business auctions. They show how first bid can be influential on closing price of the auctions through its value and its arrival time. They show how early first bids with high values result in higher final prices.

In the current dataset, which is the same used in the Pilehvar et al. (2013), we can also see the effect of the arrival of first bid on auctions dynamics and their outcome. Figure 3.1 shows the effect of first bid arrival on attracting more bidders; what is known as herding effect. Figure 3.2 shows the impact of first bid arrival on the recovery rate of auctions. We should note that in both of the plots, we have controlled for the effect of per unit price of the pallet [Declared Retail Value/Quantity] to see the sole effect of time of the first bid on he number of bidders and the recovery rate. In other words, we have divided auctions into three groups based on the distribution of 'per unit value' of the auction. In the first group (i.e. Low-Value Pallets) we put auctions that their value is in the lower quartile, in the second group (i.e. Medium-Value Pallets) we put auctions with their value in the interquartile range and in the third bucket (i.e. High-Value Pallets) we have auctions with value in the upper quartile. In Figure 3.2, we can see how auctions that receive their first bid earlier finish with a higher recovery rate. And as it is shown, regardless of the value (i.e., per unit price) of the pallet, the pattern stays the same.

In contrast to studying early bidding, some researchers have studied late and last-minute bidding. In understanding the reasons of the last-bidding or late-bidding behavior, Ockenfels and Roth (2002) argue that some bidders practice this behavior to protect information in common value auctions or to avoid a bidding war with 'like-minded late bidders' or bidders who determine the value of an item by looking to other

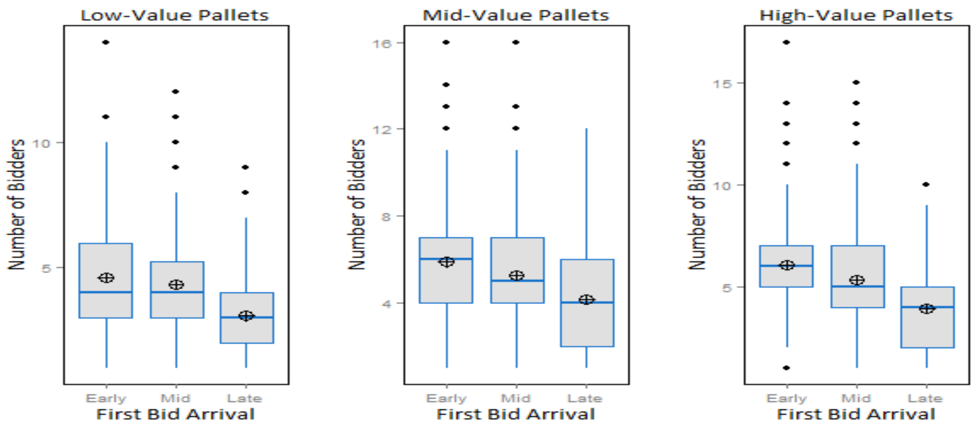

Figure 3.1: Effect of Time of the Firs Bid on Number of Bidders

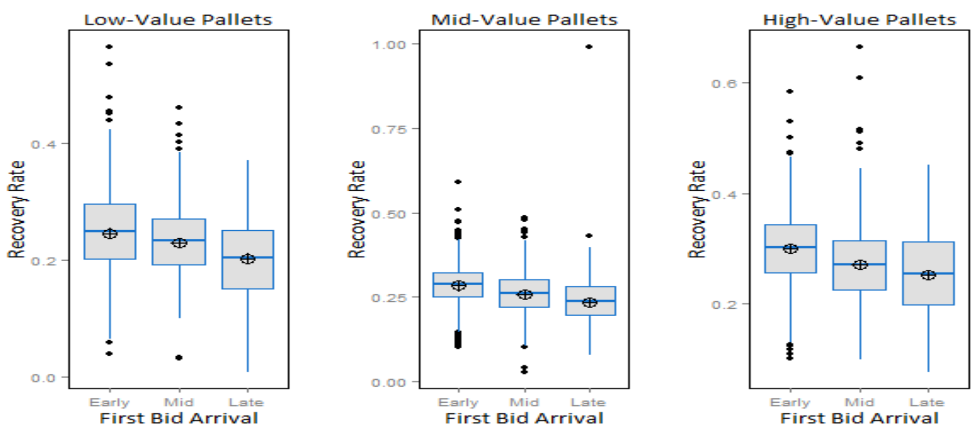

Figure 3.2: Effect of Time of the Firs Bid on Recovery Rate

bidders' bid. They also show that incremental bidding is lower among experienced bidders. On the other hand, there may be some nonstrategic reasons for late bidding in auction websites like eBay and Amazon such as procrastination or management of bidding in multiple similar auctions while the latter might be a similar explanation to some of the bidding behavior in the our studied auction platform. As we will see later, generally bidders that place their first bids earlier are bidders which are more active on the auction platform around that time and they have bids in multiple auctions. Borle et al. (2006) also study the distribution of bid timings (late bidding) and extent of multiple biddings among bidders with different levels of experience. They study some eBay auctions where they define users' recorded feedback as experience. Their findings suggest that experienced bidders are more active either in the beginning or at the end of the auctions. They also look at the "multiplicity of the bids" and find out that bidders with different levels of experience differ in the extent of multiple bidding and that experienced bidders place less bids.

The effect of auction features on arrival of the bids is also studied in literature. Using a Bayesian approach, in addition to effect of experience which we mentioned, Borle et al. (2006) evaluate the effect of different product categories on the timing of the bid. In estimating the distribution of the bid-timing (and only by focusing on the final bids of bidders), they use beta distribution with parameters being function of the product category and bidder characteristics. By applying Bayesian methodology for estimating the parameters, they show that most of the product categories vary significantly in extent of late bidding and that more experienced bidders are more active in the beginning or final hours of auctions. Authors also suggest further research regarding unsuccessful auctions (i.e., auctions that receive no bids). In Bradlow and Park (2007) time of bid arrivals is studied at the auction level (i.e., throughout the auction). Using some auction features, they model the number of potential bidders (i.e., observed and latent bidders) at each bid of the auction and assume an exponentially distributed bid arrival time for each of the potential bidders. Following this assumption, they estimate a probability model for the arrival of the next bid by finding the minimum arrival time among potential bidders. While the developed model is estimating the time between bid arrivals through the auction, it doesn't explain the time between the start of the auction and arrival of the first bid. Their results show an overall good fit but there is some underestimation in posterior prediction of bid time increments and the authors suggest further research on modeling bid arrival time.

Another important feature of online auctions is the auction duration. In the literature there has been mixed evidence in regards to the effect of the length of the auction on recovery rate. In some works it has been proposed that longer auctions receive more bidders (i.e. more bids) which may result in herding behavior and higher recovery rates (Wilcox (2000), Dholakia et al. (2002)). On the other hand, higher premiums has been observed in shorter auctions (Haruvy and Leszczyc, 2010) which may be due to fact that shorter auction draw more impatient bidders or may increase bidding intensity and competition among bidders. In the secondary retail market online auction platform that we are studying, auction duration are set to be either two or three days where longer auctions are posted toward the weekend to give bidders more time to bid. In the analyzing the impact of auction duration on the number of bidders and the recovery rate, even though longer auctions have a slightly higher recovery rate (0.2641 for longer auctions compared to 0.2605 for shorter auctions) we found no significant reason to believe that auction duration is playing a major role in the recovery rate of the auction. We found, however, that shorter auctions receive their first bid slightly earlier which change some dynamics of the auction.

In this work, we also study the effect of simultaneously opened auctions and overlapping auctions to see how they may affect dynamics of the focal auctions. In the literature, there are also some works on shared information among adjacent comparable auctions on their dynamics (e.g., bid arrival, final price, etc.). The shared information and price comparisons can be either implicit or explicit (i.e., triggered by a recommendation system on the auction platform). The effect of explicit reference prices (from similar overlapping auctions) on bidding behavior and how it affects bid arrivals in auctions is studied in Dholakia and Simonson (2005). They show how explicit comparisons of adjacent listing suggested by the auctioneer may adversely affect the bid arrival in the focal auction. This study shows that explicit reference points decreases the effect of overlapping price on the focal auctions and influencing bidders to place 'fewer, lower and later bids'.

In the proposed work, we develop a Bayesian mixture model to estimate the distribution of time of the first bid. In developing our model, we consider some of the features of the auctions and the online platform to investigate how they may affect the arrival of the first bid. We test two major mixture model with different number of components and compare their fit and predictive performances. Using the best models, we then study different mixtures to see how the auctions in each group are segmented and what are the managerial implications of the findings. To the best of our knowledge, time of the first bid hasn't been specifically modeled in the literature and Bayesian analysis of it using mixtures of distributions has not been considered before as well.

3.3 Modeling Time of the First Bid

In this section, we review time of the first and its distribution. We propose a mixture model of beta distributions to model time of the first bid. We explain the details of the models and their parameter estimation, compare their fit and their predictive performances and their outcome in terms of separating auctions into groups with similar characteristics.

Our starting point in modeling time of the first bid is its definition. In the online auction platform that we are studying, unlike some of the other auction environments, auctions have a fixed length which is set by the auctioneer before the auctions go live. The lengths of the auctions are set to be either two days or three days with the two-days auctions being more common (more than 70% of the auctions are two days long) and three-days being mostly the auctions posted late in the week to allow more participation time to the bidders. We define time of the first bid (TOFB) as a ratio of [Time Passed/Total Time], where *time passed* is the gap between start-of-the-auction-time and bid-time and *total time* is the duration of the auction. Figure 3.3 shows the histogram plot of the this variable across all auctions. Following this definition, TOFB is measured as a value in the (0,1) interval. So it is not unreasonable to assume that TOFB follows a beta distribution. This is also similar to what has been used in literature. In Borle et al. (2006) authors define concentration ratio of bids as time left/auction duration and they study time of final bid of bidders in auctions. In modeling the concentration ratio of bids, they assume it to have a beta distribution and use Bayesian approach to model log of the parameters through a covariate structure based on experience of the bidders.

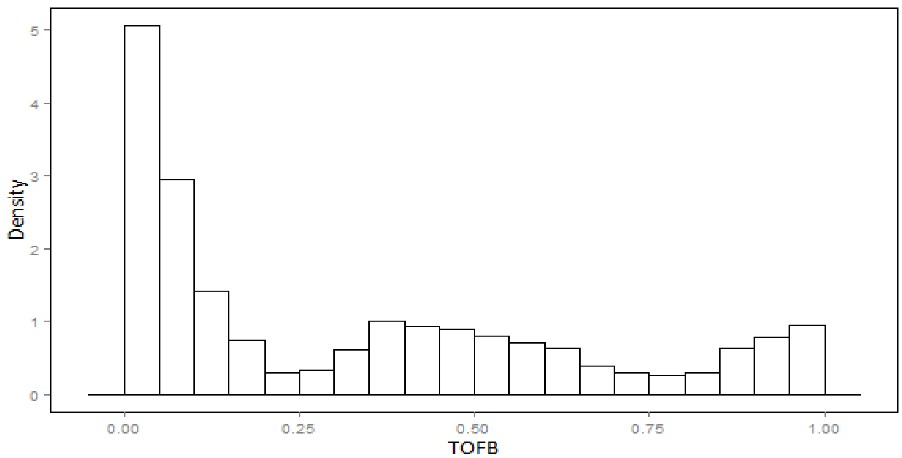

Figure 3.3: Distribution of Time of the First Bids

However, as seen in Figure 3.3, the data suggests different patters in arrival of the first bid; some auction are receiving their first bid very early in the auction and some in the middle or very late (which may lower the recovery rate and/or increase the risk the auction failure). In an attempt to capture the underlying patterns of this distribution, we propose applying a mixture modeling approach of beta distribution to model these complexities by only focusing on the features of auctions rather than their bidders. In the upcoming section, we discuss different aspects of Bayesian finite mixture models and how we use them to model time of the first bid.

3.4 Finite Beta Mixture Model

Mixture of distributions are used as flexible tools for modeling different types of data and are used in wide variety of areas such as machine learning, statistical pattern recognition web spam filtering. Mixture distributions are made up of a finite or infinite number of components (i.e., distributions) that can explain different features of data. These models are commonly used where the populations is assumed to consist of sub-populations. Bayesian analysis of mixture models are based on methods of simulation such as Gibbs sampling and using latent constructs and data augmentation (Diebolt and Robert (1994), Soyer and Xu (2010)).

In the context of online auctions, mixture distribution have been used as a tool to study both biddes and auctions. Mancha et al. (2014) use a finite mixture partial least square model to study bidders' heterogeneity in English and Vickery auctions. They find multiple segments of bidders with distinct characteristics in product categories that they choose, number of bids they place, their willingness-to-pay and etc. In Keller (2006), a mixture of two components is used to model market price of an item in sealed-offer k-doubled auctions where sellers offer its selling price and bidder offers its buying price. The authors suggests that since there is no known distribution for modeling the market price of the items, a mixture model can be used to learn from both seller's and buyer's evaluation of the price. In Jank (2005), a finite mixtures model is applied to cluster a large set of eBay auctions based on both auctions features and bidders characteristics.

In the proposed work, we model the time to the first bid TOFB (as a ratio between 0 and 1) by using mixture of beta distributions. The choice of beta distribution is based on the nature of normalized TOFB which is between 0 and 1. The beta distribution has been well studied and used in business applications and economical model building. In Bayesian inference, this distribution is commonly used as a conjugate prior probability for distributions such as Bernoulli, binomial and geometric.

Bayesian analysis of finite mixture models is mainly based on using Markov Chain Monte Carlo methods like Metropolis-Hastings to simulate the parameters of Beta distribution (Bouguila et al. (2006)). In our proposed work, we use a similar approach but for a different parameterization of the beta distribution.

3.4.1 Mixture Models

The basic principle for setting up mixture models is to use some latent indicators (i.e., unobserved variables) to specify the mixture component from which each observation is drawn. In general, we can define a M-component mixture model for observation y_i, $i = 1, ..., N$ as:

$$p(y_i \mid \lambda, \Theta) = \sum_{m=1}^{M} \lambda_m p(y_i \mid \Theta_m)$$

where Θ_m are the parameters of the m^{th} component in the mixture and $\lambda = \{\lambda_1, ..., \lambda_M\}$ where λ_m's are the parameters indicating the proportion of the population from component m with $\sum_{m=1}^{M} \lambda_m = 1$. In our development we consider mixtures beta distributions using a common reparameterization of beta distribution (van Dorp and Mazzuchi (2004)) where density function of the m^{th} component is given by:

$$p(y_i \mid \mu_m, \phi_m) \propto y_i^{\phi_m \mu_m - 1} y_i^{\phi_m (1-\mu_m)-1}, \mu_m \in [0, 1]$$

It follows from this form that:

$$E[Y \mid \mu_m, \phi_m] = \mu_m$$

$$Var[Y \mid \mu_m, \phi_m] = \frac{\mu_m.(1 - \mu_m)}{(\phi_m + 1)}$$

where $Y = \{y_1, ..., y_N\}$. This allows us to interpret μ_m as a location parameter (i.e., expected TOFB) and ϕ_m as a precision/dispersion parameter which represents our uncertainty about the expected TOFB.

Mixtures of distribution are examples of missing data models in that the data can be considered to come from multiple sub-populations with different size and characteristics. In that regard, we may be interested in finding members of each of the sub-sections or estimate the parameters of those sections. The missing data approach is core to different methods that model mixtures of distribution such as EM algorithm which is the most well-known method in dealing with mixtures of distribution. This algorithm, which is based on Expectation Maximization (EM) algorithm of Dempster et al. (1977), is developed based on a complete-data likelihood function in which the mixture likelihoods are augmented with latent random variables. These latent variables help making inferences about actual parameters of the distributions and their own estimation may also be of interest. However, according to Frühwirth-Schnatter (2006), EM algorithm has some practical difficulties. For example, a set of different starting point is needed for finding the global maximum of the likelihood (rather than a local maximum) and that the algorithm can fail in convergence when sample sizes are small. But Bayesian approach produces valid inference even with small data sets and mixtures with small component weights. Also, availability of full posterior distribution can provide better inference than the maximum likelihood approach.

Similar to the EM algorithm, Bayesian estimation of finite mixture models, which is based on data augmentation algorithm of Tanner and Wong (1987) and MCMC algorithms such as Gibbs sampling (Smith and Roberts, 1993), uses the missing data (i.e. latent structure) approach. In developing our model, since we are assuming auctions are originated from multiple distributions and we do not know from which distribution each observation is coming from, we can consider this problem as a missing data problem as well. In so doing, we introduce latent variables Z_{im} for

$i = 1, ..., N$ and $m = 1, ..., M$ such that:

$$Z_{im} = \begin{cases} 1 & \text{if } y_i \text{ belongs to class } m \\ 0 & \text{otherwise} \end{cases}$$

and for each i we have $\sum_{m=1}^{M} Z_{im} = 1$. If we define the latent vector for each observation $Z_i = (Z_{i1}, ..., Z_{iM})$, then given λ (i.e. mixture probability) we can assume that unobserved vector Z_i has a multinomial distribution of :

$$Z_i \mid \lambda_1, ..., \lambda_M \sim Multin(1; \lambda_1, ..., \lambda_M)$$

which means that only one of the components of Z_i is 1 and the remaining are 0's. Now given Z, the density of the i^{th} observation can be written as:

$$p(y_i \mid \Theta, Z) = \prod_{m=1}^{M} \Big(\lambda_m p(y_i \mid \mu_m, \phi_m)\Big)^{Z_{im}}$$

Therefore, the likelihood term (also known as complete likelihood) can be written as:

$$\mathcal{L}(\mu, \phi, \lambda \mid Y, Z) = \prod_{i=1}^{N} \prod_{m=1}^{M} \Big(\lambda_m p(y_i \mid \mu_m \phi_m)\Big)^{Z_{im}}$$

where $Z = \{Z_1, ..., Z_N\}$, $\mu = \{\mu_1, ..., \mu_M\}$ and $\phi = \{\phi_1, ..., \phi_M\}$.

The Gibbs sampler is the most commonly used approach in Bayesian mixture estimation which is based on successive simulation of Z, λ, μ and ϕ. In estimating the distribution of mixture proportion given Z and Y we have:

$$p(\lambda \mid Z, Y) = p(\lambda \mid Z) \propto p(Z \mid \lambda)p(\lambda)$$

The choice of prior for mixture proportions $p(\lambda)$ is the Dirichlet distribution:

$$p(\lambda) \propto \prod_{m=1}^{M} \lambda_m^{\psi_m - 1}$$

with specified parameters ψ_m's. For $p(Z \mid \lambda)$ we have:

$$p(Z \mid \lambda) = \prod_{i=1}^{N} \prod_{m=1}^{M} \lambda_m^{Z_{im}} = \prod_{m=1}^{M} \lambda_m^{n_m}$$

where $n_m = \sum_{i=1}^{N} \mathbb{I}_{Z_{im=m}}$. In others words, n_m's are total number of observations in each mixture m. It follows from these distribution that:

$$p(\lambda \mid Z) \propto \mathcal{D}(\psi_1 + n_1, ..., \psi_M + n_M)$$

where $\mathcal{D}$ is a Dirichlet distribution with parameters $(\psi_1 + n_1, ..., \psi_M + n_M)$.

3.4.2 Model Description

As noted earlier we are considering an alternative representation of the beta distribution with parameters μ_m and ϕ_m where $\mu_m \in [0, 1]$ can be interpreted as the average $TOFB$ of mixture component m and ϕ_m is the precision parameter which represents our uncertainty about the expected $TOFB$ meaning that for fixed value of μ_m, the larger the value of ϕ_m, the smaller the variance. However, as it is shown, the variance is a function of both mean and precision parameter. So it should be noted that even though ϕ_m is the precision parameter and influences dispersion, it is not the only factor that controls dispersion.

In modeling μ_m's, we take two approaches and we compare their fit and predictive performances. In the first setting, we choose uniform priors for them. In other words, $\mu_m \sim U(0, 1)$ where m is an index representing the m^{th} mixture. In the second approach, we try to model them using a covariate structure by using some auction-specific variables. In this setting, since μ is a parameter between $(0, 1)$, we use a link function to learn about this parameter. In so doing, μ_m's become observation-specific (i.e. μ_{im} and they will be modeled through a regression model with mixture-specific coefficients β_m:

$$\mu_{im} = h(\beta_m X_i)$$

where h is the link function such a probit or logit, β_m's are a vector of regression parameters with normal priors and X_i is a vector of explanatory variables of the i^{th} observation (i.e. auction). For the choice of explanatory variables, we use some auction features such as quantity, price-per-unit, number of overlapping comparable auctions and opening day of the auction. The posterior values of μ_{im} is interpreted as the average time of the first bid of auctions i given that it was originated from mixture m. The value of regression parameters explain the significance (influence) of each of the covariates on time of the first bids in that mixture.

In the development of the model, we use a common precision parameter for all of the m components. In other words, we suppress index m from ϕ_m and we will have:

$$\phi_1 = \phi_2 = ... = \phi_M = \phi$$

In choosing prior for the precision parameter, we use a Gamma distribution with known positive scalars of a and b as its parameters: $\phi \sim Gamma(a, b)$. We then use MCMC algorithms such as Metropolis-Hastings to simulate from the posterior of the parameters (Bouguila et al., 2006).

As it is also reviewed in Frühwirth-Schnatter (2006), there are multiple approaches discussed in the literature for identifying the number of components in a finite mixture. While most of the approaches are data driven, some focus on the posterior density of the mixtures. An example of the latter approach is mode hunting in the mixture posterior density. In this approach, a random permutation Gibbs sampler is used to draw from posterior. The number of modes are then calculated to find the true number of components and identify overfitted models. Another informal approach is mode hunting is the sample histogram. Even though this method is very simple and may be appropriate for certain datasets, it may be misleading as well. Another method is through the method of moments. where discrepancies between the theoretical moments and sample moments are evaluated to find the number of components that has more impact on reducing this gap. A Bayesian variant of this method compares the posterior densities of the moments rather than their point estimates. However, in this work we focus on Bayesian model comparison and fit measures. Deviance Information Criterion (DIC) of Spiegelhalter et al. (2002) is a model comparison criterion that takes into account both the fit and complexity of the model. Having deviance is defined as $D = -2log\mathcal{L}(\Theta)$, DIC in its general form it is defined as:

$$\text{DIC} = \overline{D} + p_D$$

where $\overline{D}$ is the posterior mean of the deviance and $p_D = \overline{D} - D(\hat{\Theta})$ where $D(\hat{\Theta})$ is a point estimate of the deviance obtained by substituting in the posterior means for Θ. In DIC formulation p_D is a penalty measure for the complexity of the model by its effective number of parameters. In Celeux et al. (2006) authors discuss that DIC may lead to negative estimates for p_D in its general form and they propose different forms of it for missing data and mixture models including the following form which they call it DIC_3:

$$\text{DIC} = -4E_{\Theta}[log\mathcal{L}(\Theta)] + 2log\hat{f}(Y)$$

where

$$E_{\Theta}[log\mathcal{L}(\Theta)] = \sum_{g=1}^{G}\sum_{i=1}^{N} log\Big[\sum_{m=1}^{M} \lambda_m^g f(y_i \mid \Theta_m^g)\Big]$$

and

$$\hat{f}(Y) = \prod_{i=1}^{N} \hat{f}(y_i) = \prod_{i=1}^{N} \Big\{ \frac{1}{G} \sum_{g=1}^{G} \sum_{m=1}^{M} \lambda_m^g f(y_i \mid \Theta_m^g) \Big\}$$

where m denotes the mixture component number, i denotes the auction number and g the number of MCMC simulations. In this setting, $\hat{f}(Y)$ is an estimator of $D(\hat{\Theta})$ which is invariant under permutation of component labels and is approximated by an MCMC evaluation. Alternatively to compare predictive performance of the models, we can obtain marginal likelihoods over a holdout dataset which can be obtained by decomposing the original data into two parts: original data d_0 and future data d_F. The marginal likelihood of the new data has the following form

$$p(d_F \mid d_0, M_i) = \int p(d_F \mid \Theta, M_i) p(\Theta \mid d_0) d\Theta$$

and can be approximated using a Monte Carlo average using posterior samples. After estimating the likelihood terms we can compute log of Bayes Factors for two competing models in the following form

$$log(BF) = log\big(p(d_F \mid M_1)\big) - log\big(p(d_F \mid M_2)\big)$$

and we will refer to it as log of Predictive Bayes Factor. According to Kass and Raftery (1995) the values between 1 and 3 are positive evidence and the values between 3 and 5 are strong evidence in favor of M_1.

As it was discussed earlier, for finite mixture models there is often uncertainty regarding the number of mixture components that should be included in the model. In the following section, we fit a series of models with increasing number of components and two different forms (with covariates and without covariates) and we select the most credible model based on both actual fit and predictive performance.

3.5 Results

In order to compare the models, we randomly select 5000 auctions. The data is analyzed with 8 competing models, 4 with increasing number of mixtures (from no mixture to 4 mixtures) without using the covariates to learn about the parameters and 4 with increasing number of mixtures and use of covariates to model the parameters. In comparing the models, we will use DIC as well as PBF. In our analysis, we use proper but diffused priors for all the parameters. More specifically, covariate coefficients β_m's are assumed to have independent normal priors with mean 0 and precision 0.01 and we specify a gamma prior with shape and scale parameters 0.01 and 0.01 on the precision parameter ϕ. The choice of priors for mixture proportions λ_m's is Dirichlet with parameters set to 1. As for the link function h, we have used logistic function. For the cases where μ's are not modeled through a covariate structure, we set a uniform prior on them (between 0 and 1). However, we order them through a truncation method to avoid possible label switching issues. In other words, we will have the following structure for priors:

$$\mu_1 \sim U(0, 1)$$

$$\mu_2 \sim U(\mu_1, 1)$$

$$.....$$

$$\mu_M \sim U(\mu_{M-1}, 1)$$

This step guarantees that mixture distributions are ordered. Label switching arises from invariance of the mixture likelihood function under relabeling of the components of a mixture model (i.e. unobserved categories) which happens in the process of sampling from the mixture posterior distribution. In our model, the truncation helps to avoid the label switching and orders mixture distributions in a way that that the first mixture has the minimum μ and the last mixture the maximum μ. In addition to this step, trace plots and density plots were visually tested to make sure they don't suffer from this problem. As the density plot, trace plot and auto-correlation plots (check appendix) we do not have convergence issues and simulation are mixing well. In addition, Table 6.6 in the appendix provides both Gweke diagnostics z-score with their associated p-values and Raftery and Lewis dependence factors. The fact that p-values are large and dependence factors are mostly smaller than 5 suggests that there no major convergence issues. However, MCMC runs of the parameters associated with the second mixture (in the case of the 3-mixture model) are not mixing as good the rest of the parameters. This is due to few auctions which are in the lower and upper tail of this part of the distribution which may switch labels during the MCMC simulation to be part of the first mixture or the third mixture.

All the computation were done in WinBUGS and R. The final results are based on running a Gibbs sampler with a burn-in sample of 10000 iterations and collecting 5000 posterior samples after thinning by 5. No convergence problem was experienced but a general observation was that the parameters belonging to smaller mixtures don's mix as good as parameters of larger components. All the final trace plots and density plots are available in the appendix. A random sample of 5000 auctions is chosen to test the fit of the models. A separate set of 500 auctions is set aside to test the predictive performance of the models.

The results of DIC comparisons are shown in Table 3.1. DICs are calculated according to DIC_3 of Celeux et al. (2006). There are two things to learn from this table. Overall, models with the covariate structure are doing better than models without the covariate structure. In addition, the models with 4 components fir the data better than models with less. In other words, according to DIC, the covariate structured model with 4 mixtures is outperforming other models in terms of fit. But this relation doesn't necessarily hold when we test these models against a new set of auctions.

To further investigate the performance of the models, we calculate the log pre-

	Mixture	pD	DIC
Without Covariates	**1**	2.16	-2496.45
	2	4.14	-2871.95
	3	5.83	-3695.78
	4	7.79	-3972.68
With Covariates	**1**	6.23	-2594.52
	2	12.42	-3148.73
	3	23.99	-3995.94
	4	26.75	-4292.04

Table 3.1: DIC and Corresponding Effective Parameter p_D

dictive likelihood of the models over a holdout set of 500 auctions. Figure 3.4 shows how covariate structured models with 3-mixtures and 4-mixtures are fitting the new data and Table 3.2 shows the results of this analysis. As the results suggest, the covariate structured model with three components is superior to model with four components; the log predictive Bayes factor is 3.24 $(216.65 - 213.41 = 3.24)$ which is a strong evidence in favor of the three-mixture model. Also, in both categories, the major increment in the likelihood occurs when we move from two-component models to three-component ones suggesting that three-mixture model may be a better fit. In the next step, we visually inspect the top two competing models to see how they compare against each other and how they separate different sections of the distribution.

	Mixture	Log Predictive Likelihood
Without Covariates	**1**	124.18
	2	147.59
	3	212.06
	4	211.69
With Covariates	**1**	125.94
	2	160.88
	3	216.65
	4	213.41

Table 3.2: Log Predictive Likelihood Comparisons (i.e. over new data)

Figure 3.5 compares the posterior predictive density of both models and Figure 3.6 compares distributions of each mixture. As it shown in both figures, the extra

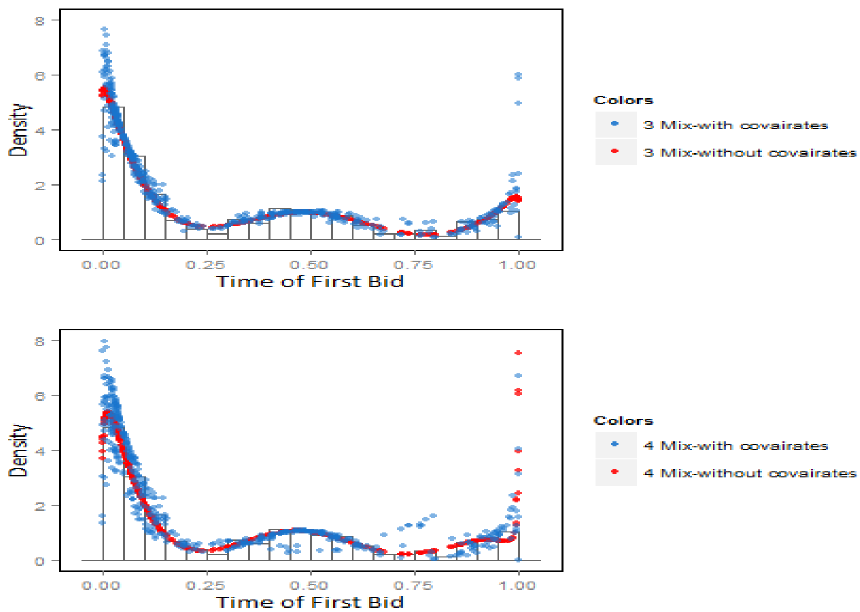

Figure 3.4: Fitted Distributions to the New Data: 3 Mixtures (up) - 4 Mixtures (bottom)

components in the 4-mixture model is a result of a split of the middle component (of the 3-mixture model) into two components. As suggested by Frühwirth-Schnatter (2006) this can be a sign of overfitting in the mixture models. As a robustness check, we fit mixture models to two-day and three-day auctions separately. The results (provided in the appendix) are similar in the sense that 4-mixture model performs better in terms of fit but 3-mixture model performs better over the new data. This may also be a suggestion that DIC_3 is favoring more complex models.

Following these comparisons, we choose the covariate-structured model with 3 mixtures to further analyze different segments of the distribution. Also, the fact that this model is more parsimonious than the 4-mixtures model makes the interpretations of the results more meaningful. In what follows, we study each mixture in terms of their auctions heterogeneity. We then go one step further to see how they differ in terms of bidders' characteristics as well. In assigning mixture membership to the auctions, we take advantage of the latent variables Z_i's. For each auction, we look at

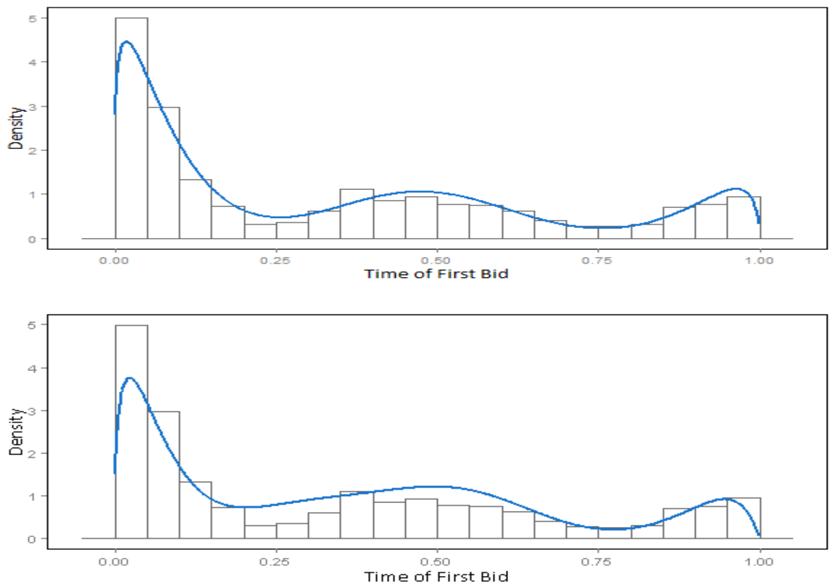

Figure 3.5: Posterior Predictive Distribution: 3 Mixtures (up) - 4 Mixtures (bottom)

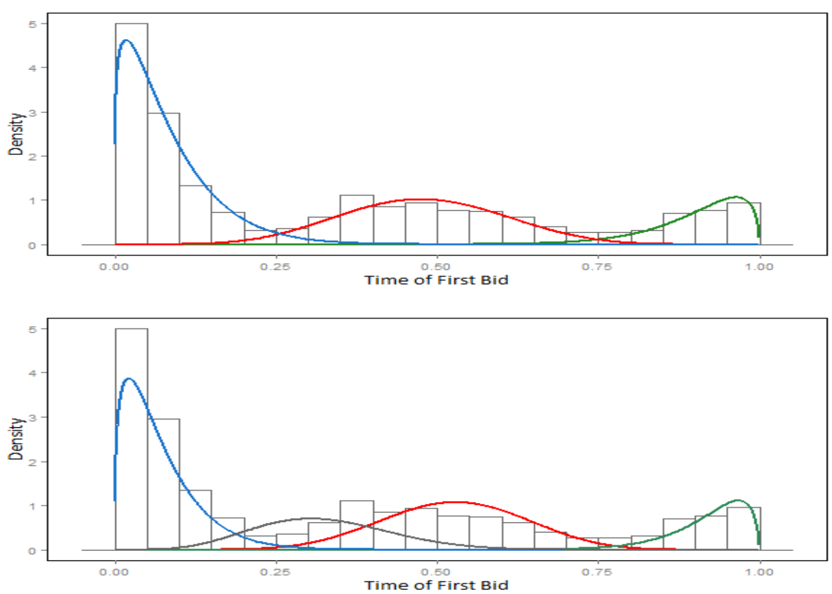

Figure 3.6: Distribution of Each Mixture: 3 Mixtures (up) - 4 Mixtures (bottom)

the posterior distribution of Z_i and we assign the auction to the group (i.e. mixture number) that has the highest probability. In order to make the interpretations easier, mixture labels have been re-ordered in terms of their average TOFB. In the following summary tables, mixture 1 refers to auctions that receive their first bid early, mixture 2 in the middle and mixture 3 refers to the auctions that receive it late.

Table 3.3 summarizes different properties of the auctions in each segment. We can see that around 52% of the auctions are falling into the group that have received their first bid earlier (average TOFB: 0.07), 33% into the second group (average TOFB: 0.48) and the rest in the group that have received the first bid late (average TOFB: 0.91). This segmentation is consistent with the posterior distribution of mixture proportions shown in Figure 3.7. While quantity of the items in the pallet doesn't seem to play a big role, retail value of items in the pallet looks to be an important factor: more valuable items motivate bidders to enter the auction earlier. In addition, existence of other similar auctions at the time of the focal auctions turn out to be a significant factor.

	Mixture Number		
	1	**2**	**3**
Number of Auctions	2588	1678	734
Number of Items (quantity)	32.91	29.70	34.10
Retail Value (per item)	210.7	188.0	181.7
Number of Overlapping Auctions	10.36	13.93	14.16
Number of Simultaneous Auctions	8.14	10.17	10.32
Winter (proportion)	0.35	0.37	0.37
Number of Bidders (per auction)	5.68	4.91	3.95
Number of Bids (per auction)	9.19	7.34	5.53
Time of First Bid	0.07	0.48	0.91
Time of Last Bid	0.95	0.97	0.99
Time of Winning Bid	0.86	0.90	0.98
Value of First Bid	0.17	0.17	0.18
Recovery Rate	0.28	0.25	0.23

Table 3.3: Summary of Auctions in Each Mixture

On the other hand, as it is shown in the table, auctions that receive their first bid very late, have more similar[3] auctions opened with them at the same time (simultaneous opening) or overlapping with them. This is important because having multiple openings at the same time is a common practice in this auction platform. Clearly posting multiple similar auctions at the same time increases the chance of the auctions receiving their first bid late which can negatively affect the outcome of an auction. A look at the rest of the table confirms this effect. Auctions in the third mixture have on average less bidders and less bids since the first activity in the auction is happening very late. Consequently, they have considerably lower recovery late. In fact, auctions in the first mixture have on average 21% higher recovery rate compared to the auctions in the third mixture and given the overall low recovery rate in this market, both auctioneers and retailers can highly benefit from it. While the effect of higher recovery rate in the first mixture may be associated with the value of pallets, as we showed earlier in Figures 3.1 and 3.2, arrival of the first bid plays a major role in this dynamics. Also, we should note that time of the year[4] does not seem to be a major factor in this segmentation. After this analysis at the auction level, we also look to see how bidders differ in each mixture.

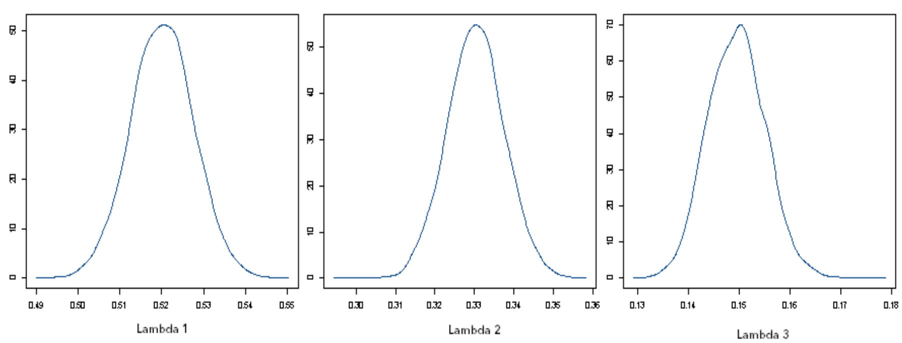

Figure 3.7: Posterior Distributions of Mixture Proportions : $\lambda_1, \lambda_2, \lambda_3$

[3]For each auction, a set of similar/comparable auctions are open auctions which their quantity and retail value is within one standard deviation from the focal auction.

[4]Whether they are posted in winter which is the busiest season of this market.

Table 3.4 summarizes bidders' characteristics in each of the mixtures. While this table does not necessarily establish a causal relation between bidders's heterogeneity and arrival of the first bid, it helps us to better understand this auction platform. In particular, the results show that auctions in mixture 1 receive their first bids from less-experienced bidders. These are bidders with less participations, less number of wins and less money spent in the previous months. Also, in comparison with bidders at other mixtures they are less active on the auction platform at the time of their bid placement. But why is it that some auctions do not receive early bids from this group and receive it later from super bidders? In order to answer this question, we extend the analysis to look not only at first bidders but on all potential bidders that are present on the auction platform (i.e. they have bids in similar open auctions) at the opening time of the auctions.

	Mixture Number		
	1	**2**	**3**
Wins (last 3 months)	8.69	14.01	14.30
Win Rate (last 3 months)	0.13	0.15	0.20
Participation (last 3 months)	56.27	75.57	74.83
Total Money Spent (last 3 months)	7747.6	12282.8	16008.4
Average Time of First Bid (last 3 months)	0.48	0.58	0.71
Average value of First Bid (last 3 months)	0.18	0.19	0.20
Wins in Just Finished Auctions (at bid time)	0.08	0.28	0.40
Participation in open Auctions (at bid time)	2.74	3.68	4.60

Table 3.4: Summary of Bidders in Each Mixture

Table 3.5 is a summary of bidders that are active around opening time of the auctions[5]. The results show that most of the bidders that were active around the opening time of auctions in the first mixture are less experienced bidders. This finding suggests that in addition to auction features, presence of some bidder groups at

[5]We consider bidders 'active' at the start of an auction if they have placed a bid in similar auctions one hour prior and one hour after the start of the focal auction.

the time auctions are posted can be influential. In fact, most of the auctions in mixture 1 (35%) are posted on Wednesday while Friday has the highest proportion for auctions in the thirds mixture (37%). This is consistent with our findings given that the data shows that experienced bidders are more active toward the beginning or end of the week[6]. This can guide auctioneers to study activity patterns of less experienced and experienced bidders to find an optimized schedule for posting time of the auctions.

	Mixture Number		
	1	**2**	**3**
Wins (last 3 months)	6.436	7.422	8.38
Win Rate (last 3 months)	0.18	0.19	0.22
Participation (last 3 months)	36.07	41.03	38.08
Total Money Spent (last 3 months)	7418.5	7505.2	8713.6
Average Time of First Bid (last 3 months)	0.57	0.62	0.69
Average Value of First Bid (last 3 months)	0.18	0.17	0.18
Wins in Just Finished Auctions (at bid time)	0.17	0.28	0.41
Participation in Open Auctions (at bid time)	1.91	2.51	4.60

Table 3.5: Summary of Active Bidders at Start of the Auctions in Each Mixture

The posterior summaries including 95% Bayesian CCI for parameters of the model are shown in Table 3.6. From the table we can see the difference in effect of covariates in different mixtures. For example, while larger number of overlapping auctions means later arrival of the first bid in mixture 1, the effect is negative in the third mixture. The same relation (but in the opposite direction) holds for opening season of the auction. However, the CCI suggests that the effect of some of the parameters such as retail value and quantity is not significant.

At the end, we should note that one main advantage of Bayesian mixture modeling of the time of the first bid is that it allows us to calculate probability of an auction receiving the first bid after a specific time t. This is useful since it can be applied to

[6]Monday/Friday are two of the busiest days on the auction website and experienced bidders are more active on these days.

	Mixture Number		
	1	**2**	**3**
λ	0.520	0.330	0.150
	0.007	0.007	0.006
	(0.505,0.534)	(0.316,0.344)	(0.139,0.160)
$\beta_{Overlap}$	0.755	0.412	-1.827
	0.091	0.069	0.153
	(0.574,0.938)	(0.280,0.541)	(-2.123,-1.529)
$\beta_{Qunatity}$	-0.202	0.266	0.440
	0.216	0.154	0.291
	(-0.641,0.197)	(-0.019,0.560)	(-0.107,1.036)
$\beta_{Weekend}$	-0.129	0.004	0.093
	0.039	0.050	0.072
	(-0.204,-0.0527)	(-0.093,0.097)	(-0.050,0.238)
β_{Value}	-0.129	-0.228	-0.226
	0.163	0.199	0.484
	(-0.444,0.185)	(-0.620,0.138)	(-1.134,0.789)
β_{zero}	-2.640	-0.187	2.964
	0.049	0.037	0.090
	(-2.738,-2.542)	(-0.250,-0.105)	(2.790,3.149)
ϕ	15.290	15.290	15.290
	0.393	0.393	0.393
	(14.52,16.06)	(14.52,16.06)	(14.52,16.06)

Table 3.6: Posterior Summaries of Parameters (Mean, Standard Deviation, 95% CCI)

identify auctions with high probability of receiving a late first bid. We can also use this model to compare a set of simultaneously posted auctions in terms of their bid arrivals.

3.6 Managerial Implication

There are a few important managerial implications to take away from this study. First is that Bayesian mixture model can help auctioneers to understand the underlying effects of features of the auctions on their bid arrival. In addition, it supports making more accurate predictions of future auctions outcomes through a better estimation of bid arrival time.

In this essay, we also showed the presence of different auction segments in terms of their bid arrival: auctions that receive their first bid early with better final outcomes, some auctions in the middle and some auctions with late first bid arrival which on average are less successful than the other two groups. It was found that in comparison to component two and three, first component consists of busier auctions (i.e. more bids and more bidders) with higher value and higher recovery rate. In addition, they have less overlapping auctions and fewer auction have been posted with them simultaneously. In contrast, auctions in the last segment have more overlapping auctions, a fact that may be one of the reasons that delays their first bid arrival. In a busier auction environment there is a higher chance that some auctions get less attention. Also, among the overlapping auctions, an auction that receives the bid earlier may look more attractive to other bidders as they may interpret that as a positive signal toward the value of that auction. We also mentioned that the model can be used to study bid arrival of future auctions which can help managers identifying risky auctions in advance.

Another key finding of this paper is on posting time of the auctions and how managers can benefit from it. We showed how bidders with different levels of experience show different bidding behaviors and how they are distributed among the segments. In particular, it was found that less-experienced bidders tend to enter auctions earlier and that auctions of the first group were mainly posted around the time that most of the active bidders are inexperienced bidders. Knowing that experienced bidders also have a pattern in terms of their activity time, this finding can help managers to optimize their posting time of the auctions. In addition, they should explore ways to motivate experienced bidders to enter auctions earlier. This can be done through reducing entry barriers such as lowering or removing the minimum price or through some targeted offers such as coupons or email campaigns. In addition, the predictive aspect of the model can identify less attractive auctions and promote them to bidders that have bid in similar auctions in the past through an online recommender system.

3.7 Concluding Remarks

In this essay we proposed a Bayesian model to estimate the distribution of the time of the first bid in the retail secondary market business to business online auctions. Our approach was based on Bayesian finite mixture models of beta distributions. The proposed approach uses the MCMC simulation methods to make inference about the parameters. To the best of our knowledge, the proposed work is the first to use Bayesian mixture approach to model time of the bids.

Two different approaches were implemented and different number of mixture components were tested. The results suggest that the model with three mixtures fits this distribution better and has higher predictive performance. In learning about the parameters of this distribution, we used a covariate structure and included some auctions features as explanatory variables. We showed the effect of auctions heterogeneity on their first bid arrival and what makes some auctions more attractive to bidders. Factors like the value of the items, opening time or the number of simultaneous and overlapping auctions turn out to be the most significant factors. Furthermore, our analysis revealed some existing patterns in bidders' behavior and the time they tend to place their bids. We also showed that in addition to the heterogeneity of the auctions, characteristics of online/available bidders at the opening time of the auction can have an effect on the first bid arrival time. In that regards, further analysis of bidders' activity patterns and the days or hours they go online can help in understanding different bid arrival patterns and modeling this distribution. Finally, as we discussed, this study can help managers to a get a better understanding of their marketplace. In addition, they can benefit from it by designing a more efficient auction platform with a better auction scheduling.

4 Discussion

The two essays together illustrate a comprehensive approach in studying formation of the first bid in retail secondary market online auctions.

The methods discussed in the first essay can be utilized to study bidders' participation in auctions and to identify the first bidder of an auction by predicting all potential bidders' time of the first bid. On the other hand, the method discussed in the second paper aims at modeling time of the first bid by segmenting auctions and estimating distribution of the first bid arrival time for a given auction. We should note, however, that models from both essays are linked in a way that both can be used to estimate time of first bid. While this estimation is straightforward in the mixture model (as it is the main objective of the model), it is not immediately available from the beta regression model of the first essay.

In order to demonstrate how beta regression model can also be used to estimate time to the first bid we refer to the example discussed in the results section 2.5.4. There we showed the posterior predictive distribution of time to the first bid of the two potential bidders:

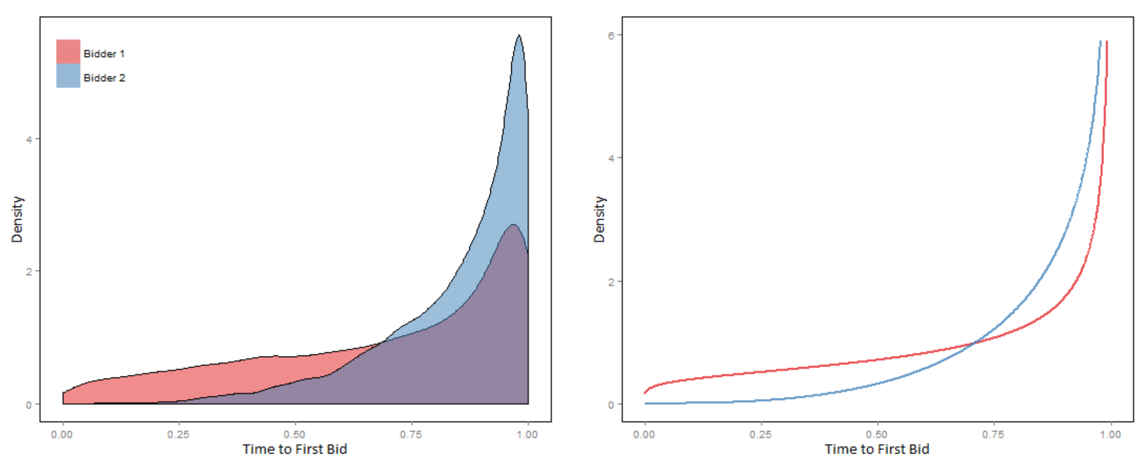

Figure 4.1: Posterior distribution of TOFB of the two participated bidders of a given auction and posterior predictive distribution of their TOFB (right)

However, we can also use bidders' distribution of the time to their first bid to estimate distribution of time of the first bid in the given auction. In so doing, at each run of the simulation, we find the minimum of y_{ij_i}'s for all bidders to have the following distribution where the red line is the observed time of the first bid of the auction:

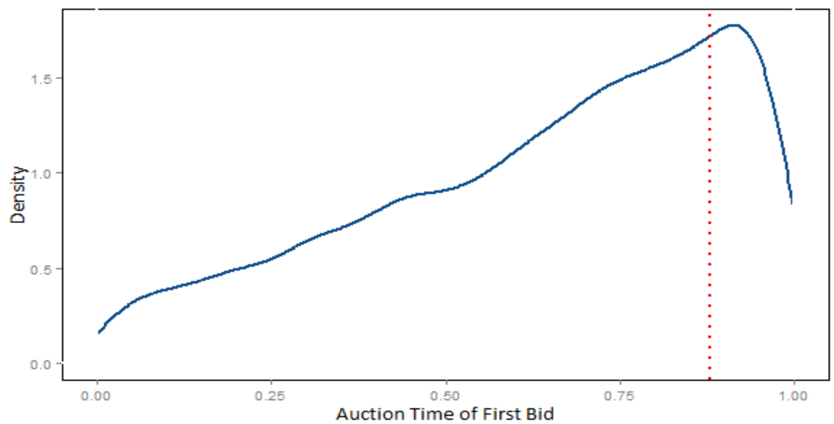

Figure 4.2: Posterior distribution of TOFB for the given auction

To finalize, we should note that an extension to this work would be comparing the performance of both models in estimating the time of the first bid of auctions using a beta regression model which is mainly based bidders' heterogeneity and a finite mixture model which is based on auctions' heterogeneity. The results may also reveal whether first bid arrival time is more influenced by auction features or bidders' characteristics.

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6 Appendices

6.1 Participation Model

Cutoff	Accuracy Rate	True Positive Rate	False Negative Rate
0	0.304	0.000	1.000
0.05	0.366	0.989	0.906
0.1	0.441	0.959	0.786
0.15	0.515	0.914	0.659
0.2	0.583	0.854	0.535
0.25	0.641	0.781	0.421
0.3	0.686	0.701	0.321
0.35	0.717	0.616	0.238
0.4	0.737	0.529	0.172
0.45	0.748	0.446	0.121
0.5	0.751	0.370	0.083
0.55	0.749	0.303	0.055
0.6	0.745	0.245	0.037
0.65	0.739	0.197	0.024
0.7	0.733	0.157	0.015
0.75	0.727	0.125	0.010
0.8	0.722	0.098	0.006
0.85	0.717	0.076	0.003
0.9	0.712	0.057	0.002
0.95	0.707	0.038	0.001
1	0.304	0.000	1.000

Table 6.1: Cutoff Sensitivity Analysis - Dynamic Model Performance Over the Main Dataset

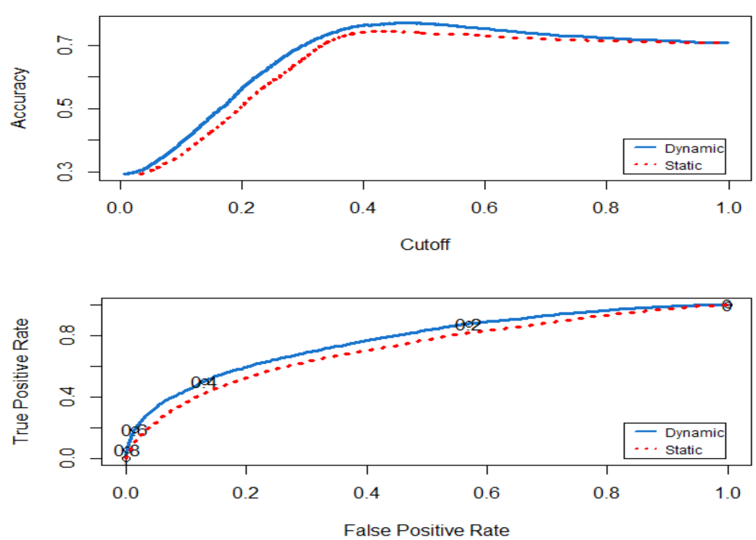


Figure 6.1: Accuracy(top) and ROC(bottom) Performance Curves Over Main Dataset

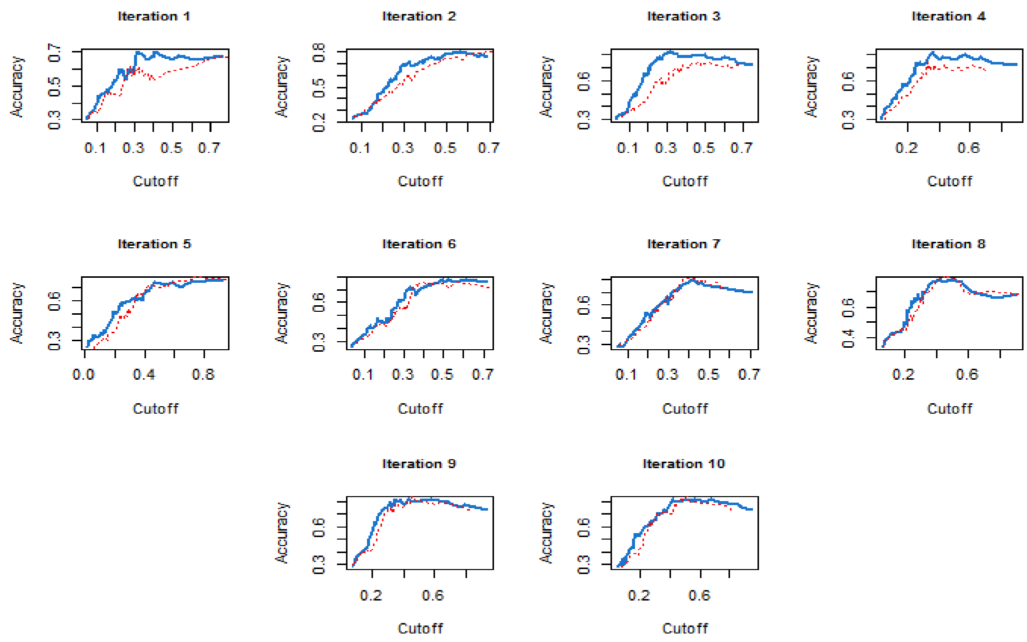


Figure 6.2: Accuracy Curves - All Prediction Iterations

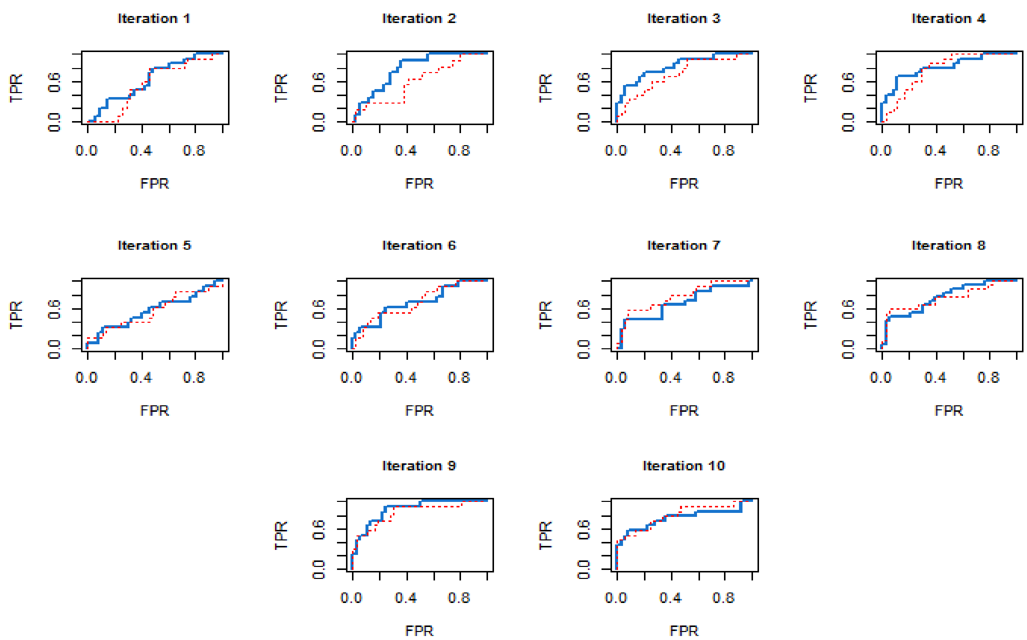


Figure 6.3: ROC Curves - All Prediction Iterations

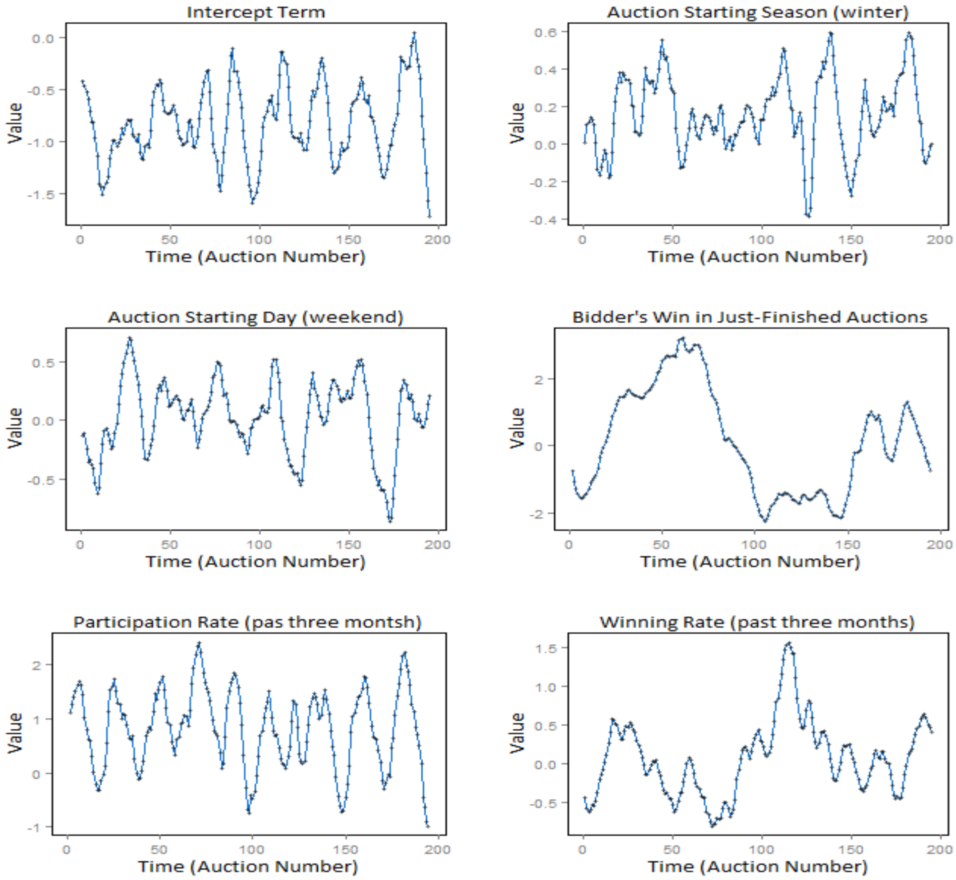


Figure 6.4: Posterior Mean of the Regression Parameters

6.2 First Bidder Model

	Average Number of Bidders	
Run.	Main Set	Holdout Set
1	5.44	5.44
2	5.35	5.66
3	5.38	5.10
4	5.42	5.58
5	5.33	5.70
6	5.42	5.34
7	5.50	5.38
8	5.37	5.22
9	5.38	5.54
10	5.46	5.56

Table 6.2: Average Number of Bidders in Each Sample

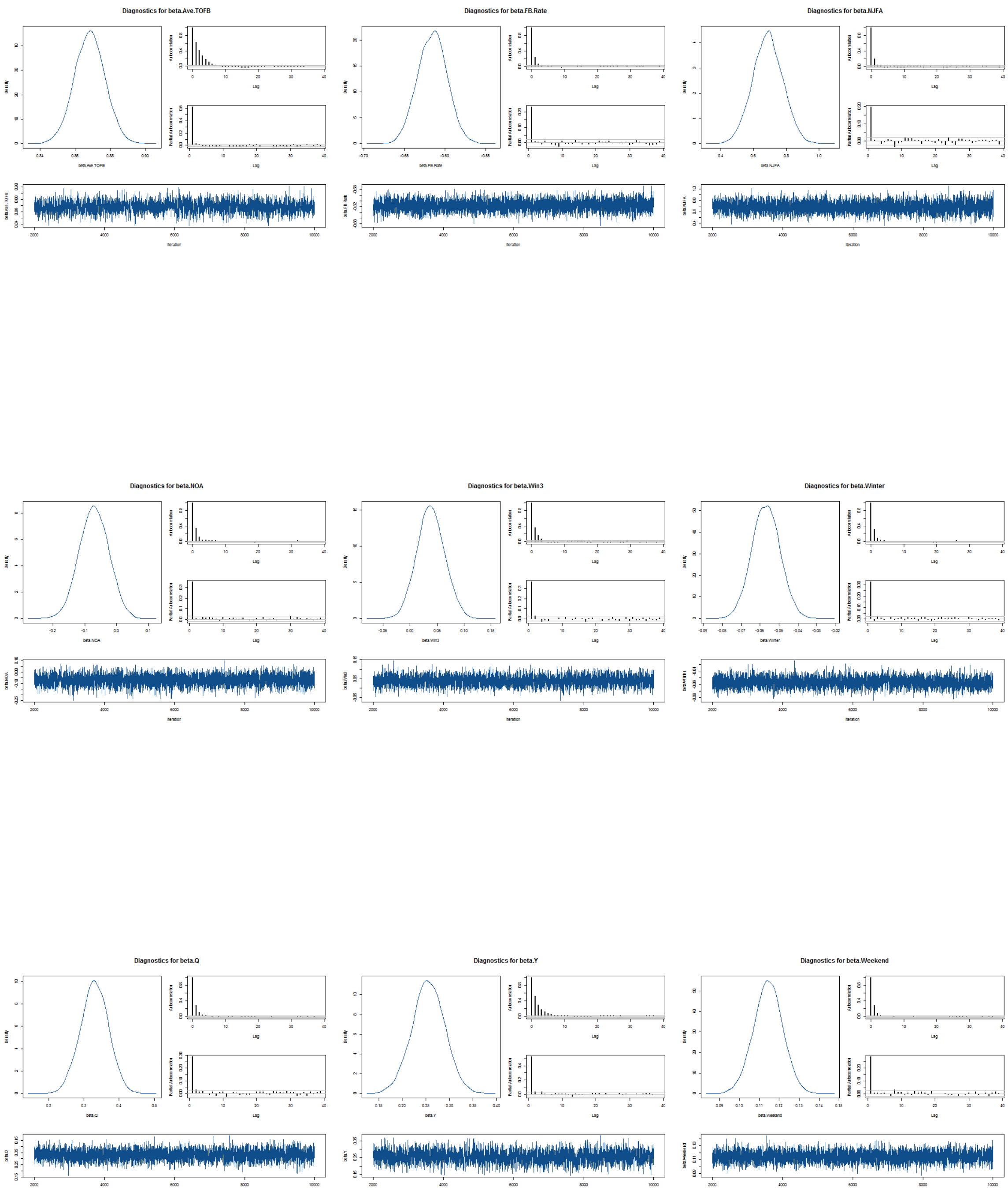
Parameter	**Mean**	**StDev.**	**95% CCI**
$\beta_{Ave.TOFB}$	0.868	0.009	(0.851,0.885)
$\beta_{FB.Rate}$	-0.616	0.018	(-0.650,-0.580)
β_{NJFA}	0.687	0.092	(0.507,0.866)
β_{NOA}	-0.071	0.046	(-0.161,0.017)
β_{Q}	0.330	0.040	(0.252,0.406)
$\beta_{Weekend}$	0.114	0.007	(0.100,0.128)
β_{Win3}	0.039	0.025	(-0.010,0.089)
β_{Winter}	-0.057	0.007	(-0.071,-0.042)
$\beta_{Retail.Value}$	0.255	0.035	(0.187,0.322)
ϕ	1.004	0.006	(0.992,1.016)

Table 6.3: Posterior Summaries of Model 1

	Geweke Diagnostic		Raftery & Lewis
	z-score	p-value	Dependence Factor
$\beta_{Ave.TOFB}$	-1.72232	0.0850	1.96
$\beta_{FB.Rate}$	1.1423	0.2533	1.24
β_{NJFA}	1.1425	0.2533	1.36
β_{NOA}	0.3706	0.7109	1.36
β_{Q}	0.9375	0.3485	1.31
$\beta_{Weekend}$	0.3433	0.7314	1.44
β_{Win3}	1.0549	0.2915	1.50
β_{Winter}	-1.9455	0.0517	1.44
$\beta_{Retail.Value}$	1.6382	0.1014	2.03
ϕ	0.3079	0.7582	1.04

Table 6.4: Convergence Diagnostics Statistics - Model 1

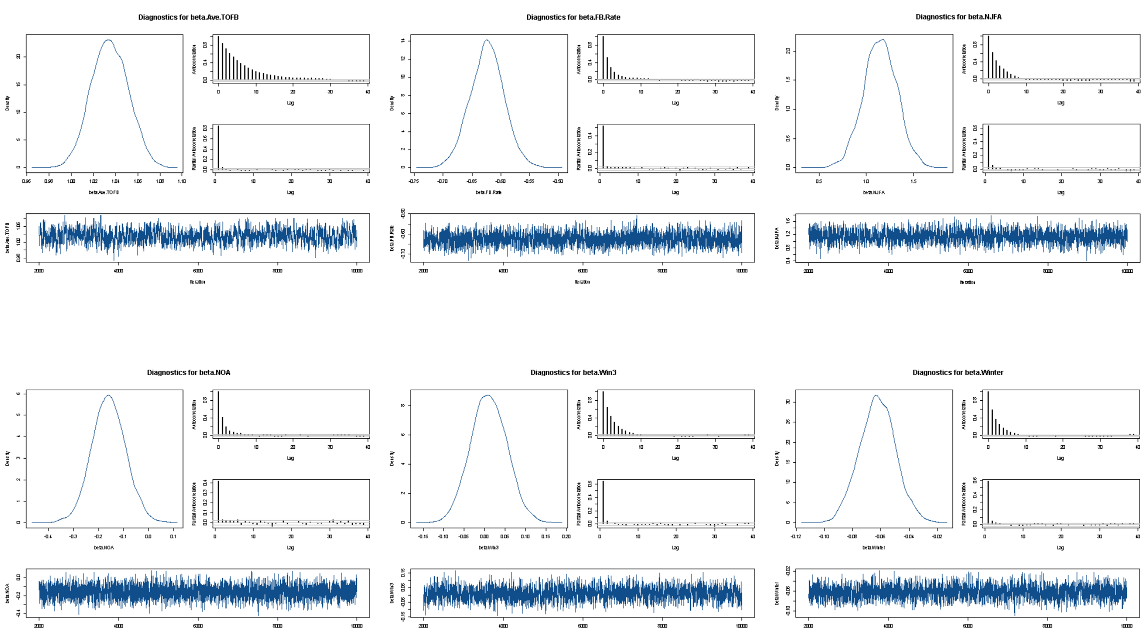
Figure 6.5: Convergence Diagnostics Plots - Model 1

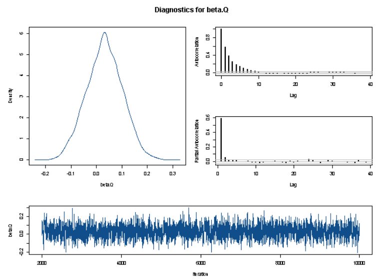


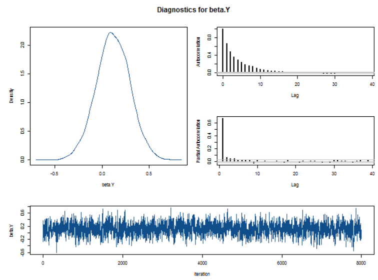
	Geweke Diagnostic		Raftery & Lewis
	z-score	p-value	Dependence Factor
$\beta_{Ave.TOFB}$	-0.0183	0.9854	3.97
$\beta_{FB.Rate}$	-1.5523	0.1206	1.54
β_{NJFA}	-0.5717	0.5675	1.80
β_{NOA}	0.7288	0.4661	2.17
β_{Q}	-1.0445	0.2963	1.53
$\beta_{Weekend}$	-1.6042	0.1087	1.51
β_{Win3}	-1.2566	0.2089	2.69
β_{Winter}	0.7724	0.4399	1.76
$\beta_{Retail.Value}$	1.0896	0.2759	2.39
$\gamma_{Ave.TOFB}$	0.5051	0.6135	2.39
$\gamma_{FB.Rate}$	0.0624	0.9502	1.38
γ_{NJFA}	-0.8073	0.4195	1.74
γ_{NOA}	0.3916	0.6953	1.91
γ_{Q}	0.4026	0.6873	1.42
$\gamma_{Weekend}$	-0.0652	0.9480	1.57
γ_{Win3}	0.5144	0.6070	2.03
γ_{Winter}	-0.3442	0.7307	1.65
$\gamma_{Retail.Value}$	0.3152	0.7526	2.81

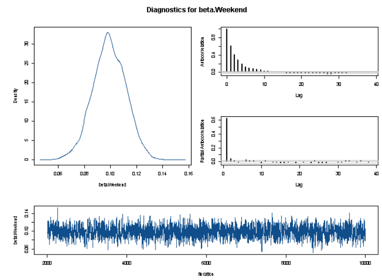
Table 6.5: Convergence Diagnostics Statistics - Model 2

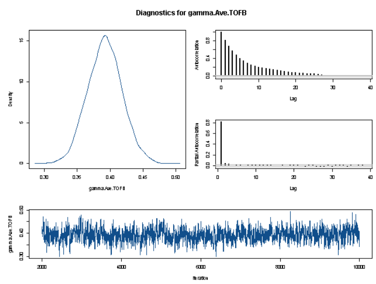
Figure 6.6: Convergence Diagnostics Plots - Model 2

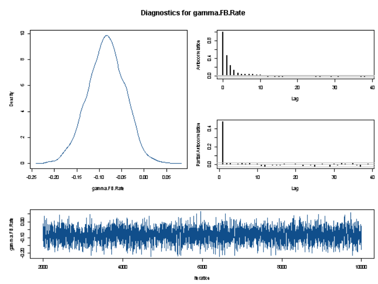


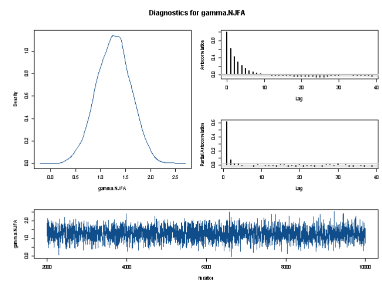

Diagnostics for beta.Q

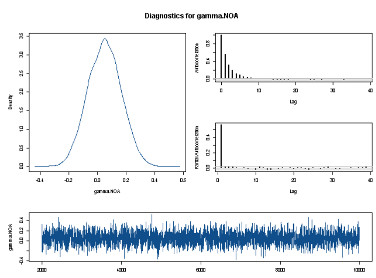

Diagnostics for beta.Y

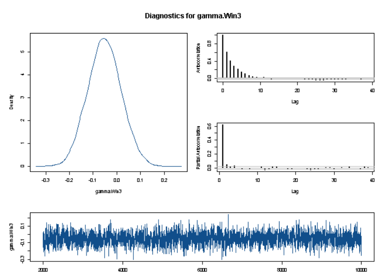

Diagnostics for beta.Weekend

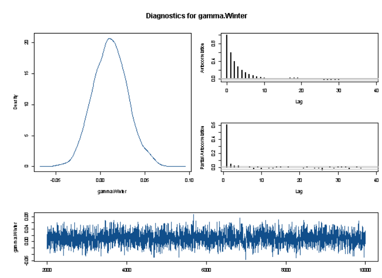

Diagnostics for gamma.Ave.TOFB

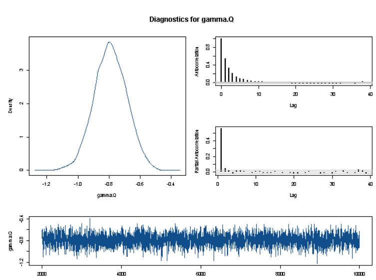

Diagnostics for gamma.FB.Rate

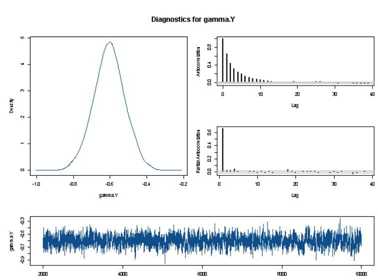

Diagnostics for gamma.NJFA

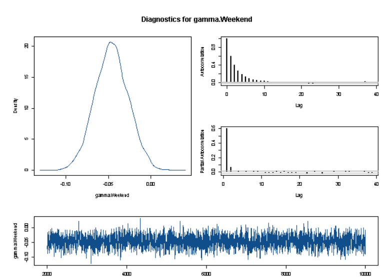

Diagnostics for gamma.NOA


Diagnostics for gamma.Win3


Diagnostics for gamma.Winter


Diagnostics for gamma.Q


Diagnostics for gamma.Y

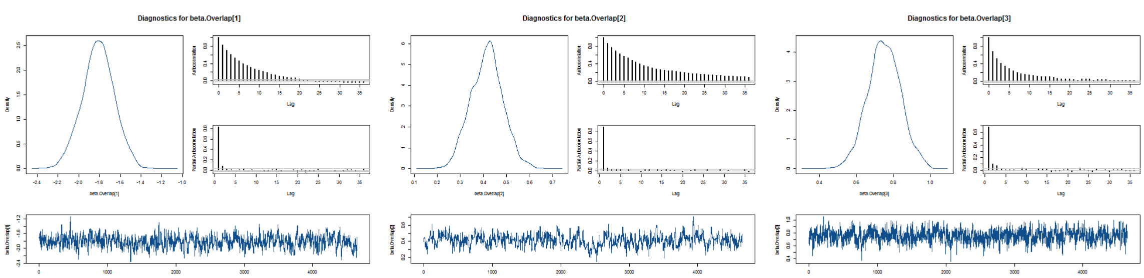

Diagnostics for gamma.Weekend

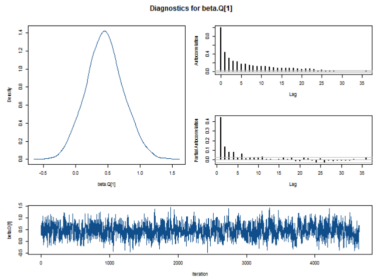
6.3 Time of the First Bid

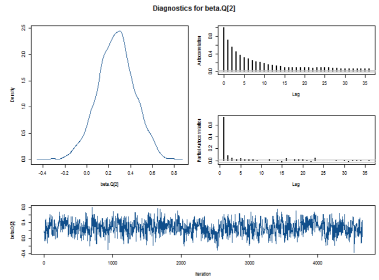
	Gweke Diagnostic		Raftery & Lewis
	Z-score	**p-value**	**Dependence Factor**
λ_1	0.523	0.601	1.03
λ_2	0.476	0.634	1.09
λ_3	-0.936	0.349	0.99
$\beta_{Overlap1}$	1.516	0.130	3.95
$\beta_{Overlap2}$	1.695	0.090	5.17
$\beta_{Overlap3}$	0.431	0.666	4.01
β_{Q1}	1.881	0.060	1.86
β_{Q2}	1.836	0.066	3.7
β_{Q3}	0.327	0.744	1.67
$\beta_{Weekend1}$	-0.771	0.441	3.17
$\beta_{Weekend2}$	-1.056	0.291	3.78
$\beta_{Weekend3}$	0.593	0.553	1.31
β_{Value1}	-0.122	0.903	2.76
β_{Value2}	0.771	0.441	4.39
β_{Value3}	1.309	0.190	2.32
β_{zero1}	-0.508	0.612	4.67
β_{zero2}	-1.220	0.223	5.33
β_{zero3}	-0.846	0.398	3.94
ϕ	0.012	0.990	1.08

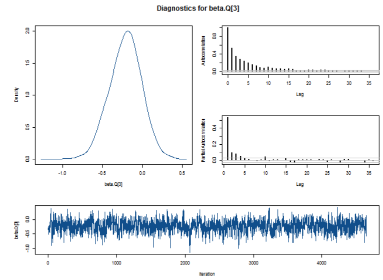
Table 6.6: Convergence Diagnostics Statistics, 3-Mixture Model

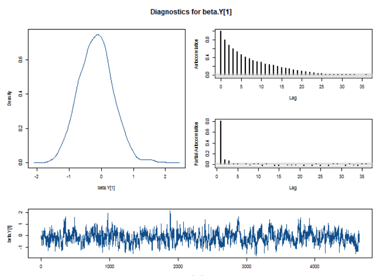
Figure 6.7: Convergence Diagnostics Plots, 3-Mixture Model

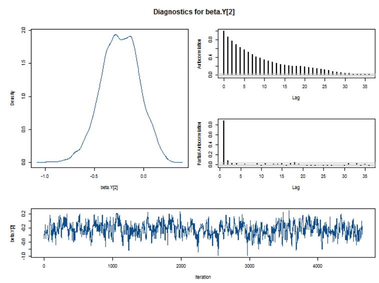


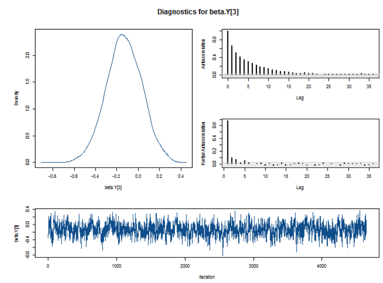

Diagnostics for beta.Q[1]

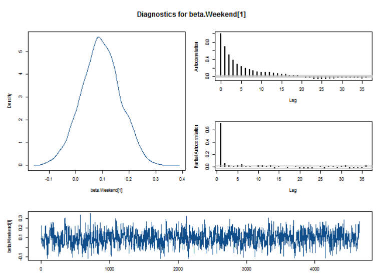

Diagnostics for beta.Q[2]

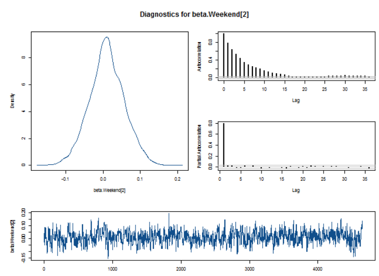

Diagnostics for beta.Q[3]

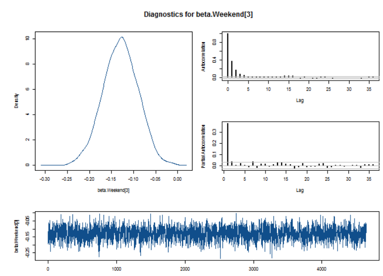

Diagnostics for beta.Y[1]

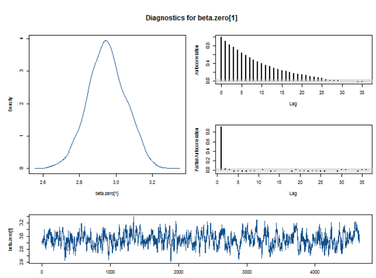

Diagnostics for beta.Y[2]

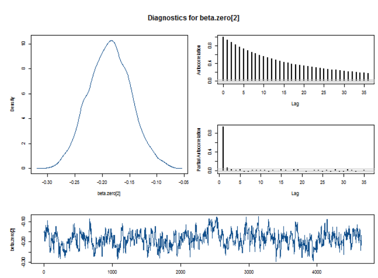

Diagnostics for beta.Y[3]

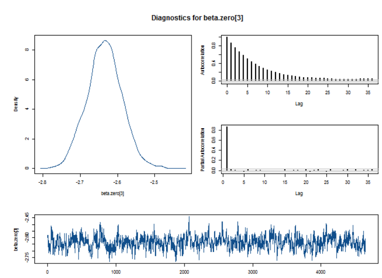

Diagnostics for beta.Weekend[1]


Diagnostics for beta.Weekend[2]


Diagnostics for beta.Weekend[3]


Diagnostics for beta.zero[1]


Diagnostics for beta.zero[2]


Diagnostics for beta.zero[3]