BIOMECHANICAL MODELING OF GLOTTAL AERODYNAMICS AND VOCAL FOLD VIBRATION DURING PHONATION

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DEDICATION

To my mother and father
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ABSTRACT

Biomechanical Modeling of Glottal Aerodynamics and Vocal Fold Vibration During Phonation

Phonation is a complex biological phenomenon which results from the coupled biomechanical interaction between glottal aerodynamics and vocal fold tissue. Due to the complexity and nonlinearity of flow-tissue interaction, reduced fidelity models such as inviscid/irrotational flow, lumped mass vocal fold models, stationary or specified vocal fold motion, etc. have been widely used in the past for modeling of phonation. While these models are able to capture some of the basic characteristics of phonation, they cannot provide quantitative results of a quality and fidelity required for clinical diagnosis or treatment. Furthermore, these methods provide only limited insights into the biophysics of phonation. In the current study, a new, high-fidelity computational tool has been developed for modeling the biophysics of phonation in all its complexity. While the immediate objective is to use this tool is to gain new insights into the biophysics of phonation, the long term goal of this project is develop a computer-based tool for laryngeal surgical planning.

The key components of the tool include (a) an immersed boundary method (IBM) based Navier-Stokes solver for modeling the glottal flow, (b) a finite-element method (FEM) based solver for modeling the viscoelastic deformation of vocal-fold tissue, (c) a penalty coefficient method for modeling the vocal-fold contact and (d) a loosely coupled IBM-FEM methodology for modeling fluid-structure interaction in the human larynx. In addition, a high resolution CT scan has been used to provide guidance for construction of a realistic laryngeal model.
Two- as well as three-dimensional simulations have been performed to gain insights into the biophysics of phonation. Self-sustained vocal fold vibrations with vibratory modes that correspond to physiological observations have been captured (Zemlin, 1988) and the vibration frequency is in the correct phonatory frequency range (Titze, 1994). The glottal flow has been shown to form a pulsatile turbulent jet and this jet flow is highly asymmetric. The effect of subglottal pressure on phonation onset and fundamental frequency has been investigated. The predicted phonation frequency shows a nonlinear increase at the lower end of the phonatory pressure range but settles to a nearly constant value at or above the normal frequency. The effect of false vocal folds on phonation has investigated through a systematic comparison of two models, one with false vocal folds and one without. It has been found that false-vocal folds tend to aid phonation by reducing the effort required to phonate and by increasing the sound intensity for a given effort. While similar effects have been noted in the past, a key contribution of the current study is to clearly delineate a physical mechanism for this effect – a reduction in viscous losses associated with reduced tendency of the glottal jet flapping in the presence of false vocal folds.

The current study shows that while two-dimensional laryngeal models are amenable to comprehensive analysis through simulations, the computational requirements for three-dimensional models are significantly larger. This effectively puts such analysis using three-dimensional models out of reach unless significant advances can be made in the computational speed and efficiency. In order to achieve significant improvement in computational speeds, a new local-grid refinement approach that employs a hierarchical nested grid approach has been developed and applied to a sharp interface immersed boundary solver. The key feature of the methodology is that the structured grid approach is
retained at all the refinement levels and this allows one to use powerful line-SOR schemes and a geometric multigrid method. A set of simulations of canonical flows have been conducted and these indicate that the solver accurately reproduces the key features of the flows and holds promise for phonation modeling with complex three-dimensional models.
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## NOMENCLATURE

### English Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A_g$</td>
<td>Area of Glottis</td>
</tr>
<tr>
<td>$a_i$</td>
<td>Nodal Tissue Acceleration in ith Direction</td>
</tr>
<tr>
<td>$C$</td>
<td>Damping Matrix</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Drag Force Coefficient</td>
</tr>
<tr>
<td>$C_L$</td>
<td>Lift Force Coefficient</td>
</tr>
<tr>
<td>$D_i$</td>
<td>Nodal Tissue Displacement in ith Direction</td>
</tr>
<tr>
<td>$E_p$</td>
<td>Material Plane Young’s Modulus</td>
</tr>
<tr>
<td>$E_{zp}$</td>
<td>Material Longitudinal Young’s Modulus</td>
</tr>
<tr>
<td>$F_0$</td>
<td>Fundamental Phonation Frequency</td>
</tr>
<tr>
<td>$G$</td>
<td>Material Shear Modulus</td>
</tr>
<tr>
<td>$G_{TVF}$</td>
<td>Glottal Gap Width</td>
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<tr>
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<td>Jacobi Matrix</td>
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<td>$K$</td>
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<tr>
<td>$P_{sub}$</td>
<td>Sub-glottal Pressure</td>
</tr>
<tr>
<td>$\Delta P$</td>
<td>Glottal Pressure Drop</td>
</tr>
<tr>
<td>$P_k$</td>
<td>Kinetic Pressure</td>
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Glottal Volume Flow Rate

Reynolds Number

Coefficient of Correlations in Vocal Fold Motion in x direction

Coefficient of Correlations in Vocal Fold Motion in y direction

Strouhal Number

Time

Cell-center Fluid Velocity in ith Direction

Face-center Fluid Velocity in ith Direction

Volume Velocity

Nodal Tissue Velocity in ith Direction

Cartesian Coordinate in the ith Direction

Glottal Flow Resistance

Strain of Tissue

Engineering Shear Strain

Tissue Viscosity

Dynamic Viscosity

Kinematic Viscosity

Material Plane Poisson Ratio

Material Longitudinal Poisson Ratio
\( \rho_f \)  
Air Density

\( \rho_s \)  
Tissue Density

\( \sigma \)  
Stress of Tissue

\( \tau_{xy} \)  
Fluctuation of flow shear stress
1.1 Phonation

Phonation – the production of sound in the larynx through the vibrations of vocal folds - is a commonly occurring biological phenomenon. Vocal folds are a pair of soft tissues located inside the larynx (shown in Figure 1-1(a)). Above these true vocal folds (also referred to simply as the vocal folds), there is another pair of soft tissues which are called the false vocal (or ventricular) folds (shown in Figure 1-1(b)). During breathing, the true vocal folds

(a) (b)

Figure 1-1 (a) Vocal tract configuration with raised soft palate for articulating non-nasal sound. (b) Coronal section of larynx and upper part of trachea (adopted from Gray’s Anatomy)
remain relaxed and the glottis (which refers to the vocal folds and the space between the folds) allows the air to pass through without any obstruction. At the onset of phonation, the vocal folds are brought to the middle (“medialized”) to obstruct the airway and the vocal folds are tightened up by the arytenoid muscles, - a process termed as “vocal folds posturing”. The lung then expels the air through the glottis and this expulsion of air tends to force apart the medialized vocal folds. When the translaryngeal pressure drop is larger than about 2-3cm of H$_2$O, the so-called phonation threshold pressure (Titze, 1988), the vocal folds experience self-sustained flow-induced vibrations. The vocal fold vibrations produce a pulsatile turbulent glottal jet flow in the supra-glottal region and this jet is the primary source of phonatory sound. For a typical human larynx the following parameters characterize the process of phonation: Mean glottal jet velocity: 30-60 m/s, mean volume flow rate 200 – 600 ml/s, Mean glottal jet Reynolds number (based on mean volume flow rate) 1000- 3000 (Titze, 1994).

According to source-filter theory (Müller, 1848), the sound produced by the glottal jet is filtered as it passed through the vocal tract (oral and nasal pharynx) and is also modified by the articulators, like lips, teeth and tongue. The outcome of this process is voice, which is used by humans for talking, singing, laughing, crying, screaming, etc. Human beings can produce different pitches or types of voice to express different meanings and emotions. The voice is composed of harmonic waves, which have one fundamental frequency (the phonation frequency F0), accompanied by harmonic overtones which are multiples of the fundamental frequency. The pitch of the human voice is determined by the fundamental
frequency, which can be altered by changing the muscle tension, pressure drop and location of vocal folds, larynx, hyoid bone, etc. (Ladefoged, 1996)

Humans can also change the states of glottis to produce different types of voices, such as unvoiced sound, breathy voice, slack voice, modal voice, stiff voice and creaky voice. For voiceless speech, the vocal folds are not postured, the glottis remains fully open. The vocal folds therefore do not vibrate and sound is only produced by the flow passing through the articulators like teeth and tongue. For example, the [s] in English is spoken voiceless. For all voiced speech, the vocal folds are fully brought to the center and vibrate at maximum amplitudes during phonation. This type of voice is called modal voice and example include the vowels [a], [e], [i], [o] and [u] and some consonants, like [m], [n], [l] and [r] in English. Breathy voice and slack voice refer to the voice in which during phonation the vocal folds still vibrate but are further apart from each other than the modal voice. An example for breathy voice is [h] in English. The stiffer voice refers to the voice in which during phonation the vocal folds are stiffer (are elongated more) and has a narrower glottal opening than modal voice. There is no example of those types of voices in English, however in other languages, like Korean, Thai, some consonants are belonging to those types (Ladefoged, 1996).

1.2 Vocal Fold Pathologies

As noted in the previous section, fluent communication requires human being to be able to produce different voices and tones and this requires precisely stretching and posturing of the vocal folds. However, sometimes due to the excessive uses of vocal folds or vocal
folds diseases, vocal folds can not be postured correctly. This can lead to an unpleasant
voice or in severe cases, to loss of ability to phonate.

1.2.1 Vocal Fold Paralysis

It is estimated that 7.5 million people in the United States have a voice disorder
(NIDCD, 2008). Based upon the clinical experience at the George Washington University
Voice Treatment Center, one-third of new patients with voice disorders are diagnosed with
vocal fold paresis or paralysis, yielding an estimate of 2.5 million people with vocal fold
paresis or paralysis in the United States. Vocal fold paralysis refers to the disease in which
the patient has a complete denervation of nerve supply to the vocal folds. This effectively
leads to an immobile vocal fold. Vocal fold paresis refers to partial or reduced nerve
activity of the vocal fold. In this case, the affected vocal fold can move, but movement is
limited. Usually vocal fold paralysis or paresis results from surgery related damage on the vocal fold nerve, virus infection and idiopathic causes. Figure 1-2 shows a laryngoscopic view of the patient’s vocal folds, where the right vocal fold is paralyzed. The patient with unilateral vocal folds paralysis will not be able to bring the vocal folds to center and there will be a gap between the two vocal folds during attempted phonation. This type of patient usually has a weak or breathy voice and may even be completely unvoiced.

1.2.2 Medialization Laryngoplasty

The common treatment for vocal fold paralysis is thyroplasty which improves the ability to medialize the paralyzed/paretic vocal fold. There are three common surgical procedures to move the vocal fold to the midline (Medialization Laryngoplasty), namely, arytenoid adduction, vocal fold injection and thyroplasty. Arytenoid adduction moves the vocal fold by moving the entire arytenoid cartilage to the center. This surgery is usually used to treat the patients who have an immobile arytenoid cartilage. Vocal fold injection places material (such as fat) into the paralyzed vocal fold to augment the vocal folds. Among these three, the most popular treatment of vocal fold paralysis is thyroplasty. This technique was first developed by Ishiki in 1974 (Ishiki, 1974). During the procedure, the surgeon opens a window in the patient’s thyroid cartilage and inserts a surgical implant into the paralyzed vocal folds to push it to the center. This surgical implant is typically comprised of a rigid material, such as Silastic, that can be modified in configuration during the surgery (Shown in Figure 1-3). A thyroplasty implant is a patient-specific device that must be properly aligned in reference to the underlying
vocal fold and has a size and shape such that it medializes the vocal fold and alters the vibratory characteristics of the vocal fold to a state that most closely resembles that of the undiseased vocal fold. During surgery, the surgeon will ask the patient to phonate after inserting the implant and based on the patient’s phonation, the surgeon will modify the size and shape of the implant. This procedure is highly depended on the experience of the surgeon and revision rates of up to 24% have been reported for this type of surgery.

Any tool that can help reduce the revision rate would have a significant impact on the treatment of patients that undergo this procedure and the current research derives its motivation from this observation. Given that phonation is mostly a biomechanical process, there is hope that appropriately constructed biophysics models of the phonation could indeed lead to a computer based tool that can assist surgeons plan this phonosurgery and reduce revision rates. The long term objective of the current research is therefore to develop a computational tool which can predict the position and configuration (shape and size) of the thyroplasty implant based on a patient’s CT scan data before the surgery. The result produced by this tool should be able to guide surgeons to produce the optimal voice outcomes with reduced trial-and-error during surgery as well as reduced revision rates. In
the near term, we expect to use this tool validated through qualitative and quantitative comparisons against available data, to gain further insights into the biophysics of phonation.

1.3 Literature Review

Phonation research is inherently multidisciplinary since it involves biomedical science, fluid mechanics and solid mechanics. From an engineering point of view, phonation is a very complex, nonlinear biomechanical phenomena that results from the interaction between glottal aerodynamics and vocal fold tissue. Phonation research has been carried out using a variety of approaches ranging from ex-vivo studies to computational modeling, theoretical analysis and experiments. In the following section, we provide a detailed overview of this research from four different aspects, namely glottal aerodynamics, vocal fold mechanics, flow-tissue interaction modeling and glottal aero-acoustics.

1.3.1 Glottal Aerodynamics

The supra-glottal flow is a highly unsteady turbulent flow and this is due to the complexity of the human airway lumen and inherent unsteadiness imposed by the vocal folds vibration. During phonation, the flow propagates through a time-varying orifice produced by periodic vocal folds vibration and forms a pulsatile jet which transitions to turbulence. Thus glottal flow has some similarities to jet flow in a confined tube. Past studies have focused on several key features of this flow, such as the pressure-flow relationship, trans-laryngeal pressure, flow resistance, vortex structure, turbulence and acoustics, and several key parameters, such as sub-glottal pressure, glottal gap width and larynx configurations.
1.3.1.1 Pressure-Flow Relationship, Translaryngeal Pressure and Flow

Resistance

For phonation, the trans-laryngeal pressure is directly related to the driven surface force for vocal fold vibration and change of volume velocity is considered to be one of the main sound sources (McGowan, 1988). So studying pressure-flow relationship is one of the main interests of phonation research. For channel or pipe flow, if flow is assumed to be irrotational, inviscid and incompressible, according to the Bernoulli equation, the pressure-flow relationship can be expressed as follows:

\[ P + \frac{1}{2} \rho \left( \frac{V_{\text{vol}}}{A} \right)^2 = C \]  \hspace{1cm} (1-1)

where \( P \) is pressure, \( \rho \) is the flow density, \( V_{\text{vol}} \) is the volume velocity, \( A \) is section area and \( C \) is constant. This indicates that the lowest pressure always occurs at smallest area section and if constant volume velocity is provided and inlet and outlet has same area, there is no pressure loss, \( \Delta p = 0 \). Thus when flow passes through the glottis, a pressure drop will be produced across the glottis and this pressure drop will push the vocal folds inward to initiate the vibration. This implies the Bernoulli effect plays a major role in vocal fold vibration. However since the glottis is very narrow compared to the trachea and supra-glottal lumen, viscous effects will have a significant effect at the glottis and the jet flow will become unsteady and transition to turbulence due to the sudden expansion downstream of the glottis. At the very least, losses due to viscous effects in the glottis as well as turbulent dissipation in the supra-glottal region, will lead to head losses. So the nature of the pressure-flow relationship under the viscous and turbulent effects needs to be examined.
The first study which addressed this question was the experimental study by Wegel (1930). In his study, a brass larynx model was built in which a straight-walled 45° converging section is connected to a 2.54 cm brass tube and there was no supra-glottal tubing. In this model, the glottal gap has a rectangular shape, approximately 1.9 cm × 0.45 cm. A series of experiments was carried with different glottal widths ranging from 0.01 to 0.10 cm. Based on his experimental results, Wegel gave the first empirical pressure-flow relationship as follows:

\[ \Delta P = (7.2 \times 10^{-6}) U A_g^{-2} d^{1.7} \rho^{-1} P_k \]  

(1-2)

where, \( \Delta P \) is the pressure drop, \( U \) is the glottal jet velocity, \( A_g \) is the area of the glottal section, \( d \) is the diameter of glottal gap, \( \rho \) is the air density and \( P_k \) is dynamic (or kinetic) pressure which is given by \( P_k = \frac{1}{2} \rho \left( \frac{V_{vol}}{A_g} \right)^2 \).

Berg (1957) carried out experiments on a more realistic static larynx model which was made by casting dental material into a normal fresh human cadaver larynx in which arytenoids were brought into the position of a closure of glottis. The glottal width of the model was adjusted by putting different sizes of closing stripes, and ranged from 0.1 cm to 3.2 cm. A constant flow condition was provided at the sub-glottal entrance. The static pressure was measured at different locations (shown in Figure 1-4). Based on the experimental results, the following empirical equation for the flow resistance, \( Z_i \), which is defined as sub-glottal pressure \( P_{sub} \) divided by volume velocity \( V_{vol} \), i.e. \( Z_i = \frac{P_{sub}}{V_{vol}} \), was obtained:
In the above equation, the first term represents the viscous loss in the glottis and second term represents the turbulent loss in the supra-glottal lumen. The total pressure drop is given as:

\[ \Delta P = 1.375P_k + 24XP_k - 0.5P_k \]  

(1-4)

In 1972, Ishizaka and Mastudaira (1972b) gave a more general expression for the pressure-flow relationship in a static larynx. In their study, they used two equations to describe the flow inside the larynx. The equation for turbulent flow was given as:

\[ \Delta P = 1.375P_k + 24XP_k - 2N(1 - N)P_k \]  

(1-5)

and the equation for laminar flow was:

\[ Z_l = \frac{12\mu h}{ld^2} + 0.875 \frac{\rho V_{vol}}{2l^2d^2} \]  

(1-3)
Following their studies, several experimental works (Scherer et al., 1981, 1983a, 1983b and Gauffin et al., 1983) have been performed to validate and compare these empirical equations. Their results showed that all of the empirical equations are valid only for a restricted range of glottal shapes, dimensions, and flows, especially only for the rectangular-shaped glottis. However, during vibration, the glottis is shown to have different shapes: convergent, rectangular and divergent (Hirano, 1977) (shown in Figure 1-5). Therefore even the static or quasi-steady glottal dynamic studies still need to include all of the different glottis shapes.

More recently Scherer and Guo (1990) gave a more general empirical equation to accommodate different glottis shapes. Based on their experimental results, the equation is given as:

\[
P^* = \left( \frac{A_1}{\text{Re}} \right) + A_2 \tag{1-7}
\]

where \(P^*\) is the pressure coefficient, which describes the relationship between the actual pressure across the larynx and pressure drop expected from the Bernoulli equation alone,
Re is the Reynolds number and $A_1, A_2$ are constants determined by the glottal shape. In their subsequent paper Guo and Scherer (1993) performed a numerical study of pressure-flow relationship for different glottal shapes. In their study, the 2D steady Navier-Stokes equations were solved using penalty finite-element methods and simulations were performed for a 2D static larynx with different glottal widths, different glottal angles (from $-40^\circ$ to $40^\circ$) and different Reynolds number. Based on the simulations, they compared pressure coefficient between numerical simulation and empirical results (Scherer and Guo, 1990) for different glottal widths and glottal angles. The results showed that the difference between the empirical solution and numerical estimates was less than $\pm 10\%$. They also provided trans-laryngeal pressure profiles for different glottal angles.

More recently, Scherer, et al. (2001) investigated the trans-laryngeal pressure profile for a symmetric and oblique glottis with a divergence angle of 10 degrees. They found that the pressure varied across the glottis for both symmetric and oblique configurations. This implies that pressure asymmetries during normal, as well as abnormal phonation, could cause phase differences between the two vocal folds. Flucher and Scherer (2006) have investigated the relationship between volume flow, geometry and pressure in a static physical model of the glottis. Based on these experiments they were able to give analytical representation of volume flow as a function of geometry and pressure.

Agarwal and Scherer (2004) investigated the effects of the false vocal folds on trans-laryngeal airflow resistance based on a static larynx model. They found that the presence of false vocal folds can reduce the trans-laryngeal airflow resistance. However, there is critical false vocal folds gap width, and when the gap width is smaller than the critical value, the
resistance will increase nonlinearly. All of studies above were able to give some characteristics of glottal flow. However, all of them were based on a quasi-steady model and phonation is a dynamic process where vocal folds movement likely has an effect on the pressure-flow relationship. Motivated by this Alipour et al. (1995, 2000) measured the pressure drop in an ex-vivo study of an excised canine larynx which was allowed to develop sustained flow-induced vibrations due to imposed sub-glottal pressure. The temporal variation of pressure measured at different locations through the glottis showed that the pressures varied along the vertical dimension of the glottis during the phonatory cycle and also varied significantly along the longitudinal direction of the glottis. The greatest pressure variation was found to be near the location of the maximum amplitude of motion (Alipour et al. 2000). Alipour et al. (2007) have also investigated aerodynamic and acoustic effects of the false vocal folds and the epiglottis in the excised larynx models. Interestingly, even though this experiment was based on measurement of flow through a vibrating model of an excised canine larynx, the effect of the false vocal folds on translaryngeal flow resistance were similar to those for a the static laryngeal model (Agarwal et al. 2004).

1.3.1.2 Flow Separation, Vortex Structure and Turbulence

As indicated in the previous section, past numerical and experimental studies have shown that viscous and turbulent effects play an important role in phonation. In order to fully understand these effects, detailed studies of the fluid field, such as flow separation, vortex structure and turbulence, are required. Furthermore, vortex structures and turbulence are also directly related to the sound field and studies of these can be used to analyze
acoustics. In the early studies (Ishizaka, 1972), the flow was assumed to be one-dimensional, the flow separation point was simply chosen at the end of the glottis, and turbulent modification for pressure of the Bernoulli equation was used beyond the separation point. This indicated a free jet at the end of glottis. However, this assumption is not suitable for diverging and converging shapes of glottis. Alipour (1991) simulated 2D glottal flow with a diverging but static glottal shape. He found that the separation point for this configurations is inside the glottis and moves downstream as the Reynolds number is increased.

Pelorson and Hirschberg (1994) investigated flow separation theoretically and experimentally. In their study, a theoretical criterion for separation prediction was provided based on Pohlhausen’s cubic method. In the experiments, asymmetric jet flow was discovered inside the supra-glottal region. This jet asymmetry, known as the Coanda effect (Coanda, 1936), has been also reported by Teager and Teager (1983), Kaiser (1983) and Alipour (1996). However Pelorson and Hirschberg (1994) argue that since this Coanda effect occurs only after the flow has fully settled down, it is not possible for this to occur in real phonation. More recent experiments of Scherer and his group (Scherer et al, 2000, Shinwari et al, 2003) have shown that inside the oblique glottis, the separation points are at different locations on the two vocal folds and the diverging vocal fold has an early separation. An asymmetric jet velocity also has been observed for a symmetric glottis in this study. They have found that even in a short transient period, the Coanda effects can still occurred.

Hofmans et al. (2003) have performed experimental and numerical studies of the flow through a static laryngeal model. In the experimental study, the unsteady pressure was
measured and flow transition to the turbulence was also captured. The pressure value at vocal fold surface was shown to have two distinctly different values when the experiments were repeated several times. This indicates that the Coanda effect did occur and the direction of asymmetric jet was random. They also found that the separation point moved downstream abruptly when the flow changed from laminar to turbulent. In the numerical study, the vortex-blob method was used to simulate two-dimensional flow and the vortex structure near the glottis was shown.

More recently, Nomura and Funada (2007) conducted a numerical study to investigate the effects of false vocal folds on sound generation. In this study, the unsteady glottal jet flow was simulated inside a two-dimensional rigid wall model of the larynx. The flow was found to be asymmetric for both with and without false vocal folds cases. They found that the existence of the false vocal folds increases the amplitude of pressure fluctuation near as well as far away from the glottis, and give rise to the broadbanding of the pressure spectrum throughout the fluid domain.

The advent of particle image velocimetry (PIV) technique has improved the ability to gain quantitative insights into the glottal flow field. Erath and Plesniak (2006a, 2006b, 2006c) studied the pulsatile flow passing a static, three-dimensional (without longitudinal shape variation), divergent glottis using this technique. They found that flow will attach to one side wall over the other at small diverging angles of 10° and 20°, corresponding to various phases of the phonation cycles. This jet also exhibited bimodal behavior: the jet randomly flipped from one side to the other over different flow cycles. For the larger divergent angle of 40°, the jet did not attach to either wall and massive flow separation
occurs above the separation point. Triep, et al. (2005) studied the flow with constant pressure drop passing through three-dimensional vocal folds with specified motion. The PIV results show that the flow attach to one side wall within one motion cycle and direction of jet shows strong variation over cycles. The phase-average flow was shown to be symmetric which indicates that the direction changing is random. In this study, the false vocal folds effects were also investigated by comparing flow in two models, one with and one without false vocal folds. They found that the radial spread of the jet is more pronounced in the case of the glottis without false vocal folds.

A more realistic model has been used by Neubauer et al. (2007) who performed experiments with a deformable Plexiglas vocal fold model. A DPIV (Digital Particle Image Velocimetry) system has been used to capture the flow structure and downstream sound has been measured using a microphone. The experiments successfully captured the coherent flow structures and transition from laminar flow to turbulent flow (shown in Figure 1-6). They found that in the laminar core region, the jet showed an unsteady Coanda effect (inclined jet center line) and flow downstream is deflected by the recirculation left by the previous cycle. In the turbulent region, a “flapping jet” was discovered due to large scale coherent structures organized in an antisymmetric array of counter-rotating vortices.
Very recently, Drechsel and Thomson (2008) have investigated the influence of the supra-glottal structure on the glottal jet exiting a two-layer synthetic, self-oscillating vocal fold model with a PIV system. They have found that presence of false vocal folds obstructed the downstream convection of the starting vortex and jet core centerline was closer to the channel center when compared to the case without false vocal folds. The velocity fluctuations have also been found to be decreased with the presence of false vocal folds.

### 1.3.2 Vocal Fold Mechanics

Phonation involves the interaction between glottal flow and vocal fold tissue. Vocal folds anatomical structures, mechanical properties and initial stress are the key factors to determine the fundamental phonation frequencies and vocal fold vibration pattern. Also the pathological outcomes of abnormal vocal folds are usually related to the changes of mechanical properties or initial stress of vocal folds. Anatomically vocal fold is composed
of three different tissue layers. From superficial to deep, they are epithelium, lamina propria and vocalis muscle as shown in (Hirano, 1975, 1977, 1981). The epithelium layer contains softer, fluid-like tissue encapsulated by an elastic membrane. The lamina propria can conveniently be divided into three layers: superficial, intermediate and deep. The superficial layer is made of loosely organized elastin fiber surrounded by interstitial fluid. The intermediate layer is composed of uniformly organized elastin fiber oriented in the anterior-posterior direction and collagen fibers. The deep layer is mainly made of collagen fibers (Titze, 1994). The muscle layer is composed of the thyroarytenoid muscle. Just like other skeleton muscles, thyroarytenoid muscle has a hierarchical bundle structure: muscle - muscle fascicles - muscle fiber- myofibril- myofilaments - myosin and actin, from macro scale to micro scale. According to the physiology, two different labeling schemes have been employed to group the vocal fold tissue layers. In the three layer model, the cover consists of the epithelium and the superficial layer of the lamina propria, the transition consists of the intermediate and deep layers of the lamina propria, the body is the thyroarytenoid muscle (Hirano, 1974, 1975, 1977, 1981, 1993). In the two-layer scheme, the body consists of deep layer of the lamina propria and muscle and the cover layer consists of epithelium, superficial and intermediate layers of the lamina propria (Hirano, 1985).
The thickness of different layers also has variations along anterior-posterior direction. The ligament layer is thicker at the two ends to withstand the massive stress and the mucosa layer is thicker in the middle to act as a cushion for vocal folds collision (Hirano, 1981). Like other human soft tissues, vocal fold tissue is believed to be composed of nonlinear viscoelastic material (Fung, 1993). Since the ligament and muscle layers have uniform fiber direction, they can be also considered as transversal isotropic material.

The mechanical property and constitutive law of different layer of vocal folds tissue has been studied since the 1980s. Kakita et al. (1981) measured the mechanical properties of epithelium, lamina propria and muscle of excised canine vocal folds using a low (~0.1Hz) frequency load. However, due to the questionable viability of their tissue which was excised post mortem, their experimental data showed a wide range and they were only able to determine the order of the elastic modulus. In the following studies at the University of Iowa (Perlman, et al. 1984, 1985a, 1985b, Alipour, et al. 1991), elastic properties of cover...
and body layer of canine vocal folds were measured in-situ. The nonlinear strain-stress relationship was captured and fitted by polynomial and exponential models. The mechanical property constants at low-strain, which can be considered as linear elastic, were given. The data they reported were about two orders of magnitude smaller than the first data reported by Kakita, (1981). Since the ligament layer does not exist for canines, the ligament properties were measured for human being by Min (1995).

Due to the viscoelastic nature of vocal folds, quantification of the viscoelastic properties is also important for vocal fold vibration analysis. One of the early studies is the measurement of canine vocalis muscle viscoelastic properties by Alipour and Titze (1995). In subsequent studies Chan, et al. (1999, 2004) measured viscoelastic properties of human mucosa subject to low (less than 50 Hz) frequency sinusoidal loading as well as their shear elastic modulus and dynamic viscous shear modulus. The viscous modulus was shown to be frequency dependent. In order to extrapolate the viscous modulus at the phonatory frequency (more than 100 Hz), qualinear viscoelastic constitutive modeling (Fung, 1993) and statistical network constitutive modeling (Zhu et al., 1991) was used to reconstruct the dynamic viscous shear modulus and frequency curve based on the empirical data (Chan et al., 2000, Zhang et al., 2006). More recently, Zhang et al. (2008) proposed a biphasic theory modeling, which modeled the fluid-solid interaction inside tissue to study the viscoelastic behavior of vocal folds. All of studies above were able to give some characteristics of vocal fold properties. However due to limited tissue samples and large intersubject property variations - for instance 2 or 3 orders of magnitude difference shown by Chan, et al. (1999) - the vocal tissue properties and constitutive modeling are still an open questions.
An additional key factor that determines the phonatory frequency is the initial stress of vocal folds, which is controlled by vocal folds muscle activities. A study has shown that different muscle activities produced different fundamental phonation frequencies (Titze, 1988). However the relationship between muscle activation and fundamental phonation frequency is still not answered.

In order to capture the dynamic characteristics of vocal folds, the eigen mode analysis has been tempted. Berry and Titze (1996) investigated the eigen frequencies and eigen modes of vocal folds with a finite element method. In their study, the vocal fold was a simple three dimensional brick shape and only made of one transverse isotropic material without consideration of the three-layer structure. A linear elastic constitutive law was used and fixed boundary constraints given to the anterior posterior and lateral wall. The first three eigen modes are shown in Figure 1-8, which correspond to the lowest three eigen frequencies. The first eigen mode represents the vertical motion of vocal folds (superior and inferior direction in anatomical position). The second mode is the lateral motion of vocal folds which presents vocal fold adduction and abduction. The third mode is also the lateral motion with 180° phase difference between the top and bottom. This motion represents the so-called mucosal wave (Titze, 1994) traveling on the vocal fold surface.

![Figure 1-8](image)

Figure 1-8 The lowest eigen modes at the middle plane. (adopted from Berry, 1996) (a) Mode-1 (b) Mode-2 (c) Mode-3.
The real vocal fold motion (shown in Figure 1-5) can be approximated well through a combination of mode-2 and mode-3. This means that during phonation both mode-2 and mode-3 occur as part of vocal motion at the same frequency. This phenomenon is called one-to-one mode entrainment (Zemlin, 1988). The tissue incompressibility effect was examined by Berry and Titze (1996). They found that as the material approaches the incompressible state, the frequency of mode-2 decreases and that of mode-3 increases. At the nearly incompressible region, mode-3 and mode-2 switch their position in the eigen-mode sequence.

Following this study, a more realistic vocal fold shape which also has a thickness variation along longitudinal direction was used and three-layer structure was presented with different material properties based on experimental data (Alipour et al. 2000). The eigen modes reported in this study are similar to the study by Berry and Titze (1996). More recently, the geometric parametric study (Cook et al. 2006) was performed on the same model of Berry and Titze (1996). The relationship between frequency and vocal fold geometry, thickness, length and depth was measured numerically and results show that the length of vocal folds has a more dominant effect on the eigen frequencies (Cook et al. 2006).

1.3.3 Flow-Tissue Interaction Modeling

As stated in the previous two subsections, the glottal aerodynamics and tissue mechanics have been investigated extensively. These results have revealed some interesting characteristics of phonation. However, full understanding of phonation requires modeling the interaction between glottal aerodynamics and vocal folds tissue. The first flow-tissue
interaction model for phonation was proposed by Flanagan and Landgraf (1968). In their model, one lumped mass was connected to a spring and dashpot to represent the vocal fold, a model which was later termed as the “one-mass model”. The empirical expression (Eq. 1-3, 1-4) from Berg (1957) was used for glottal flow and this was coupled with the one-mass vocal fold model. In spite of the simplicity of this model, self-sustained vibrations in the modeled vocal folds were captured and the model produced a plausible glottal wave form. However, since the vocal fold model only has one degree of freedom, the shear stress effects could not be represented by this model and it could not predict the traveling mucosa wave.

![Schematic diagram of two-mass model approximation of vocal fold](image)

Figure 1-9 Schematic diagram of two-mass model approximation of vocal fold
(adopted from Ishizaka, 1972)

Shortly after the one-mass model was presented, the seminal work of Ishizaka and Flanagan (1972) on the two-mass model was published. In this model, as shown in Figure 1-9, two masses were connected by a spring and each of them was connected to a spring and dashpot. Thus, shear effects could be represented by the spring connecting the two masses. The Bernoulli equation with Berg’s viscous and turbulent modification terms (Berg, 1957) was used for fluid modeling. With this model, the self-sustained vibration was
captured and the glottal wave form showed an asymmetric shape (comparing with the symmetric shape wave produced by one-mass model), which indicates a longer vocal fold opening time and a shorter closing time. The vibration pattern produced by this model was found to be very similar to the motion captured by high-speed pictures (Farnsworth, 1940).

Material nonlinearity was also investigated in this study through the inclusion of nonlinear springs which had a cubic force-displacement law. They found that within the low sub-glottal pressure region (less than 10 cm H2O), phonation frequency increased with pressure and nonlinearity had little impact on phonation frequency. However under higher sub-glottal pressure, the frequency was found to increase with sub-glottal pressure and higher nonlinearity produced higher frequencies. For the linear spring case, the frequency does not change with sub-glottal pressure. This indicates that material nonlinearity plays an important role for high pitch phonation, like falsetto registration (Titze, 1994).

Following the two-mass model, higher dimensional lumped mass models were also studied. For example, a 16-mass model has been used to present three dimensional vocal folds (Titze, 1973, 1974). In this model, three dimensional vocal fold is made of eight sections and each section is made of two lumped mass and springs, representing the body and cover layer separately. This model was coupled with the 3D wave equation and 3D self-sustained vibrations were captured with this model. For this model, each cross section only two lumped masses were include, this was not enough to present the mucosa wave traveling along the surface, which is represented by a phase difference in the superior and inferior direction.

In order to capture the mucosal wave with the presence of cover-body structure, a three-mass model was proposed by Story and Titze (1993). In this model, two
interconnected lumped masses are used to represent the cover layer and one lumped mass was used to represent the body layer. An empirical equation (Ishizaka and Matsudaira, 1972) has been used for modeling the fluid. The simulation of this model showed sustained vibration and the intra-glottal pressure was shown in this study.

While the above discussion shows the improvements made in modeling of the vocal fold mechanics, the modeling of glottal aerodynamic has also undergone a steady improvement in complexity and fidelity. LaMar et al. (2003) used the 1-D Euler equation with viscous modification from an empirical equation and coupled it with a two-mass model. Duncan et al. (2006) coupled a 2-D unsteady Naiver-Stokes immersed boundary solver with a two-layer material vocal fold model, where the vocal folds are modeled as a two-layer system of particles. Tao et al. (2007) also coupled the unsteady 2D Navier-Stokes equations with a two-mass vocal fold model. In this study, the so called Coanda effects in the glottal jet were successfully captured and they showed that this also triggered the asymmetric vocal fold vibrations.

A more realistic model has been employed by Rosa et al. (2003). In this study, the 3D continuum transversal isotropic viscoelastic vocal folds model was coupled with 3D steady Navier-Stokes equation and the simulation has been performed in an ideal cylindrical pipe with modeled false vocal folds. The self-sustained divergent and convergent vocal folds vibrations were captured. However due to low grid resolution, no detailed fluid field information was provided in this study.

More recently, in our group, the Navier-Stokes fluid model was coupled with 2D linear elastic solid model using immersed boundary method for both fluid and solid solvers (Luo et al. 2008). The “flip-flop” behavior of glottal jet has also been captured in this study.
However, because this method solves the solid equation at fixed Eulerian grid, it is relatively difficult to extend this method to large deformation problem and history dependent materials.

In addition to fluid-structural interaction (FSI) simulations, eigen studies of the coupled FSI system have also been attempted. One of the early studies was performed by Ishizaka on his two-mass model (Ishizaka, 1981). In his study, it was found that with increase of glottal jet velocity, the frequencies of mode 2 (adduction and abduction motion) and mode 3 (mucosal wave) approach each other and finally merge to one frequency. This captured a critical jet velocity of 1:1 entrainment of vocal folds vibration. More recently, Zhang et al. (2006) performed FSI eigen analysis on a continuum 2D vocal fold model coupled with the 1D Bernoulli equation. In this study, the relationship between the glottal jet velocity and the presence of 1:1 entrainment was again captured. A proper-orthogonal decomposition (POD) was also employed to examine the vibration pattern and its modulation by the jet velocity. Also based on the eigen analysis of the one-mass model and the 1D Bernoulli equation, Jiang (2007) and Tao (2008) provided the critical condition for the initiation of vocal fold oscillation.

1.3.4 Glottal Aero-acoustics

During phonation, vocal folds act as a generator of sound which is then modified by the upper airway, like nasal passage and oral cavities. Thus, sound modeling inside the larynx is needed for both phonation and speech studies. One of the early sound modeling studies is the theoretical study by McGowan (McGowan, 1988). In this study, a cylindrical tube was used to represent the vocal tract and Howe’s equation (Howe, 1975) was solved by
integration over the tube. It was found that sources of sound in phonation can not be completely characterized by the volume velocity, which is corresponding to a monopole source, and a dipole source also needs to be considered. The dipole source results from the unsteady forcing applying by the solid boundary on the fluid. A more realistic model with which includes the effect of vocal fold like obstruction has been used in the numerical study by Zhao et al. (2002). In this study, 2D unsteady, compressible the Navier-Stokes equations were solved with a high-order finite-volume scheme in a 2D larynx, wherein the of the vocal folds has a specified sinusoidal motion. The strength of the monopole, dipole and quadrupole sources was calculated according to the acoustic analogy (Zhao et al. 2001). They found that the dipole sources come from an unsteady force from the wall and may be dominant at low frequencies. The monopole source was found to be significant at higher frequencies above about 400 Hz. In their subsequent study (Zhang et al. 2002), false vocal folds were added to the model and the dipole source strength was found to increase due to the interaction between air and false vocal folds walls. This phenomenon has been also observed by Drechsel and Thomson (2008) in their PIV experimental study.

1.4 Statement of Purpose

Detailed reviews are given in the previous sections and these reviews provide a good view of the history and the current status of this field. Here we provide a concise summary of this research, especially those studies that are relevant to the current research. Experiments in this arena can be categorized based on the approaches that have been employed and these various approaches are summarized in the table below:
Table 1-1 various experimental approaches employed for phonation modeling

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<th>Simple/Low-Fidelity</th>
<th>Medium Fidelity</th>
<th>Complex/ High-Fidelity</th>
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</thead>
<tbody>
<tr>
<td>Glottal Aerodynamics</td>
<td>No Flow</td>
<td>Steady Inflow</td>
<td>Pulsatile Inflow</td>
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<td></td>
<td></td>
<td>VF driven Flow</td>
<td></td>
</tr>
<tr>
<td>Vocal Fold Model</td>
<td>Stationary</td>
<td>Prescribed Motion</td>
<td>Flow-Induced Vibration</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geometry</td>
<td>Nominally Two-Dimensional</td>
<td>Three-Dimensional (Axisymmetric)</td>
<td>Three-Dimensional (Non-Axisymmetric)</td>
</tr>
</tbody>
</table>

Currently, most experimental studies are focused on three-dimensional turbulent flow with either stationary or deformable vocal folds. For these experimental studies, unsteady turbulent flow can be investigated in detail with PIV technique and Reynolds number can be kept same with the realistic physiological Reynolds number. However, there are some difficulties, generally encountered in the experiments. In-vivo vocal fold samples are difficult to obtain and to keep viable. Also it is difficult to find synthetic materials which have mechanical properties similar to vocal fold tissues, especially with the three-layer vocal fold inner structures. Furthermore, it is difficult to perform an experiment in a realistic shaped airway. In addition, experimental methods cannot be applied for clinical diagnosis or treatment.

For computational methods, realistic material properties and airway shape can be in principle be implemented with relative ease. Furthermore, for our purposes, computational methods also can be employed for directly clinical diagnosis or treatment. Thus computational modeling is a good substitute and/or supplement for experimental studies. As indicated in previous section, there are two key constituents in a computational model of phonation, the model for the glottal aerodynamics and the model for vocal-fold deformation and vibrations. A wide array of approaches have been used for both of these
that span the range from simple, low-fidelity to complex, high-fidelity approaches. The table 1-2 provides an overview of the different types of approaches that have been used to date:

<table>
<thead>
<tr>
<th></th>
<th>Simple/Low-Fidelity</th>
<th>Medium Fidelity</th>
<th>Complex/ High-Fidelity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Glottal Aerodynamics</strong></td>
<td>None (no fluid flow)</td>
<td>Bernoulli Equation</td>
<td>Inviscid Flow</td>
</tr>
<tr>
<td><strong>Vocal Fold Deformation &amp; Vibration</strong></td>
<td>Stationary</td>
<td>2 Mass Model</td>
<td>&gt;2 mass model</td>
</tr>
<tr>
<td><strong>Dimensionality</strong></td>
<td>One dimensional</td>
<td>Two Dimensional</td>
<td>Three Dimensional</td>
</tr>
</tbody>
</table>

In the past, a modified two-dimensional Bernoulli Equation coupled with two-mass model is usually employed for computational modeling. Very recently, a few studies (Tao et al. 2007, Duncan et al. 2006) have employed two dimensional Navier-Stokes coupled with lumped mass model. For three-dimensional modeling, only one work has been documented (Rosa et al. 2003). Due to low grid resolution, no fluid field information was provided in this work. These models are able to capture some of the basic characteristics of phonation, they cannot provide quantitative results of a quality and fidelity required for clinical diagnosis or treatment. Furthermore, these methods provide only limited insights into the biophysics of phonation. In the current study, a new, high-fidelity computational tool has been developed for modeling the biophysics of phonation in all its complexity. While the immediate objective is to use this tool to gain new insights into the biophysics of phonation, the long term goal of this project is develop a tool for planning laryngeal surgery.
The key components of the tool include (a) an immersed boundary method (IBM) based Navier-Stokes solver for modeling the glottal flow, (b) a finite-element method (FEM) based solver for the viscoelastic deformation of vocal-fold tissue, (c) a penalty coefficient method for modeling the vocal-fold contact and (d) a loosely coupled IBM-FEM methodology for modeling fluid-structure interaction in the human larynx. In addition, a high resolution CT scan has been used to provide guidance for construction of a realistic laryngeal model.
CHAPTER 2 NUMERICAL METHOD

Modeling of phonation requires a coupled fluid-solid solver that can handle the complex geometry and motion associated with phonation. Here an existing immersed boundary solver, ViCar3D, originally developed in our group (Mittal, 2008), is chosen for modeling the glottal aerodynamics. This solver is coupled with a newly developed viscoelastic solid mechanics solver. Both the solid mechanics solver as well as the coupling between fluid and solid solvers has been developed as part of the current research and these will be discussed in detail in subsequent sections. However, for the sake of completeness we describe concisely, some salient features of the ViCar3D immersed boundary solver. The current discussion is based significantly on Mittal et al. (2008).

2.1 Fluid Dynamics Solver

As mentioned in section 1.3.1, the glottal flow is highly unsteady due to the complexity of the human airway and inherent unsteadiness imposed by periodic vocal fold vibration. Because typical glottal jet velocity is about 30m/s which corresponds to a Mach number less than 0.1, the flow can safely be assumed to be incompressible. The governing equations for this type flow are unsteady the incompressible Navier-Stokes equations:

\[
\frac{\partial \mathbf{u}_i}{\partial x_i} = 0 \tag{2-1}
\]

\[
\frac{\partial \mathbf{u}_i}{\partial t} + \frac{\partial (u_i \mathbf{u}_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial}{\partial x_j} \left( \frac{\partial \mathbf{u}_i}{\partial x_j} \right) \tag{2-2}
\]

where \( i, j = 1, 2, 3 \), \( u_i \) are the velocity components, \( p \) is the pressure, and \( \rho \) and \( \nu \) are the fluid density and kinematic viscosity.
Since this is a moving boundary problem with a very complex geometric shape, the numerical solver must be able to quickly resolve the boundary motion and easily handle the grid generation for complex boundary shapes. This requirement makes the immersed boundary method (IBM) a perfect candidate for the fluid solver. Immersed boundary method was first proposed by Peskin (Peskin, 1972) to model cardiac mechanics. By solving equations on a fixed Cartesian grid, the immersed boundary method results in great simplicity in terms of grid generation. Thus it is more suitable for moving boundary problems and complex flows compared to traditional boundary conformal grid methods (Mittal and Iaccarino 2005). Due to this advantage of IBM, it has been widely used to simulate biological flows which usually involves significant motion/deformation and flow-tissue interaction (Mittal, 2006, Vargas, 2008, von Loebbecke, 2008, Zheng, 2008).

2.1.1 Finite Difference Scheme

The Navier-Stokes equations (2-2) are discretized in space using a cell-centered collocated (non-staggered) arrangement of the primitive variables \( u \) and \( p \). In addition, the face-center velocities, \( U \), are computed and stored (see Figure 2-1). This separate computation of face velocity, which was initially proposed by Zang et al. (1994), results in discrete mass-conservation to machine accuracy and leads to a more accurate and robust solution procedure. The fractional-step method of Van-Kan (1986) is used to integrate the equations in time, which consists of three sub-steps. In the first sub-step, a modified momentum equation is solved to get an intermediate velocity \( u^* \). A second-order, Adams-Bashforth scheme is employed for the convective terms while the implicit Crank-Nicolson
scheme is used to discretize the diffusion terms and this eliminates the viscous stability constraint. The following modified momentum equation is solved at the cell centers.

\[
\frac{\delta U^*}{\delta t} - \frac{\delta p^*}{\delta x_i} + \frac{1}{2} \left[ 3N_i^n - N_i^{n-1} \right] = -\frac{1}{\rho} \frac{\partial p^*}{\partial x_i} + \frac{1}{2} \left( D_i^* + D_i^{n*} \right)
\]  

(2-3)

where \( N_i = \frac{\delta(U_j u_j)}{\partial x_j} \) and \( D_i = \nu \frac{\delta}{\delta x_j} \left( \frac{\partial u}{\partial x_j} \right) \) are the convective and diffusive terms respectively, and \( \frac{\delta}{\delta x_j} \) represents a second-order central difference. The intermediate face velocities \( U^* \) is computed by averaging the neighboring intermediate cell-center velocities \( u^* \) and only the face velocity component normal to the cell-face is calculated and stored.

Figure 2-1 Schematic describing the name conventional and location of velocity components employed in the spatial discretization of the governing equation. (adopted from Mittal et al. 2008)

The averaging procedure is as follows:
\[
\tilde{u}_i = u^*_i + \Delta t \frac{1}{\rho} \left( \frac{\partial \tilde{p}^*}{\partial \tilde{x}_i} \right)_{cc}
\]  
(2-4)

\[
\tilde{U}_1 = \gamma_w \tilde{u}_{1p} + (1 - \gamma_w)\tilde{u}_{1w}
\]  
(2-5)

\[
\tilde{U}_2 = \gamma_s \tilde{u}_{2p} + (1 - \gamma_s)\tilde{u}_{2s}
\]  
(2-6)

\[
\tilde{U}_3 = \gamma_b \tilde{u}_{3p} + (1 - \gamma_b)\tilde{u}_{3b}
\]  
(2-7)

\[
U^*_i = \tilde{U}_i - \Delta t \frac{1}{\rho} \left( \frac{\partial \tilde{p}^*}{\partial \tilde{x}_i} \right)_{fc}
\]  
(2-8)

where \(\gamma_w\), \(\gamma_s\) and \(\gamma_b\) are the linear interpolation weights for the west, south and back face velocity components respectively. In the above equations, \(cc\) and \(fc\) stands for cell-center and face-center respectively. This procedure eliminates odd-even decoupling that occurs with non-staggered methods, which will produce large pressure variation in space.

The second sub-step is the pressure correction equation:

\[
\frac{u_{i}^{n+1} - u^*_i}{\Delta t} = -\frac{1}{\rho} \frac{\partial \tilde{p}^*}{\partial \tilde{x}_i}
\]  
(2-9)

Integrating the equation (2-9) over a single cell and put the mass conservation constraint over the cell for \(n+1\) time step. The resulting pressure Poisson equation is obtained as follows:

\[
\frac{1}{\rho} \frac{\delta}{\delta \tilde{x}_i} \left( \frac{\delta p^*}{\delta \tilde{x}_i} \right) = \frac{1}{\Delta t} \frac{\delta U^*_i}{\delta \tilde{x}_i}
\]  
(2-10)

The third sub-step is the pressure and velocities update. The pressure and velocities (cell-center, face-center) are updated separately. The procedure is as follows

\[
p^{n+1} = p^n + \tilde{p}^*
\]  
(2-11)

34
\[
\begin{align*}
    u_i^{n+1} &= u_i^* - \Delta t \frac{1}{\rho} \left( \frac{\partial p^*}{\partial x_i} \right)_{cc} \\
    U_i^{n+1} &= U_i^* - \Delta t \frac{1}{\rho} \left( \frac{\partial p^*}{\partial x_i} \right)_{fc}
\end{align*}
\] (2-12) (2-13)

2.1.2 Immersed Boundary Representation

Since the immersed boundary method does not require the mesh to be body conforming, imposition of boundary condition is the key feature of immersed boundary method. Immersed boundary methods have been categorized into two different groups (Mittal, 2005), smeared-interface methods and sharp-interface method. For smeared-interface methods, boundary conditions are implemented by adding a forcing term into continuous Navier-Stokes equations before they are discretized. This method is relative independent of spatial discretization due to the formulation and it can be easily employed for existing Navier-Stokes solver. However, the main drawback is that this method will create a smeared (or diffuse) immersed boundary.

For sharp interface methods, the boundary conditions are specified at the exact location of the boundary and there is no smearing of the boundary. However, the construction of this methodology is intrinsically connected with the spatial discretization and therefore, more effort is required incorporate this method into an existing Navier-Stokes solver. The current falls into sharp-interface methods category. The solver employs a 3D ghost-cell methodology for prescribing the boundary conditions on the immersed boundary (Mittal, 2008). In this method, the body surface is represented by unstructured grid with triangular elements and this surface is immersed into the Cartesian volume grid. The Cartesian cells
are solid-cells or fluid-cells depending on whether they are inside or outside the immersed body. This method proceeds by identifying the ghost cells (denoted by "GC") which are solid cells which have at least one fluid cell neighbor. A "probe" is then extended from one of these ghost cells onto an "image-point" (denoted by “IP”) inside the fluid such that it intersects normal to the immersed boundary and the boundary intercept (denoted by "BI") is midway between the ghost-node and the image-point. Next a bi-linear interpolation (trilinear in 3D) is used to express the value of a generic flow variable at the image-point in terms of the surrounding nodes. Following this, the value of variable at the ghost-cell is computed by using a central-difference approximation along the normal probe such that the prescribed boundary condition at the boundary intercept is incorporated. Using this procedure, the boundary conditions are prescribed to second-order accuracy and this, along with the second-order accurate discretization of the fluid cells leads to local and global second-order accuracy in the computations. The formulation for Dirichlet and Neumann boundary conditions are shown as follows

\[ \phi_{IP} = \sum \beta_i \phi_i \]  

(2-14)

\[ \phi_{GC} + \sum \beta_i \phi_i = 2 \phi_{BI} \]  

(2-15)

\[ \phi_{GC} - \sum \beta_i \phi_i = \Delta \left( \frac{\delta \phi}{\delta n} \right)_{BI} \]  

(2-16)
Where $\phi$ is the generic flow variable, $i$ is from 1 to 4 (for 2D) or 8 (for 3D) and represents the $i$th surrounding node for IP, $\beta$ is the interpolation weight and $\Delta l$ is the probe length.

Boundary motion can now be included into this formulation quite easily. Since the equations are written in the Eulerian form, the boundary can be moved at a given time-step, the body-intercepts and image-points recomputed and subsequently the flow is advanced in time. The boundary motion is accomplished by moving the nodes of the surface triangles in a prescribed manner. The general framework can therefore be considered as Eulerian-Lagrangian, wherein the immersed boundaries are explicitly tracked as surfaces in a Lagrangian mode, while the flow computations are performed on a fixed Eulerian grid.

One issue associated with moving boundary case for sharp interface method is the so-called "fresh cell" problem. "Fresh cell" refers to the cell that was solid cell at previous
time step and became fluid cell at current time step due to the boundary motion. As shown in Figure 2-3, two fresh-cells have been created due to the boundary motion. For these cells, the convective term $N_i^n, N_i^{n-1}$, pressure gradient $\partial p^n / \partial x_i$ term and diffusive term $D_i^n$ from previous time step are not available. In order to get values for those terms, a procedure, similar to the ghost cell calculation, has been adopted here (Mittal et al. 2008). A probe is place from the fresh-cell toward and perpendicular to the new boundary surface. Then this probe is extended in the opposite direction into the fluid to create an image point, IP. The value of image pint will be interpolated from surrounding nodes (expect the fresh-cell) and boundary incept point BI, the gray region shown in Figure 2-3. Once the value of IP is obtained, the value of fresh-cell can be interpolated between IP and BI.

Figure 2-3 Schematic showing the formation of fresh-cells due to boundary motion and the interpolation stencil (in gray) for one representative fresh-cell
2.1.3 Fast Solution Procedure and Validation

Line-SOR method has been employed as the underlying solver for both convection-diffusion (Eq. (2-3)) and pressure Poisson equation (Eq. (2-10)). Due to its elliptic nature, the Poisson equation usually takes much longer time to converge than the convection-difference equation and the performance of the whole solver is highly dependent on the performance of Poisson equation solver. For incompressible flow, especially when the moving boundary problems are solved, the performance of Poisson solver is usually considered to be the bottle neck and determines how large a problem can be simulated with this solver.

In order to save the computational time, a geometric multi-grid method has been developed to solve Poisson equation (Bozkurttas, et al., 2005, Dong et al. 2006). In this method, the sharp interface immersed boundaries are represented only at finest grid level.
At the coarse grid levels, the interfaces are represented through the volume fraction of coarse cells without boundary reconstruction. The performance for this method has been tested on a 2.8 GHz Pentium-IV cluster for a canonical case (two circular cylinders immersed in a square computational domain (Bozkurttas et al., 2005). The performance of current multi-grid method has been compared with PetSc, which is a sophisticated GMRES based solver (Balay et al., 2003). As shown in Figure 2-4, the current multi-grid method showed a performance that CPU times scaled $N^{1.17}$ to total number of grids. Ideally the Jacobi and Gauss-Seidel iterative method has a $N^2$ performance. The PetSc package showed a $N^{1.78}$ performance and is better than the Jacobi and Gauss-Seidel. The current multi-grid is much better than the PetSc method and is very close to the ideal multi-grid performance $N$.

Figure 2-5 $L_1$, $L_2$, and $L_\infty$ norms of the error for the streamwise velocity $u_1$ and transverse velocity $u_2$ components versus the computational grid size for flow passing 2D cylinder. (taken from Mittal et al. 2008)
The fluid solver itself has been extensively validated with different canonical cases, such as flow past 2D cylinder, flow past 3D sphere, flow passing suddenly accelerated cylinder. The details can be found in Mittal et al. (2008). The solver also has shown both local and glottal second-order accuracies (shown in Figure 2-5).

2.2 Solid Mechanics Solver

A new 3D solid solver has been developed and this has been coupled with the IBM fluid solver described in the previous section. For this solver, the finite-element method is employed to model the multi-layer viscoelastic vocal folds and a penalty coefficient contact model has been integrated to model the vocal fold collision effects. The details of this solid solver are discussed in the following subsections.

2.2.1 Constitutive Law

As shown in the section 1.3, vocal folds are made of multi-layered, nonlinear viscoelastic materials. However, during normal phonation, vibration only causes a small deformation in the vocal folds and the linear relationship assumption between stress and strain and strain rate is valid due to this small deformation (Alipour 2000). Thus the stress can be expressed as a linear combination of strain and strain rate.

\[ \bar{\sigma} = \bar{C} \bar{\epsilon} + \bar{A} \bar{\dot{\epsilon}} \]  

(2-17)

where stress \( \bar{\sigma} \), strain \( \bar{\epsilon} \) and strain rate \( \bar{\dot{\epsilon}} \) are second-order tensors. For small deformation, \( \epsilon \) and \( \dot{\epsilon} \) can be written as follows:

\[ \bar{\epsilon} = \frac{1}{2} \left( \nabla \bar{d} + \left( \nabla \bar{d} \right)^T \right) \]

(2-18)
\[
\ddot{\epsilon} = \frac{1}{2}\left(\ddot{\vec{V}} + (\dddot{\vec{V}})^T\right)
\]  

(2-19)

where \(\ddot{d}\) is the displacement vector and \(\vec{V}\) is the velocity vector. Due to the symmetry of stress, strain and strain rate tensor, only 6 components of the tensor are independent for each tensor.

The elastic constant coefficient \(\tilde{C}\) and viscous constant coefficient \(\tilde{A}\) are fourth-order tensors with 81 constants (Everstine, 2008). For a transversally isotropic material, the number of the constants of elastic coefficient \(C\) reduces to 5, due to the symmetry of strain, stress and materials (Everstine, 2008). If the six components of stress tensor are rearranged into a vector form, for the transversal isotropic elastic material, the relationship between strain and stress can be written as follows:

\[
\begin{bmatrix}
\epsilon_{xx} \\
\epsilon_{yy} \\
\epsilon_{zz} \\
\gamma_{yz} \\
\gamma_{zx} \\
\gamma_{xy}
\end{bmatrix} =
\begin{bmatrix}
1 & -\frac{v_p}{E_p} & -\frac{v_{zp}}{E_z} & 0 & 0 & 0 \\
\frac{v_p}{E_p} & 1 & -\frac{v_{zp}}{E_z} & 0 & 0 & 0 \\
\frac{v_{zp}}{E_p} & \frac{v_{zp}}{E_p} & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{zp}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{zp}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{zp}}
\end{bmatrix}
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\tau_{yz} \\
\tau_{zx} \\
\tau_{xy}
\end{bmatrix}
\]  

(2-20)

where the x-y plane is the symmetry plane, z is the longitudinal direction. \(\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}\) are the normal strains in x, y, z direction, \(\gamma_{yz}, \gamma_{zx}, \gamma_{xy}\) are the engineering shear strains in y-z plane, z-x plane, and x-y plane, which are equal to \(2\epsilon_{yz}, 2\epsilon_{zx}, 2\epsilon_{xy}\) respectively,
\( \sigma_{xx}, \sigma_{yy}, \sigma_{zz} \) are the normal stresses in x, y, z direction, \( \tau_{yz}, \tau_{zx}, \tau_{xy} \) are the shear stresses in y-z plane, z-x plane, and x-y plane. Furthermore, \( E_p \) is the plane Young’s modulus, \( \nu_p \) is the plane Poisson’s ratio, \( E_z \) is the longitudinal Young’s modulus, \( \nu_{pz} \) is the longitudinal Poisson ratio and \( G_{zp} \) is the shear modulus in the longitudinal direction. The constant \( \nu_{pz} \) is determined through the following compatibility condition:

\[
\frac{\nu_{pz}}{E_p} = \frac{\nu_{zp}}{E_z} \tag{2-21}
\]

If the stress is written in term of strain, the equation (2-20) will become:

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\tau_{yz} \\
\tau_{zx} \\
\tau_{xy}
\end{bmatrix} = \begin{bmatrix}
\frac{1 - \nu_{pz} \nu_{zp}}{E_p E_z} & \frac{\nu_p + \nu_{pz} \nu_{zp}}{E_p E_z} & \frac{\nu_{zp} + \nu_{zp} \nu_p}{E_p E_z} & 0 & 0 & 0 \\
\frac{\nu_p + \nu_{pz} \nu_{zp}}{E_z E_p} & \frac{1 - \nu_{zp} \nu_{pz}}{E_z E_p} & \frac{\nu_{zp} + \nu_{pz} \nu_p}{E_z E_p} & 0 & 0 & 0 \\
\frac{\nu_{pz} + \nu_{pz} \nu_p}{E_{pz} E_z} & \frac{\nu_{pz} (1 + \nu_p)}{E_{pz} E_z} & \frac{1 - \nu_p^2}{E_{pz} E_z} & 0 & 0 & 0 \\
\frac{E_{pz}}{E_p} & \frac{E_{pz}}{E_p} & \frac{E_{pz}}{E_p} & 0 & 0 & G_{zp} \\
0 & 0 & 0 & 0 & 0 & G_{zp} \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\gamma_{yz} \\
\gamma_{zx} \\
\gamma_{xy}
\end{bmatrix} \tag{2-22}
\]

where \( \Delta = \frac{(1 + \nu_p)(1 - \nu_p - 2\nu_{pz} \nu_{zp})}{E_p E_z} \)

According to Alipour (2000) and Rosa (2003) the viscous coefficient \( A \) can be modeled by the tissue viscosity \( \eta \). Thus for a purely viscous material, the relationship between the stress and strain rate can be written as follows
Thus for transversal isotropic viscoelastic material, stress is composed of two parts: the elastic stress and the viscous stress. This model is called the Kelvin-Voigt model (Fung, 1993) and has been used extensively for modeling deformation of biological materials.

2.2.2 Finite Element Formulation

In this work, the Galerkin method (Belystchko, 2000) is employed for the finite element formulation. The solid dynamics is governed by the following Navier equation

\[
\frac{\partial \sigma_{ij}}{\partial x_j} + \rho f_i = \rho \frac{\partial^2 d}{\partial t^2}
\]  

(2-24)

where \(i\) and \(j\) range from 1 to 3, \(\sigma\) is the stress tensor, \(\rho\) is the tissue density, \(f\) is the body force, and \(d\) is the tissue displacement.

Figure 2-6  Schematic showing a solid subjected to constraints and loads.
For an arbitrary solid body subjected to constraints and loads (as shown in Figure 2-6), multiplying Eq(2-24) by virtual displacement $\delta l_i$, and integrating over the volume $v$ results in the following equation:

$$
\int_v \sigma_{ij,j} \delta l_i dv + \int_v \rho f_i \delta l_i dv - \int_v \rho \ddot{l}_i \delta l_i dv = 0
$$

(2-25)

Using integration by parts, the first term $\int_v \sigma_{ij,j} \delta l_i dv$ will become

$$
\int_v \sigma_{ij,j} \delta l_i dv = \int_v \left( \sigma_{ij} \delta l_i \right)_j - \sigma_{ij} \delta l_{i,j} dv = \int_{sD} n_j \sigma_{ij} \delta l_i ds - \int_v \sigma_{ij} \delta l_{i,j} dv
$$

(2-26)

The surface integral term, $\int_{sD} n_j \sigma_{ij} \delta l_i ds$, can be further divided into two parts, $\int_{sD} n_j \sigma_{ij} \delta l_i ds$ which is an integral over the displacement boundaries, $sD$ and $\int_{s\sigma} n_j \sigma_{ij} \delta l_i ds$ which is an integral over traction boundaries, $s\sigma$ (see Figure 2-6). For displacement boundary condition, since $\delta l_i \equiv 0$, $\int_{sD} n_j \sigma_{ij} \delta l_i ds = 0$. For traction boundary condition, since $n_j \sigma_{ij} = \sigma_s$, where $\sigma_s$ is the surface traction, $\int_{s\sigma} n_j \sigma_{ij} \delta l_i ds = \int_{s\sigma} \sigma_s \delta l_i ds$. Thus Eq(2-25) can be rewritten as follows

$$
\int_{s\sigma} \sigma_s \delta l_i ds - \int_v \sigma_{ij} \delta l_{i,j} dv + \int_v \rho f_i \delta l_i dv - \int_v \rho \ddot{l}_i \delta l_i dv = 0
$$

(2-27)

The volume $v$ is discretized into the so called finite elements. Different types of elements can be employed for the discretization, such as triangular, quadratic and tetrahedral elements. The generic variable at any point inside the element can be expressed in terms of element nodal value through the linear interpolation. As shown in Eq. (2-28), the
displacement at arbitrary point \( d_i \) can be obtained by summing the weighted nodal displacement \( D^\alpha \) as follows:

\[
d_i = \sum_{\alpha=1}^{n} D^\alpha N_{i\alpha}
\]  

(2-28)

where \( N_{i\alpha} \) is the weight function, usually called the shape function, which is the function of nodal location \((x, y, z)\). Furthermore, \( n \) is total node number which is equal to 3 for a triangular element and 4 for a quadratic or tetrahedral element.

According to Eq. 2-28, the virtual displacement, and virtual velocity can be written as following (\( \alpha \) is summation index, \( i \) and \( j \) are free indices):

\[
\delta d_i = \delta D^\alpha N_{i\alpha}
\]  

(2-29)

\[
\delta \ddot{d}_{i,j} = \delta D^\alpha N_{i\alpha,j} = \delta D^\alpha B_{j\alpha}
\]  

(2-30)

Substituting Eq.(2-28, 2-29, 2-30) into Eq.(2-27) and Eq. (2-27) gives

\[
\delta D^\alpha \left( \int \sigma_s N_{i\alpha} \, ds - \int \sigma_j B_{j\alpha} \, dv + \int \rho f_i N_{i\alpha} \, dv - \left( \int \rho N_{j\beta} N_{i\alpha} \, dv \right) \dot{\delta}^\beta \right) = 0
\]  

(2-31)

For Eq. (2-31), the first integration term \( \int \sigma_s N_{i\alpha} \, ds \) is the traction force on node \( \alpha \), here denoted by \( F^s_\alpha \), the third integration term \( \int \rho f_i N_{i\alpha} \, dv \) is the body force on node \( \alpha \), denoted by \( F^b_\alpha \). For the second integration term, \( \int \sigma_j B_{j\alpha} \, dv \), substituting the constitutive equation, Eq.(2-17) into \( \sigma_j \), gives the following:
\[
\int \sigma_{ij} B_{ija} \, dv = \int \left( C_{ijkl} \varepsilon_{kl} + A_{ijkl} \dot{\varepsilon}_{kl} \right) B_{ija} \, dv \quad (2-32)
\]

Substituting Eq(2-28) into Eq.(2-18), (2-19), \( \varepsilon_{kl} \) and \( \dot{\varepsilon}_{kl} \) can written in terms of nodal values as follows:

\[
\varepsilon_{kl} = \frac{1}{2} (d_{k,j} + d_{i,k}) = \frac{1}{2} (D^\beta N_{kj\beta} + D^\beta N_{ij\beta}) = D^\beta B_{kj\beta} \quad (2-33)
\]

\[
\dot{\varepsilon}_{kl} = \dot{D}^\beta B_{kj\beta} \quad (2-34)
\]

Substitute Eq.(2-33), (2-34) into Eq.(2-32)

\[
\int \sigma_{ij} B_{ija} \, dv = \left( \int C_{ijkl} B_{kj\beta} B_{ija} \, dv \right) D^\beta + \left( \int A_{ijkl} B_{kj\beta} B_{ija} \, dv \right) \dot{D}^\beta \quad (2-35)
\]

Finally, substituting Eq.(2-35) into Eq.(2-31), inserting the nodal traction force and body force on right hand side of equation and canceling the \( \delta D^\alpha \) term, give the following equation:

\[
\left( \int \rho N_{ij\beta} N_{ja\alpha} \, dv \right) \ddot{D}^\beta + \left( \int A_{ijkl} B_{kj\beta} B_{ija} \, dv \right) \dot{D}^\beta + \left( \int C_{ijkl} B_{kj\beta} B_{ija} \, dv \right) D^\beta = F^i + F^b \quad (2-36)
\]

where \( \int \rho N_{ij\beta} N_{ja\alpha} \, dv \) is the mass matrix, denoted by \( M_{ij\alpha} \), \( \int A_{ijkl} B_{kj\beta} B_{ija} \, dv \) is the damping matrix, denoted by \( C_{ij\alpha\beta} \), and \( \int C_{ijkl} B_{kj\beta} B_{ija} \, dv \) is the stiffness matrix, denoted by \( K_{ij\beta} \). Thus the equation becomes a second-order ordinary differential equation, shown as following:

\[
M_{ij\alpha} \ddot{D}^\beta + C_{ij\alpha\beta} \dot{D}^\beta + K_{ij\beta} D^\beta = F^i + F^b \quad (2-37)
\]

In order to obtain \( M \), \( K \) and \( C \) matrices, an integration must be performed over the different elements. Usually, computing the integration under the original Cartesian
coordinates would lead to complex and different integration limits for each element. A better option is the use of isoparametric coordinates (Belystchko, 2000) which employ area-fraction as coordinates. The \((\xi, \eta)\) isoparametric coordinate used here is shown in Figure 2-7. Thus the integration of arbitrary function \(f(x, y)\) over the element can be performed with parametric coordinates with a uniform integration limit as follows:

\[
I = \int_0^1 \int_0^1 f(x, y) J d\xi d\eta
\]  
(2-38)

where \(J\) is the Jacobian given by \(J = \frac{\partial (x, y)}{\partial (\xi, \eta)}\). The 3D case is very similar to the 2D case, except using there we use volume-fraction as parametric coordinate. Following this, a standard numerical integration can be employed and for current code, upto 5th order numerical integration options are provided. Details of numerical integration can be found in Appendix I.

Figure 2-7 Isoparametric coordinates mapping for 2D triangular elements.

During normal phonation, the Reynolds number is about 3000. Under this Reynolds number, the shear stress is relatively small compared to the normal stress. Thus for calculation of traction force, only the normal stress is computed and is assumed to be linearly distributed load perpendicular to the surface. The equivalent nodal force can be
calculated through the integration based on shape functions. The formulation for 2D bodies is given by

\[ F_i^r = L \left( \frac{p_i + p_j}{3} + \frac{6}{6} \right) \]  

(2-39)

Where \( L \) is the length of the surface line segment, \( i \) and \( j \) are the nodal indices and \( p_i \) and \( p_j \) are the surface pressures at nodes. For 3D bodies, the expression is as follows:

\[ F_i^r = A \left( \frac{p_i + p_j + p_k}{6} + \frac{12}{12} \right) \]  

(2-40)

where \( A \) is the area of surface triangular element, \( i \), \( j \) and \( k \) are the nodal indices, and \( p_i \), \( p_j \) and \( p_k \) are the surface pressures at the nodes. The derivation of these formulations can be found in Appendix II.

### 2.2.3 Contact Model

Contact - which refers to two bodies which touch each other - is very common in mechanics problem. During phonation, the two vocal folds contact each other and appropriate modeling of this contact is important in order to ensure realistic dynamics of the vocal folds. Four different methods (Belystchko, 2000) have been employed in contact modeling, namely, the Lagrangian multiplier method, the penalty method, the augmented Lagrangian method and the perturbed Lagrangian method. Among these methods, the penalty coefficient method is easy to implement and does not change size of mass and stiffness matrices (Belystchko, 2000). The penalty coefficient method has successfully been employed in phonations models (Rosa, et al, 2003). Motivated by this we have also implemented a penalty coefficient method for modeling the vocal fold contact.
Figure 2-8 2D illustration of contact force discretization. The upper object intrudes into the lower object. The boundary of upper body is discretized as line segments. The squares are nodal points.

Figure 2-8, shows two objects contacting each other. Without contact model, the upper object intrudes into the lower object. According to the penalty coefficient method (Belyaevskho, 2000), the contact force is modeled as follows:

\[
F_{a}^{\text{contact}} = \int_{\partial V^c} \gamma \nabla g \nabla N_a \, d_a
\]  

(2-41)

where \( F_{a}^{\text{contact}} \) is the nodal contact force, \( \partial V^c \) is the contact area, \( \gamma \) is the penalty coefficient, \( g \) is the penetration distance and \( N_a \) is the shape function employed in the FEM. The above model creates a contact force that opposes penetration of one vocal fold into another and this force is proportional to the penetration distance with \( \gamma \) as the constant of proportionality. By choosing different values for \( \gamma \), one can enforce different types of contact with higher values of \( \gamma \) leading to a more "hard" contact condition. In the current simulations we employ a \( \gamma \) value of \( 10^4 \) times maximum stiffness coefficient. The above contact force is added to the right hand side of Eq. (2-37) in the current method in order to incorporate the contact effect into the dynamics of the vocal folds.
2.2.4 Solution Procedure and Validation

Eq.(2-37) is a second-order, ordinary differential equation in time and has been discretized using a second-order Newmark scheme (Belystchko, 2000). The resulting discretized equation is shown as follows:

\[
\left( K + \frac{1}{\beta \Delta t^2} M + \frac{\gamma}{\beta \Delta t} C \right) D^{n+1} = F^{n+1} + M \left[ \frac{1}{\beta \Delta t^2} D^n + \frac{1}{\beta \Delta t} \dot{D}^n + \left( \frac{1}{2 \beta} - 1 \right) \ddot{D}^n \right] + C \left[ \frac{\gamma}{\beta \Delta t} D^n + \left( \frac{\gamma}{\beta} - 1 \right) \dot{D}^n + \left( \frac{\gamma}{2 \beta} - 1 \right) \Delta t \ddot{D}^n \right]
\]

where \( \beta \) and \( \gamma \) are constants. When \( \beta = 0.25 \) and \( \gamma = 0.5 \), the scheme has second-order accuracy. The equation is solved by a banded LU decomposition solver and the Cutilli-McKee and Gibbs-Poole-Stockmeyer methods are used to re-index the nodes to produce a banded matrix (Everstine, 1972).

Two validation cases have been tested and compared with analytical solutions. The first case is that an infinitely long annulus is subjected to a displacement boundary condition. The annulus has a inner radius \( R_1 \) and a outer radius \( R_2 \). The inner surface of annulus is displaced in the radial direction by distance \( s \), and the outer surface is free (i.e. zero traction). For this condition, an exact solution to the above problem can be obtained if it is limited to a static, linearly elastic problem (Fung, 1965). In this case, the elastostatics is reduced to the axisymmetric plane-strain Lamé equation whose exact solution for the radial displacement \( d \) at radius \( r \) is given by

\[
d(r) = -\frac{A}{2G} \frac{1}{r} + \frac{2\nu G}{\lambda} r
\]
where \( \lambda = \nu E/(1 + \nu)(1 - 2\nu) \), and \( G = E/2(1 + \nu) \) are the two Lamé constants, \( E \) and \( \nu \) are Young’s modulus and Poisson’s ratio, respectively and \( A \) and \( C \) are constants given by

\[
A = \frac{2GR_i^2R_s}{R^2 - R_s^2} s, \quad C = \frac{A}{2R_i^2(1 - 2\nu)}
\]  

(2-44)

Figure 2-9 (a) Contours of radial displacement computed using finite element method. (b) the radial displacement as a function of \( r \) (red line: exact solution; green line: numerical solution).

Figure 2-9(a) shows the contours of the radial displacement and \( E = 20 \), \( \nu = 0.33 \), \( R_i = 1 \), \( R_s = 2 \), \( s = 0.05 \). The radial displacement function is shown in Figure 2-9(b) where it is found that the numerical solution has excellent agreement with the exact solution.

The second test case is that an isotropic spherical shell of internal radius \( R_0 \) and external radius \( R_i \) is subjected to internal and external pressure \( p_0 \) and \( p_1 \). The displacement for this problem has been given by Everstine (2008) and is as follows:

\[
D_i = Ar^{-3}x_i + Cx_i
\]  

(2-45)

where \( A \) and \( C \) are constants given by
\[ A = \frac{(p_0 - p_1) R_0^3 R_1^3}{4 \mu (R_1^3 - R_0^3)}, \quad C = \frac{p_0 R_0^3 - P R_1^3}{(3 \lambda + 2 \mu) (R_1^3 - R_0^3)} \]  

(2-46)

where \( \lambda = \nu E/(1 + \nu)(1 - 2\nu) \), and \( G = E/(2(1 + \nu)) \) are the two Lamé constants,  \( E \) and \( \nu \) are Young’s modulus and Poisson ratio, respectively.

Figure 2-10 shows contours of radial displacement for 1/8th section of the shell this figure is for \( E = 20000 \), \( \nu = 0.33 \), \( R_1 = 1 \), \( R_2 = 2 \), \( p_0 = 1000 \), \( p_1 = 0 \). The comparison in the figure shows again that the finite element solution is in excellent agreement with the analytical solution.

2.3 Flow-Structure Interaction Coupling Scheme

As described in sections 2.1 and 2.2, the fluid solver and solid solvers have been developed separately and these need to be coupled in an appropriate manner to perform FSI calculations. The coupling scheme is implemented through the marker points which are
used to describe the flow-structure interface. The marker points are required to be coincided with the nodes of finite element mesh on the surface to avoid the extra interpolation. For fluid solver, at the interface, non-penetration, no-slip velocity boundary conditions and zero Neumann pressure boundary condition are given. According to the ghost cell methodology, the boundary condition can be implemented through the boundary intercept (BI) points. The locations and velocities of these points are directly interpolated from the nodal values of finite-element solution, since marker points are coincided with surface nodes of finite element mesh. For pressure boundary condition, since zero gradient boundary condition is prescribed, the pressures at the body intercept points are interpolated only from fluid cells. For the solid solver, the traction boundary condition is given at flow-structure interface. Fluid dynamic pressure (normal stress) is computed for each marker point. Since the mesh is non-body-conformal, pressure is not available directly at the marker points and interpolation must be employed to compute those values. Each marker point is surround by 8 (4 for 2D) Cartesian nodes. These nodes are either fluid nodes or ghost nodes at which the value of pressure is explicitly stored. Tri-linear interpolation (bilinear interpolation for 2D) is used to compute the pressure on marker points. Once pressure at marker point is computed, nodal traction force can be computed based on Eq.2-39, 2-40.

In order to pass the correct values between fluid solver and solid solver, the two sets of equations must be nondimensionalized together with a common set of dimensional parameters. Here glottal pressure drop $\Delta p$, air density $\rho_f$ and centimeter $l$ have been
chosen to be characteristic variables. Thus the resulting equations in the nondimensional form are as following:

\[
\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial \tilde{x}_j} = -\frac{\partial \tilde{p}}{\partial \tilde{x}_i} + \frac{1}{\text{Re}} \frac{\partial}{\partial \tilde{x}_j} \left( \frac{\partial \tilde{u}_i}{\partial \tilde{x}_j} \right) \tag{2-47}
\]

\[
\tilde{M}_{ab} \tilde{D}_a^\beta + \tilde{C}_{ab} \tilde{D}_a^\beta + \tilde{K}D_a^\beta \tilde{F}_a = \tilde{F}_a \tag{2-48}
\]

Where

\[
\tilde{u}_i = \frac{u_i}{\sqrt{\Delta p / \rho_f}} , \quad \tilde{t} = t / \left( \sqrt{\Delta p / \rho_f} \right) , \quad \tilde{x}_i = x_i / l , \quad \tilde{p} = p / \Delta p , \quad \text{Re} = \left( \sqrt{\Delta p} / \sqrt{\rho_f} \right) , \quad \tilde{M}_{ab} = M_{ab} / \left( \rho_f l^2 \right) , \quad \tilde{C}_{ab} = C_{ab} / \left( \sqrt{\Delta p \rho_f l^2} \right) , \quad \tilde{K}_{ab} = K_{ab} / (\Delta p l) ,
\]

\[
\tilde{D}_a^\beta = D_a^\beta / \left( \Delta p / \rho_f l \right) , \quad \tilde{D}_a^\beta = D_a^\beta / \left( \sqrt{\rho_f / \Delta p} \right) , \quad \tilde{D}_a^\beta = D_a^\beta / l , \quad \tilde{F}_a = F_a / (\Delta p l^2)
\]

Thus, Eq.(2-47) and (2-48) are solved together to perform FSI simulations. There are two coupling strategies for FSI, namely loose coupling and strong coupling. The major difference between loose coupling and strong coupling is that they respectively integrate the governing equations of structure explicitly and implicitly in time. For the explicit coupling, the structure governing equation can be written as follows:

\[
M\dot{D}^{n+1} + C\dot{D}^{n+1} + KD^{n+1} = F^n \tag{2-49}
\]

The force term \(F^n\) is computed using previous time step fluid load and usually, the explicit coupling scheme is subject to a numerical instability constraint. To illustrate this problem, we take flow past an elastic mounted sphere as an example, shown in Figure 2-11. If the flow is assumed as incompressible potential flow, the aerodynamic load of the sphere is computed as follows:

\[
F_x = -\frac{2}{3} \pi a^3 \rho_f (\dot{U}_B - \dot{U}) + \frac{4}{3} \pi a^3 \rho_f \dot{U} \tag{2-50}
\]
where \( a \) is the radius of sphere, \( \rho_f \) is the fluid density, \( \dot{U}_b \) is the body acceleration, \( \dot{U} \) is free stream flow acceleration.

The first term of Eq.(2-50) is so called “added mass force” which is due to the sphere velocity relative to the free stream. The second term is the drag due to imposed pressure gradient. If assume that free stream velocity is constant \( U \) and we neglect the damping of spring of the sphere, the kinematic equation of sphere is:

\[
\frac{4}{3} \pi a^3 \rho_s \dot{X} + KX = -\frac{2}{3} \pi a^3 \rho_f \ddot{X}
\]  

(2-51)

where \( X \) is the location of sphere center, \( \rho_s \) is the solid density. If weakly coupling is employed, Eq. (2-51) is written as

\[
\frac{4}{3} \pi a^3 \rho_s \dot{X}^n + KX^n = -\frac{2}{3} \pi a^3 \rho_f \ddot{X}^{n-1}
\]  

(2-52)

Here 2\(^{nd}\) order central difference scheme is employed to discretize Eq.(2-52) in time. The resulting finite difference equation is

\[
\frac{4}{3} \pi a^3 \rho_s \frac{X^{n+1} - 2X^n + X^{n-1}}{\Delta t^2} + KX^n = -\frac{2}{3} \pi a^3 \rho_f \frac{X^n - 2X^{n-1} + X^{n-2}}{\Delta t^2}
\]  

(2-53)
Rearranging eq. (2-53) into the following form:

\[
X^{n+1} + \left( \frac{1}{2} \frac{\rho_f}{\rho_s} + \Delta t^2 \omega - 2 \right) X^n + \left( 1 - \frac{\rho_f}{\rho_s} \right) X^{n-1} + \frac{1}{2} \frac{\rho_f}{\rho_s} X^{n-2} = 0
\]  

(2-54)

where \( \omega = K/(4/3 \pi a^3 \rho_s) \) is the natural frequency of the sphere. Assuming numerical error \( \varepsilon_n = \lambda \varepsilon_{n-1} \) and substituting this into Eq. (2-54), the error equation is:

\[
\lambda^3 + \left( \frac{1}{2} \frac{\rho_f}{\rho_s} + \Delta t^2 \omega - 2 \right) \lambda^2 + \left( 1 - \frac{\rho_f}{\rho_s} \right) \lambda + \frac{1}{2} \frac{\rho_f}{\rho_s} = 0
\]  

(2-55)

The roots of eq. (2-55) satisfy the following equations:

\[
\lambda_1 + \lambda_2 + \lambda_3 = -\left( \frac{1}{2} \frac{\rho_f}{\rho_s} + \Delta t^2 \omega - 2 \right)
\]  

(2-56)

\[
\lambda_1 \lambda_2 \lambda_3 = \frac{1}{2} \frac{\rho_f}{\rho_s}
\]  

(2-57)

\[
\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_1 \lambda_3 = 1 - \frac{\rho_f}{\rho_s}
\]  

(2-58)

The criteria for a stable solution is

\[
|\lambda_i| \leq 1 \quad \text{for} \quad i = 1, 2, 3
\]  

(2-59)

Thus we can get the following constraints for \( \frac{\rho_f}{\rho_s} \) and \( \Delta t^2 \omega \)

\[
\left| \frac{1}{2} \frac{\rho_f}{\rho_s} + \Delta t^2 \omega - 2 \right| = |\lambda_1 + \lambda_2 + \lambda_3| \leq 3
\]  

(2-60)

\[
\left| \frac{1}{2} \frac{\rho_f}{\rho_s} \right| = |\lambda_1 \lambda_2 \lambda_3| \leq 1
\]  

(2-61)
\[ 1 - \frac{\rho_f}{\rho_s} = |\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_1| \leq 3 \quad (2-62) \]

The constraint for \( \frac{\rho_f}{\rho_s} \) and \( \Delta t^2 \omega \) is computed as follows:

\[ \frac{1}{2} \frac{\rho_f}{\rho_s} + \Delta t^2 \omega \leq 5 \quad (2-63) \]

\[ \frac{1}{2} \frac{\rho_f}{\rho_s} \leq 1 \quad (2-64) \]

\[ \frac{\rho_f}{\rho_s} \leq 4 \quad (2-65) \]

Eq.(2-64) and (2-65) require \( \rho_f \leq 2 \rho_s \) and \( \rho_f \leq 4 \rho_s \) respectively. Thus density ratio between fluid and solid must be less than two. From Eq.(2-66), it can be found that the time constraint comes from both density ratio and natural frequency. For density ratio part, the larger of density ratio requires the smaller time step.

For strong coupling, the aerodynamic load is computed implicitly, thus the governing equation is:

\[ M \ddot{D}^{n+1} + C \dot{D}^{n+1} + KD^{n+1} = F^{n+1} \quad (2-66) \]

Since the load is computed implicitly, there is no density ratio constraint. However, since the velocity and pressure at the FSI interface are unknown a priori, the strong coupling requires iteration between the fluid and solid solvers. Usually this can lead to a very intensive computation and thus should not be used if not needed. In the current case,

\[ \frac{\rho_f}{\rho_s} = \frac{\rho_{air}}{\rho_{mixture}} \approx \frac{1 \text{ kg/m}^3}{1000 \text{ kg/m}^3} \approx 0.001 \text{ and therefore the constraint on the density ratio are} \]

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easily satisfied. Furthermore, the constraint on the time becomes: \( \Delta t \leq \sqrt{\frac{5}{\omega}} \). However, the most energetic time-scale in the vocal fold is significantly large and as will be shown later, the overall time-step is limited by the CFL criterion coming from explicit treatment of non-linear convection terms in the Navier-Stokes equations.

![Figure 2-12 Schematic of flow-structure interaction coupling scheme.](image)

In the current study, the Newmark scheme (Belyaev, 2000) is employed which is unconditionally stable with respect to the time-advancement of the Eq.(2-42). This implies that there are no additional stability constraints beyond the condition on the density ratios as given in Eq. (2-64). In explicit coupling (shown in Figure 2-12), the flow is marched by one step with current deformed shape and velocities of fluid/solid interface as the boundary conditions. The aerodynamic forces imparted on the VF are then calculated at current location of the vocal fold surface via an interpolation scheme on the flow grid. Finally, the solid is marched by one step with the updated forces, and the deformation and velocities on the solid grid are interpolated onto the vocal fold surface, so that the fluid/solid interface is updated. This explicit coupling is quite simple, robust and efficient. Implicit coupling, if
needed, can be easily incorporated by iterating between the fluid and solid solvers at each
time-step.
CHAPTER 3 ANATOMICAL MODEL OF VOCAL FOLDS AND EIGENMODE ANALYSIS

In this chapter, the physical characteristics of the vocal fold models are described first, which have been employed in the current study. The vocal fold models derived from a high resolution (0.5mm plane-to-plane resolution) laryngeal CT scan of a 30 year old male subject with normal (undiseased) vocal folds taken at The George Washington University hospital. Following this, we perform eigenmode analysis of a hierarchy of vocal fold models. This study is a useful first step towards conducting fluid-structure interaction modeling since it allows us to determine the fundamental vibratory modes of the vocal fold in the absence of fluid forcing and also provides some understanding of the expected sensitivity of the vibratory characteristics of the vocal folds to its physical properties.

3.1 Anatomical Model of Vocal Folds

3.1.1 Vocal Fold Geometric Model

The shapes of the vocal folds and the false vocal folds are based on Figure 3-1(a) which shows a close-up coronal view of the CT scan at the anterior-posterior midline of the glottis. The focus of the current project is to develop and test the fluid-tissue interaction ability of the solver and to perform quantitative analysis of the biophysics of phonation for canonical glottal models. Many past studies have employed two-dimensional or idealized 3D laryngeal models and have found that these models provide fairly good insight into the biomechanics of phonation. It should also be noted that there is significant variation in the laryngeal geometry among individuals but since we are interested in studying a
prototypical larynx, we do not attempt to match all the finer details present in the CT scan. The geometry of the 2D model attempts to match the key geometrical features in the CT. It was noted that the two vocal folds in the CT scan were somewhat asymmetric. This could be congenital, pathological or an artifact of the CT imaging process. To eliminate this asymmetry, only the shape of the right vocal fold is used and left vocal fold is created by mirroring the right vocal fold about the centerline (shown in Figure 3-1(b)). A simple, 3D model is created by extruding the 2D model in longitudinal direction (shown in Figure 3-1(c)). Thus, this particular 3D vocal fold model does not take account of the variations in laryngeal geometry in the anterior-posterior direction.

![Figure 3-1 Vocal folds and false vocal folds anatomical model. (a) Coronal view of human larynx. (b) 2D vocal folds and false vocal folds shape extracted from CT scan. (c) 2D/3D vocal fold and false vocal folds model by extruding 2D model.]

### 3.1.2 Material Properties

The false vocal folds (FVFs) are modeled as rigid bodies since they do not move during the normal phonation and the true vocal folds (TVFs) are modeled as viscoelastic bodies.
The cover-ligament-muscle model (Hirano, 1981) has been used to construct the internal structure of the TVFs. The three layer structure shown in Figure 3-2 employed here is the same as that used by Luo, et al. (2008). The material properties of vocal folds are shown in Table 3-1. Similar properties have been used in the past by Alipour et al. (2000) and Luo et al. (2008). Note that Alipour et al. (2000) did not specify the longitudinal Young’s Modulus. For the 2D simulations, the plain strain assumption has been adopted and the material is assumed to be isotropic with Poisson ratio 0.3 (Luo et al. 2008).

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho ) (g/cm(^3))</th>
<th>( E_p ) (kPa)</th>
<th>( E_p) (kPa)</th>
<th>( E_p) (kPa)</th>
<th>( G_p ) (kPa)</th>
<th>( \eta ) (poise)</th>
</tr>
</thead>
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<tr>
<td>cover</td>
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<td>2.041</td>
<td>0.9</td>
<td>20</td>
<td>0.0</td>
<td>10</td>
</tr>
<tr>
<td>transition</td>
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<td>3.306</td>
<td>0.9</td>
<td>80</td>
<td>0.0</td>
<td>40</td>
</tr>
<tr>
<td>body</td>
<td>1.043</td>
<td>3.990</td>
<td>0.9</td>
<td>40</td>
<td>0.0</td>
<td>20</td>
</tr>
</tbody>
</table>

Figure 3-2 Three layer vocal fold inner structure and triangular elements used in the current solver.
3.2 Eigen Mode Analysis

The eigen-mode analysis proceeds by solving the following eigenvalue problem:

\[
M_{\alpha\beta} \ddot{D}^\beta + K_{\alpha\beta} D^\beta = 0 \tag{3-1}
\]

The boundary conditions provided in this eigenmode analysis are described in Figure 3-3. The eigensystem is solved using power iteration method which employs Gram-Schmidt Orthogonalization Algorithm (Everstine, 2008). The eigenanalysis sorts the modes based on the eigen frequencies and in the current study, we only consider at most the first four eigenmodes.

![Figure 3-3 Schematic of three dimensional vocal fold and boundary conditions for eigenanalysis.](image)

The eigen modes and eigen frequencies represent the possible responses of the structure to the external load. During the phonation, vocal folds typically vibrate at a single frequency which is the fundamental phonation frequency (F0). Despite the non-linearity in the fluid-tissue interaction, this resonant frequency is usually not far away from one of the vocal folds eigen frequencies. Thus, eigen mode analysis can give good insight into the
vocal fold vibration during phonation. Here, eigen mode analysis have been conducted for 2D as well as 3D vocal fold models.

### 3.2.1 Eigen Mode Analysis of Two-Dimensional Vocal Fold Model.

For this 2D case, 17202 triangle elements have been used to discretize the vocal folds domain and a zero-displacement condition has been applied on the lateral (see Figure 3-3) walls. The lowest three frequencies and corresponding modes have been computed (shown in Figure 3-4). The first eigen frequency is 95.3 Hz and for this mode and the vocal folds move in the inferior and superior direction. For the second mode, the eigen frequency is 215.3 Hz and the vocal folds move in the medial and lateral direction which leads to an adduction/abduction type of motion. Finally, for the third mode, the eigen frequency is 244.6 Hz and it is represented by a wave traveling along the vocal folds (the so called mucosal wave (Titze, 1994)). It should be noted that all of three eigen frequencies are within the range of human phonation frequencies and could occur during the normal phonation.

![Figure 3-4 Three lowest eigen modes for the 2D vocal fold model (a) mode 1: 95.3 Hz, (b) mode 2: 215.3 Hz, (c) mode 3 : 244.6Hz](image)
3.2.2 Eigen Mode Analysis of Simple 3D Vocal Fold Model

For this particular eigenanalysis, the shape of vocal folds is creating by extruding the 2D model 1.5 cm in anterior-posterior direction. 58427 tetrahedron elements have been employed to discretize the 3D vocal folds and the internal 3 layer structure is kept identical to the 2D case along the longitudinal direction. The anterior, posterior and lateral walls are given zero displacement boundary condition to represent the attachment to the cartilage (see Figure 3-3).

Figure 3-5 shows the eigenmodes for the lowest four eigenmodes. The first 3D mode is similar to the first 2D eigen mode in that the vocal folds move in the inferior/superior direction. However, the frequency of this first eigenmode is 68.1 Hz which is significantly lower than the corresponding mode for the 2D case. The second mode is a traveling wave along the longitudinal direction. This mode involves inferior-superior motion and the wave length is half of the length of vocal folds. Both the third and fourth modes are corresponding to the second and third modes in the 2D vocal folds: i.e. mode 3 is an adduction and abduction mode and mode four corresponds to a mucosal wave mode. It is also useful to point out that the frequencies of the latter 3 modes are quite close and very much within the typical phonation range. This would suggest that all three modes could potentially coexist during phonation. It also should be noted that differences in eigen frequencies between two-dimensional and three-dimensional cases result from differences in the material property assumptions. For the three-dimensional case, the material is assumed to be transversally isotropic. Thus, even the in-plain Young’s modulus is very small and the longitudinal effects still keep the eigen frequencies within the range of normal fundamental phonation frequencies. For the two-dimensional case, the plain strain
assumption has been adopted and the material is assumed to be isotropic with Poisson ratio 0.3. Since the in-plain Young’s modulus is small, to keep the eigen frequencies within the correct range, the longitudinal Young’s modulus has been employed for the isotropic material.

![Figure 3-5 Four Lowest 3D eigen modes for 3D vocal fold model with no longitudinal variation. (a) mode 1: 68.1 Hz (b) mode 2: 107.0 Hz (c) mode 3: 112.9 Hz (d) mode 4: 137.5 Hz](image)

3.2.3 Longitudinal Length Effects

The longitudinal length of vocal folds shows great variations among individuals, due to different body height, sex, age (Titze, 1994). In general we would expect that longer vocal folds have a lower phonation frequency but it is useful to quantify more precisely the
impact that VF length has on the eigen frequencies. Here 10 vocal folds samples with different lengths ranging from 1 cm to 2 cm are employed. The 4 lowest eigen frequencies are computed and plotted versus the length of vocal folds (shown in Figure 3-6). Figure 3-8 shows that eigen frequencies decrease with length of vocal folds monotonically. Frequency difference between mode 3 and mode 4 is smaller with short vocal folds and larger with longer vocal folds. According to Ishizaka (1981) and Zhang et al. (2006), the frequency of mode 3 and mode 4 approach each other as the glottal jet velocity increases. Furthermore, when the jet velocity is greater than the critical value, mode 3 and mode 4 vibrate at the same frequency, a phenomenon called “one-to-one entrainment”. For shorter vocal folds, the difference of eigen frequency between mode 3 and mode 4 is smaller. It indicates that lower glottal jet velocity is required to produce one-to-one entrainment. It should be noted that with normal vocal folds length 1cm – 2cm, the eigen frequencies for mode 2, 3 and 4 are all above 100 Hz and in the normal phonation frequency range (Titze, 1994).

![Figure 3-6](image)  
*Figure 3-6  effects of length of 3D vocal fold on the 4 lowest eigen frequencies*
3.2.4 Longitudinal Shape Variation Effects

In order to account for the shape variation effects along the longitudinal direction, the 3D idealized vocal fold has been fitted into an elliptic tube. The dimension and shape of this ellipse is roughly based on the trachea of the subject and is idealized with four quadratic ellipse segments, which aspect ratio are 2:1 and 2:3, respectively. The three-layer structure of this model is same with two dimensional case. As shown in Figure 3-7 the anterior and posterior parts of the vocal folds have been fixed corresponding to the arytenoid cartilage and thyroid cartilage and the longitudinal length of the vibratory part of the vocal folds is 1.5 cm.

![Figure 3-7 3D elliptic shape trachea and 3D vocal folds with the longitudinal shape variations.](image)
Figure 3-8 4 Lowest eigen modes and frequencies of 3D vocal folds with longitudinal shape variations. (a) mode 1: 76.2 Hz (b) mode 2: 108.9 Hz (c) mode 3: 119.5 Hz (d) mode 4 131.1 Hz

Figure 3-8 shows the four lowest eigen modes and eigen frequencies for the elliptic vocal folds. These eigen modes are very similar to the eigen modes of 3D model without longitudinal shape variations. The first mode also exhibits superior/inferior motion, which corresponds to mode 1 without longitudinal shape variation. The second mode is an adduction/abduction mode which corresponds to mode 3 of the model without longitudinal shape variation. The third mode is a traveling mucosal wave which corresponds to mode 4
without longitudinal shape variation. The fourth mode is a traveling wave along the longitudinal direction, which is similar to mode 2 without longitudinal shape variation. Furthermore, the eigen frequency is also close to the model without longitudinal shape variation. Thus, the longitudinal variation in vocal fold geometry does not produce prominent effects on the eigen modes.
CHAPTER 4 MODELING FLOW-TISSUE INTERACTION DURING PHONATION: VALIDATION AND VERIFICATION

In this chapter, a detailed description of results obtained from fluid-structure interaction (FSI) calculations of phonation is provided. While most of the results pertain to the 2D glottal model, some preliminary results for a 3D glottal model are also presented. The objectives of the current chapter are to (1) explore various aspects of the vocal fold vibration and glottal aerodynamics and (2) establish the fidelity of the current modeling procedure through qualitative as well as quantitative comparisons with established data.

4.1 2D Flow-Tissue Interaction

Extensive simulations of fluid-structure interaction in a 2D glottal model have been carried out and these are described in this section.

4.1.1 2D Simulation Setup

Following the approximate dimensions found in the CT scan, the overall dimensions of the flow domain is chosen to be 12 cm long (in the direction of the flow) by 2 cm wide. The true vocal folds are 1 cm long and extend 0.99 cm towards the glottal midline. The false vocal folds are 2.3 cm long and extend 0.67 cm towards the supraglottic space. The ventricles are about 0.56 cm wide at their widest location. The false vocal fold gap is 0.667 cm, which is in the range reported by Furmanik et al. (1976), Scherer et al. (1995) and Sakakibara et al. (2004).
Dirichlet boundary conditions are applied for the pressure at both the inlet and outlet. The gage pressure at inlet is 1 kPa and at outlet is 0 kPa which is equivalent to 10.2 cm of water and this is in the physiological range for phonation. Zero normal gradient velocity boundary conditions are applied at the inflow and outflow and this allows the flow rate to settle to a value dictated by the pressure drop and the flow impedance. Finally, no-slip and no-penetration boundary conditions are applied on the walls and the flow-tissue interfaces.

The Cartesian grid for the flow solver covers the entire computational domain and TVFs and FVFS are immersed in the grid, as shown in Figure 4-2. In the $x$-direction, a non-uniform grid is employed with denser grid in the vicinity of TVFS and FVFS while in the transverse $y$-direction, a uniform grid is employed. A $289 \times 256$ Cartesian grid is chosen.
after grid refinement study (shown in Figure 4-5 and Figure 4-6). For the solid solver, the same 17202 triangle mesh used in eigen mode analysis (shown in Figure 3-2) has been employed and same tissue properties (shown in Table 3-1) are also used here.

For the flow solver, the size of the time-step is restricted by numerical stability of the flow solver which dictates a maximum Courant-Friedrichs-Lewy (CFL) number (Courant et al, 1928) of about 3.0. In the current simulation a time-step equivalent to $3.5 \times 10^{-3} \text{ms}$ is employed which leads to 1000 to 3000 time steps in every vibration cycle for phonation frequencies in the range between 100 and 200 Hz. The solid solver demands a smaller time-step size, especially during vocal folds collision. Due to the explicit coupling between the fluid and solid solver, the two can employ different time-steps and in the current simulations, the solid solver uses a time-step size of $3.5 \times 10^{-4} \text{ms}$ which is 10 times smaller than the time scales adopted by the flow solver. All simulations use a single processor of a 1.96 MHz AMD Opteron™ workstation.

Figure 4-2 Grid used in the vicinity of vocal folds and false vocal folds in the current simulation.
4.1.2 Results and Discussion

Simulation has been performed until the vocal folds reached a nearly steady limit cycle vibration. Figure 4-3 (a) shows the history of the glottal gap width and it clearly indicates a transient state that eventually developed into a stationary state representing sustained vibrations. Figure 4-3 (b) shows the glottal gap width, $G_{TVF}$, for 5 vibration cycles from 0.330s to 0.351s. From spectrum analysis, it can be found that at this stage, the vibration has a single frequency of 231 Hz (shown in Figure 4-4(b)). This is the fundamental phonation frequency for the current 2D laryngeal model and it is at the upper end of the range associated with normal phonation in humans (Titze, 1994). It also should be noted
that there is also another lower frequency occurring during the transient stage, which is about 96 Hz (shown in Figure 4-4(a)). This indicates another vibration pattern occurring at onset of phonation which dies out and gives way to the limit cycle vibration mode.

![Figure 4-4 Spectrum analysis of Time variation of glottal gap width (a) transient stage (0.0s – 0.07s), (b) steady stage (0.07-0.363s).](image)

To examine the sensitivity of the simulation results to grid resolution, the grid resolution has been doubled. Only the grid in the y-direction has been doubled, since the x-grid has been already very small in order to keep the reasonable grid ratio. Thus a $289 \times 512$ grid has been employed to simulate the same case. Due to the stochastic nature
of the glottal flow, the instantaneous flow fields are expected to be very different for these two cases. Here the statistical variables are used for this grid refinement validation. Figure 4-5 shows the comparison of time variation of glottal gap width for these two cases. It can be found that time variations of the glottal gap width are almost identical for two cases. The maximum difference in the glottal gap width between these two simulations is 3% of the maximum values of these quantities. This means that two cases produce the same phonation fundamental frequency and vocal folds have the same vibration amplitude. Figure 4-6 shows the comparison of time variation of volume flux rate. It can also be found that two cases almost produce the same volume flux rate at any time instant. The maximum difference in the volume flow rate between these two simulations is 5% of the maximum values of these quantities. This indicates that flow resistance does not change with grids. Thus the original grid $289 \times 256$ is proved to be sufficient for the current simulation. It also should be noted that the finer-grid aspect ratio took more computational time and decreased the accuracy of the scheme. However, the results did not show a noticeable difference between these two simulations.

![Figure 4-5 Comparison of time variation of glottal gap width for original grid and finer grid](image)
Figure 4-6 Comparison of flow volume flux rate for original grid and finer grid

Figure 4-7 shows a sequence of vocal fold shapes during one vibration cycle. It can be observed that the motion of vocal folds are mainly composed of adduction and abduction motion and mucosal wave motion, which are represented by mode 2 and mode 3 from eigen mode analysis (see Figure 3-4). During the opening (adduction) phase, the inferior part of vocal folds (towards the left in the plots in Figure 4-7) starts to open first and at this instant in time, the vocal fold shows a convergent shape (shown in Figure 4-7 (b)). Subsequently the superior part starts to separate from each other and both inferior and superior parts start to abduct (shown in Figure 4-7 (c)). At the maximum opening location, the inferior part adducts first and this followed by the superior part (shown in Figure 4-7 (d) and (e)). At this stage, vocal fold shows a divergent shape. This vibration pattern matches the observation from Hirano (1977) and the results predicted by the two-mass model (Ishizaka, 1972, Pelorson, 1994, Tao, 2006). Thus, the qualitative characteristics of the vibrations are correctly captured by current continuum model.
Figure 4-7 Vocal fold shapes at different instantaneous time instant within a vibration cycle: (a) 0.3384s (b) 0.3392s, (c) 0.3400s, (d) 0.3409s, (e) 0.3416s, (f) 0.3427s. Note that the time period of the vibration is 0.004329S.

The vibration pattern shown above corresponds to steady limit cycle vibration. However in the transient stage, the vocal folds also show a rotational motion (moving inferior and superior direction) superposed on the abduction-adduction vibration pattern. Figure 4-8 shows the shapes of vocal folds at two extreme inferior and superior positions in the transient stage. It can be found that inferior and superior motion is much larger.
compared to adduction and abduction at this stage. For flow-structure interaction system, the lowest mode of structure is usually the first to be triggered and system is usually resonating around the lowest eigen frequency. This explains why rotational motion, which corresponds to the first eigen mode, is dominant at the onset of phonation. However, during the phonation, the pair of vocal folds blocks the airway and is immersed inside the air within the larynx. When vocal folds are moving in the inferior and superior direction, they behave very similar to a piston moving back-forth in the tube filled with liquid. This is the classic dashpot model and this type motion will cause a large damping force exerted by the fluid. This explains why this motion has been damped out after a few cycles. For the same reason, the adduction-abduction motion does experience as significant a damping from the fluid since the mass of fluid between two vocal folds is very limited. In this case, the damping force is mainly caused by the viscoelasticity in the vocal fold tissue. Thus, sustained vibrations are produced if the pressure drop is larger enough to overcome this damping force.

In addition, the deformation in the transient stage (shown in Figure 4-8) is relative large. Thus for the fluid solver, a number of fresh and dead cells are encountered in this stage. For the solid solver, since the deformation is relative large, the linear elastic assumption might not be accurate for this stage. The material and geometric nonlinearities could be incorporated in the future work.
During phonation, vocal folds show a sustained vibration and it leads to a pulsatile jet flow. Figure 4-9 shows the time variation of volume flux rate which shows a quasi-periodic wave form. At the steady limit cycle stage (shown in Figure 4-9 (b)), the average of the volume flux rate and the peak value are respectively $0.01619 \text{ m}^2/\text{s}$ and $0.04328 \text{ m}^2/\text{s}$. If the vocal fold length is assume to be 2 cm, the average and peak volume flux rate are $322 \text{ ml/s}$ and $865 \text{ ml/s}$ which are inline with in-vivo measurement on excised larynges (Alipour, 2006).

The peak Reynolds numbers based on the flow rate is $Re_\theta = 300$. In adult humans, effective Reynolds number of the glottal flow can attain peak values of about $Re_\theta = 3000$ (Titze, 1994) and is expected to be lower in children. Past studies have employed Reynolds number (based on our current definition) ranging from about 1000 in Alipour (1996) to 3000 in Alipour (2004). However, unlike the current methodology, these
past computational studies employ dissipative numerical methods and therefore the actual effective viscosity in their simulations is expected to be significantly lower. It should also be noted that the vibratory characteristics of the vocal fold are expected to be relatively insensitive to the Reynolds number and the lower Reynolds number in the current study alleviates the grid refinements while still producing relevant results.

Figure 4-9 Time variation of flow volume flux rate (a) overall history 0s-0.4s, (b) zoom in view for 7 steady limit cycles, 0.330-0.351s.

Figure 4-10 and Figure 4-11 show a sequence of instantaneous spanwise vorticity contours which reveal details of the flow dynamics during the sustained vibration. It can be seen that when the VFs are open, the fluid is pushed out by the subglottal pressure into the supraglottal region leading to the formation of the so-called “glottal jet”. It is interesting
that the jet shows significant asymmetry and may be deflected to one side of the channel. This is because there are strong flow recirculation zones in the downstream channel created in previous cycles which tend to turn the glottal jet one way or the other. The sequence of plots also show that the direction of the jet deflection may change from one cycle to another depending on the particular condition of the downstream circulation as shown in Figure 4-11. In the current simulation, the cycle-to-cycle jet deflection is found to be stochastic in nature.

Steady channel flow with a sudden expansion is known to have a bifurcation in its solution at a critical Reynolds number which depends on the expansion ratio (Drikakis, 2006). Beyond the bifurcation point, the symmetric solution becomes unstable and the steady flow may become asymmetric even though the geometry is symmetric, similar to the flow patterns shown in Figure 4-10 and Figure 4-11. According to Drikakis(2006), the symmetry breaking takes place at the critical Reynolds number based on the flow rate
\[
\text{Re} = \left(\frac{3}{2}\right) \frac{Q}{\nu} = 26
\]
when the expansion ratio is 10. In addition, the critical Reynolds number is reduced when the expansion ratio increases. In the current simulation, the expansion ratio varies between 15 and 100 during a vibration cycle, and the jet Reynolds number is much higher than the critical Reynolds number. Therefore, the flow is operating under the conditions that would produce an asymmetric solution. Furthermore, the Reynolds number is higher enough for the present flow to be unsteady even if the geometry was stationary.
Figure 4-10 Contours of spanwise vorticity over one vocal fold vibration cycle. The eight plots correspond to eight time intervals over the vibration cycle: (a) 0.3384s, (b) 0.3392s, (c) 0.3398s, (d) 0.3400s, (e) 0.3409s, (f) 0.3416s, (g) 0.3420s, (h) 0.3427s.
Figure 4-11 Contours of spanwise vorticity over one vocal fold vibration cycle. The six plots correspond to six time intervals over the vibration cycle: (a) 0.3436s (b) 0.3441s, (c) 0.3447s (d) 0.3454s, (e) 0.3461s, (f) 0.3468s

Figure 4-12 (a) shows the time-averaged streamline pattern of the flow and Figure 4-12(b) shows the average $u$ velocity profiles at different locations along the $x$ axis. A pair of vortices resides on the two sides of the jet just above the vocal folds and a pair of smaller vortices is formed between the two big vortices and lateral walls. Thus the flow field inside the ventricle is similar to a lid-driven cavity flow. This flow circulates inside the ventricle and produces a back pressure to balance the pressure drop in $x$ direction. In addition, the
circulation in the ventricle narrows down the effective channel width and straights up the outside jet flow. A large recirculation is also formed behind the false vocal folds. It should be noted that even though the average is taken over about 10 cycles, the mean flow is still asymmetric about the channel centerline. This is due to the stochastic nature and large time-scales which are present in the supraglottal region.

![Figure 4-12 Averaged flow field between t = 0.2915 and t = 0.3346. (a) Streamlines; (b) u velocity profiles at different locations.](image)

As shown in Figure 4-10 and Figure 4-11, the glottal flow shows a high asymmetric nature and this asymmetric flow will produce asymmetric forces on vocal folds. Thus the vibration of vocal folds is expected to be asymmetric also. In order to examine the symmetry of the TVF vibration, two sets of points on the two vocal folds have been
tracked. As shown in Figure 4-13, one set of points is on the superior edge whereas the other set is on the inferior edge of the true vocal folds. Correlating the motion of these two sets of point should provide a clear understanding of the symmetry of the vocal fold vibration.

Figure 4-13 Points selected on vocal folds surface for analysis of vibration symmetry

Figure 4-14 shows the $x$- and $y$- locations of the vocal folds for the two sets of points. These plots are phase-plane plots and plot the displacement of a point on one vocal fold versus the corresponding point on the other vocal fold. Perfect correlation between the two vocal folds would correspond to a 45 degree straight line in these plots. As can be seen, there is some degree of asymmetry in the vocal fold vibration and the vocal folds are going to be slightly out of phase.
Figure 4-14 Phase plane plots of the displacement of the points on the superior side and inferior side of the two vocal folds. (a) $x$ displacement (inferior-superior) (b) $y$ displacement (abduction-adduction) from centerline.

The mechanical stress in vocal fold tissues is important since vocal folds tissue may experience fatigue and damage due to the excessive stress. In extreme case, excessive and prolonged stress can cause laryngeal pathologies such as vocal fold nodules (Titze, 1994).

Figure 4-15 Shows contour of stresses in the vocal folds during the open and closed phase of the vibration cycle. At the closing phase, the contact force produces a big compressive stress (negative $\sigma_{yy}$) on the medial surfaces of the vocal folds and the maximum elongation stress (positive $\sigma_{yy}$) occurs at the root of ligament. The maximum shear stress
\( \tau_{xy} \) occurs on the root of vocal folds. At maximum opening position, the shear stress is much smaller than during the closing phase. The normal stress \( \sigma_{yy} \) is mainly compressive during the opening phase and occurs at the superior part of the vocal fold body. The normal stress \( \sigma_{xx} \) is similar at two position and is in general smaller than \( \sigma_{yy} \).

Figure 4-15 Contours of stresses in the vocal folds during the open and closed phase of the vibration cycle.

4.1.3 Sub-Glottal Pressure Effects

As pointed out earlier, sustained vocal fold vibration required that the forces on the vocal folds be large enough to overcome the damping forces. In order to investigate the effect of sub-glottal pressure on phonation, six cases have been simulated with different sub-glottal pressures, which are 0.1, 0.3, 0.5, 0.7, 1.0, 1.5 and 2.0 kPa. In all these cases, the pressure at the outflow is set at 0 kPa.

For 0.1 kPa and 0.3 kPa subglottal pressure, no sustained vibration is produced whereas sustained vibrations are observed for a subglottal pressure of 0.5kPa. This indicates the
threshold phonation pressure should be between 0.3 and 0.5 kPa. A similar range has been reported by Baer (1975) who conducted in-vivo measurement on excised larynx and Titze (1991, 1994) who employed numerical study using two mass vocal fold model coupling with 1D Bernoulli equation.

![Figure 4-16 Variation of fundamental frequency with sub-glottal pressure.](image)

For all cases except those with subglottal pressure of 0.1 kPa and 0.3 kPa, power spectrum analysis is employed to extract the fundamental frequency from time variation of glottal width. The variation of fundamental frequency $F_0$ versus sub-glottal pressure $P_{sub}$ is shown in Figure 4-16. When sub-glottal pressure is just above the phonation threshold pressure, the fundamental frequency is found to increase non-linearly with sub-glottal pressure. When sub-glottal pressure is somewhat beyond the normal phonation pressure, the fundamental frequency is found to become almost constant. It should be noted here that usually the fundamental frequency increases with subglottal pressure when the sub-glottal is significantly than normal phonation pressure (Titze, 1994). This discrepancy is most likely due to the assumption of material linearity. However, since the focus here is on
investigating the normal phonatory behavior the current model is expected to be adequate. The similar pressure-frequency relationship has been reported by Ishizaka (Ishizaka, 1981),

4.2 3D Model of Flow-Tissue Interaction

Following the 3D eigenmode analysis, 3D flow-tissue interaction used the same 3D vocal fold geometric model which is 1.5 cm long in the longitudinal direction without shape variation. The material properties and the 3 layer inner structure are also kept the same as the model used in the 3D eigen mode analysis. A $12cm \times 2.0cm \times 1.5cm$ straight rectangular duct has been used to mimic the human airway. The locations of the vocal folds and false vocal folds inside the airway are kept the same as the 2D flow-tissue interaction study. For the true vocal folds, all of the surfaces except the flow tissue interfaces are fixed. The subglottal and supraglottal pressure are also the same as the nominal 2D case and all of the walls are given non-slip boundary conditions. This simulation employs a nonuniform $192 \times 128 \times 65$ Cartesian grid for the fluid solver and a 58427 tetrahedral element grid for the solid solver.

![Figure 4-17 Flow domain in the 3D flow structure simulation and finite element mesh for the true vocal folds.](image-url)
Figure 4-18 shows time variation of the volume flux rate. Due to the extremely large computational time, only the first three cycles are simulated. It can be found that the flux is similar to the transient flux state in the 2D simulation. At the second and third cycles, there are two frequencies in which the lower one is superposed on the higher one. The lower frequency is about 66.7 Hz, which is close to the lowest eigen frequency whereas the higher one is about 133.3 Hz which is between the third and forth eigen frequencies. It should be noted that for the 2D simulation, the transient stage also has two frequencies. The lower frequency is 96 Hz, which is close to the lowest 2D eigen frequency. The higher frequency is 231 Hz, which is between the second and third eigen frequencies (the second and third eigen modes in two-dimensional case is corresponding to the third and fourth eigen modes in three-dimensional case).

Figure 4-18 Time variation of volume flux rate for 3D flow-tissue interaction.

Thus in the transient stage, similar to the 2D simulation, the rotational motion is expected to be dominant. Figure 4-19 shows an isosurface of maximum magnitude velocity eigen value during one vibration cycle and the corresponding vocal folds shapes at the longitudinal center plane (z = 0.75cm). It can been seen from Figure 4-19 that the vocal folds undergo a rotational motion at the longitudinal center plane (z = 0.75cm), which is similar to that observed for the 2D model (shown in Figure 4-8).
Figure 4-19  Iso-surface of maximum magnitude of swirl strength and vocal folds shape at z = 5.5cm plane at five different time instants over one vocal fold vibration cycle. (a)-(b) 0.0175s, (c)-(d) 0.0203s, (e)-(f) 0.0217s, (g)-(h) 0.0238s, (i)-(j) 0.0255s.
The simulations also indicate that the glottal jet is highly three-dimensional but despite the three-dimensionality, shows a tendency to deflect towards one of the walls of the false vocal folds (shown in Figure 4-19). Thus the 3D simulation has captured some characteristics of phonation and similar results can also be found in the transient results of 2D simulations. It indicates that this 3D flow-tissue solver is able to predict the behavior of flow-tissue interaction. However due to the long transient time of current problem, the simulation can not reach the steady limit cycle vibration stage.

The large computational times required for even this relatively simple 3D computational model indicate that significant advances are needed in the computational methodology in order to make such simulations viable. One approach is to use parallel computing which would significantly reduce the turn-around time for these simulations. Another approach is to develop local adaptive grid refinement methodology which will allow us to optimize grid density and thereby increase simulations turn-around. This latter strategy has been adopted in the current study will be described in detail in Chapter 6.
CHAPTER 5 EFFECTS OF FALSE VOCAL FOLDS ON PHONATION

In previous chapter, flow-tissue interaction has been performed and sub-glottal pressure effects have been investigated. As the other phonatory key parameter, larynx configuration, here represented by presence of false vocal folds, will be discussed solely in this chapter.

5.1 Introduction

As mentioned in first chapter, false vocal folds are a pair of thick folds of mucous membrane that are located in the supra-glottal space in the larynx. These false vocal folds are separated from the true vocal folds (TVF) by the ventricular space and are not directly connected to the true vocal folds. Interference of the false vocal folds during phonation is implicated in ventricular dysphonia and this may be a result of pathologies (paralysis, tumors) or surgical intervention, such as hemilaryngectomy (Leeper et al. 1990, Maryn et al. 2003, Pinho et al. 1999, von Doersten et al. 2005). Ventricular dysphonia can sometimes be compensatory as a substitute voice in case of true-vocal fold inadequacy (Maryn et al. 2003). For instance, in patients with unilateral true vocal fold paralysis, the contralateral false vocal fold may be adducted to recover phonatory function of the larynx. The false vocal folds are also known to play an active role in some types of throat singing (Sakakibara et al. 2004).

The role of false vocal folds during normal phonation is however not well understood although a number of studies in the past have indeed examined this issue with various modeling and experimental approaches. One of the first modeling studies published on this topic was that of Zhang et al. (2002) who used a computational approach to study the flow
and sound production in a modeled larynx. The key features of the model were that it was two-dimensional, assumed flow symmetry about the medial line and the vocal fold had a prescribed motion (i.e. no fluid-structure coupling). They found that the presence of the false-vocal folds reduced the flow resistance and also introduced additional dipole sources of sound.

Agarwal et al. (2004) used an experimental setup to examine the effect of false-vocal folds on translaryngeal flow resistance. The key features of their experimental model were that the geometry was axisymmetric, the true vocal folds were assumed to be stationary (although different TVF geometries were examined) and the FVF gap was varied. This study found that for a ratio of false to true vocal fold gap width of about 2.0, the translaryngeal flow resistance reached a minimum which was 20-25% lower than the resistance in the absence of the false vocal folds.

Triep et al. (2005) have recently performed a particle-image velocimetry (PIV) study of a scaled up laryngeal model and have also looked at the effect of FVF on the glottal flow. The glottal gap in this study was designed to be realistic but the vocal folds were moved according to a prescribed motion. Their primary finding was that the glottal jet tended to attach randomly to one FVF or the other from cycle to cycle.

Alipour et al. (2007) have recently carried out a detailed study of the effect of the epiglottis and FVF during phonation. Translaryngeal flow resistance ($Z_L$) is defined as

$$Z_L = \frac{\Delta P_L}{\tilde{Q}}$$

(5-1)

where $\Delta P_L$ is the mean pressure drop across the glottis and $\tilde{Q}$ is the associated mean flow rate through the glottis. Their study employs an excised canine larynx which was suitably
modified to examine various effects including variation in adduction, pressure and flow rate. Their primary conclusion was that removal of FVF resulted in decreased translaryngeal flow resistance and a decrease in the sound intensity level. Thus it would seem that their result regarding translaryngeal resistance contradicts past studies (Zhang et al. 2002, Agarwal et al. 2004) that suggest that the presence of the FVF can decrease flow resistance. Furthermore, it is also expected that a decrease/increase in flow resistance for a given pressure would lead to increase/decrease in the sound pressure level. However it should be noted that Alipour et al. (2007) based their conclusion on differential flow resistance which is the rate at which the required mean pressure across the larynx changes with flow rate. Thus, their flow resistance was defined as $Z'_L = \frac{\partial (\Delta P)}{\partial Q}$. In fact, a closer look at their results (see Table 2 and Figs. 3 and 5 of their paper) indicates that for many cases, especially those with flow rates less than about 600 $ml/s$, the flow resistance as defined in Eq.(5-1) is actually lower with the FVFs than without.

Thus, although there is some indication that FVF might play a positive role in phonation, the issue is not fully settled. Furthermore, what is still lacking is a clear connection between the effect of FVFs and the associated glottal aerodynamics. Thus, the question as to what effect the FVFs have on the glottal jet that subsequently results in lowered translaryngeal resistance remains to be answered. Finally, the effect that FVF have on the flow induced vibration of the true-vocal folds have not been addressed in any of the past studied except for that of Alipour et al. (2007). An understanding of this role is important for a number of reasons. First, a better understanding of FVF function would provide clinicians with useful insights on surgical interventions or other pathologies
associated with the false vocal folds (Maceri et al. 1985). Second, an understanding of FVF function in phonation can lead to improvements of biomechanical models of the human larynx. Such models are being used to understand the biophysics of phonation (Alipour et al 2000, Ishizaka, 1972) and are also being considered for potential use in surgery planning and prediction (Luo et al. 2008). Whereas some of the past studies have employed laryngeal models with false vocal folds (Rosa et al. 2003), the majority of the models do not include these structures and it is not clear to what extent, the exclusion of FVFs modifies the vocal fold and glottal jet dynamics.

5.2 Simulation Setup

A simulation of two-dimensional case with presence of false vocal folds (denoted by (+) FVF) has been performed and analyzed in the previous chapter. Here, a same case except without presence of false vocal folds (denoted by (-) FVF) is carried until it reaches steady limit cycle vibration stage.

5.3 Comparison between (+) FVF and (-) FVF

Both simulations are run for over 10 cycles to ensure that initial transients are eliminated from the analysis. ten steady cycles from each of them are chosen for comparison. Due to the slight difference of vibration frequencies of two cases, the starting instantaneous time for those 10 cycles are chosen to be different: 0.31758 s for (+) FVF and 0.37299 s for (-) FVF. Table 5-1 summarizes the key flow and vibration quantities computed for the two cases. It should be noted that because the current simulations are two-dimensional, the flow rate is provided in terms of flow rate per unit anterior-posterior length of the glottal gap, and thus has units of \( (m^2/s) \). However, in order to connect this
flow rate with those seen in real larynges, the flow rate also is recomputed by assuming the anterior-posterior length of the glottal gap to be 2 cm. These values are also provided in the table in units of (ml/s)

Table 5-1 Summary of key computed quantities for cases with and without false vocal folds.

<table>
<thead>
<tr>
<th></th>
<th>(-)FVF</th>
<th>(+)FVF</th>
<th>[\frac{(+FVF - (-FVF)}{(-FVF)} \times 100]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_0$ (Hz)</td>
<td>227</td>
<td>231</td>
<td>+2.2%</td>
</tr>
<tr>
<td>Amplitude of $G_{TVF}$ (cm)</td>
<td>0.0929</td>
<td>0.1238</td>
<td>+33%</td>
</tr>
<tr>
<td>Mean value of $G_{TVF}$ (cm)</td>
<td>0.0481</td>
<td>0.0547</td>
<td>+13.7%</td>
</tr>
<tr>
<td>Peak Value of flow rate $Q (m^2/s)$</td>
<td>0.02937 (587 ml/s)</td>
<td>0.04328 (865 ml/s)</td>
<td>+47.4%</td>
</tr>
<tr>
<td>$\bar{Q}(m^2/s)$</td>
<td>0.01338 (266 ml/s)</td>
<td>0.01619 (322 ml/s)</td>
<td>+21%</td>
</tr>
<tr>
<td>Flow Impedance $\frac{\Delta P}{\bar{Q} (m^2/s)}$</td>
<td>74756</td>
<td>61767</td>
<td>-17%</td>
</tr>
</tbody>
</table>

5.3.1 True Vocal Folds Vibration

Figure 5-1 shows the time variation of the glottal gap width with time and it can be seen that both cases reach a state of nearly periodic self-sustained vibration. The phonation frequency ($F_0$) calculated from this variation indicates values of 232Hz and 227Hz for the (+)FVF and (-)FVF cases respectively. These values are at the upper end of the range associated with normal phonation in humans. It is also interesting to note that for the current case, the presence of the false vocal folds does not seem to have a significant effect on $F_0$. 
Figure 5-1 Comparison of the time-variation of the glottal gap width for the two cases.

Figure 5-1 shows the time variation of the glottal gap (gap between the two true vocal folds) for the two cases. As can be seen from the plot, both cases have reached a stationary state wherein the maximum gap is nearly constant from cycle-to-cycle for the two cases. Interestingly, the simulations indicate that the false-vocal folds have a noticeable effect on the amplitude of the true-vocal fold vibration. In particular where as the time-average value of the maximum glottal gap for the (-)FVF case is 0.0929 cm, the corresponding value for the (+)FVF case is 0.1238 cm. Thus, the presence of the FVF increases the amplitude of the TVF vibration by about 33%. Note that since most of the past studies (Agarwal et al, 2004, Kucinschi et al, 2006, Zhang et al, 2002) on FVF function have not included dynamically coupled models for the TVFs, this effect has not previously been observed. The presence of this effect has significant implications for the functional morphology of false vocal folds. First, it indicates that the false vocal folds potentially play an active role in facilitating phonation since they lead to a higher amplitude of vocal fold vibration for a given translaryngeal pressure drop. It also implies that the presence of the false vocal folds would reduce the threshold phonation pressure. Conversely, removal of false vocal folds (as is done in some supraglottic laryngectomies) would lead to a reduced ability to phonate.
and/or increased required effort for phonation. This is inline with the observations of Maceri et al. (1985).

The physical mechanisms that underlie this effect of the FVF however are far from clear but the current simulations afford the opportunity to gain insight into this phenomenon. Clearly, the increase in TVF vibration amplitude due to the presence of the FVF involves modification of the glottal flow and feedback of this onto the pressure on the true vocal folds. Now the glottal aerodynamics and the structural dynamics are examined in order to gain insight into this feedback process.

5.3.2 Translaryngeal Flow Rate and Impedance

Figure 5-2 shows the temporal variation of the flow rate through the glottis for the two cases and it can be seen that the presence of the FVF results in an increased translaryngeal flow rate. The time-mean flow rates are computed to be $1.338 \times 10^{-2} \text{ m}^2/\text{s} \ (332 \text{ ml/s})$ and $1.619 \times 10^{-2} \text{ m}^2/\text{s} \ (266 \text{ ml/s})$ for the (-)FVF and (+)FVF cases respectively. Thus, the presence of the false vocal folds increases the mean flow rate by about 21%. Furthermore, the time-averaged, peak flow rate increases from $2.97 \times 10^{-2} \text{ m}^2/\text{s} \ (587 \text{ ml/s})$ for the (-)FVF case to $4.32 \times 10^{-2} \text{ m}^2/\text{s} \ (865 \text{ ml/s})$ for the (+)FVF cases and this represents a 32% increase in this value. It should noted that while the increase in flow rate is directly connected with the increased amplitude of glottal vibration (and therefore the glottal gap), the differences between the relative increase in peak and mean values of the glottal gap and flow rate suggest that the cases with and without false vocal folds also have other
differences including the shapes of the velocity profile through the glottis as well as the temporal variation of the shape of the glottal gap.

Figure 5-2 Comparison of the time variation of the volume flow rate for the two cases.

Translaryngeal flow impedance can be defined in the usual way as in Eq. (5-1) and computed for these two cases. Current calculations show that the impedance for the (-)FVF case is $74756 \left( \frac{Pa}{m^2/s} \right)$ whereas that for the (+)FVF case is $61767 \left( \frac{Pa}{m^2/s} \right)$. In units of cm of water per ml/s of flow rate, the translaryngeal flow impedance is 0.040 and 0.033 for the (-)FVF and (+)FVF cases, respectively. Thus, there is a significant (nearly 17%) reduction in the translaryngeal flow impedance due to the inclusion of the false vocal folds.

As mentioned in the introduction, a number of previous studies have examined the issue of the effect of FVF on flow impedance and it is useful to make some comparisons with these previous studies. One of the most relevant studies in this regard is that of Agarwal et al. (2004). This study employed a static model of the larynx and determined the change in flow impedance for a number of different TVF and FVF geometries. One key parameter that was varied in this study was the ratio of the FVF gap to TVF gap (ratio denoted here by $\eta$) and reduction in translaryngeal flow impedance was found for the
various laryngeal shapes for $\eta > 1$. Thus, as long as the FVF gap was larger than the TVF gap, there was some reduction in the translaryngeal flow impedance. However, noticeable (25%) reduction in translaryngeal flow impedance was seen only in the range of $\eta$ from about 2 to 6. For the current simulations, given that for the (+)FVF case, the peak value of the TVF gap is 0.1238 cm and the FVF gap width is 0.667cm, the value of the $\eta$ for the current simulation can be estimated to be about 5.4. For a similar ratio of gap-widths, Agarwal et al. (2004) observed a reduction in flow impedance of up to about 20% for some cases, and therefore, the current results are consistent with this previous study. This is despite the fact that the current study employs flow-structure interaction between the flow and the true vocal folds, whereas the study of Agarwal et al. (2004) employed static true vocal folds.

Zhang et al. (2002) used simulations to examine, among other things, the effect of false-vocal folds on glottal aerodynamics and found that the presence of the false vocal folds produced a small reduction in translaryngeal flow impedance. However, there were two key differences between their simulations and the current study. First, the motion of the true-vocal folds was prescribed in their study and, therefore, could not respond dynamically to changes in the flow. Second, their simulations assumed axisymmetry and therefore precluded the jet from exhibiting any asymmetric behavior including bistability (Erath and Plesniak, 2006) as well as the Coanda effect. As will be shown later, this could potentially have a significant impact on the effect of the false vocal folds.

Finally current results are compared with the *in-vitro* studies of excised canine larynges by Alipour et al. (2006) Whereas they concluded that the presence of the false vocal folds
increased the *differential* flow resistance (defined as $Z'_L = \frac{\partial (\Delta P_j)}{\partial Q}$). However a closer look at their results (see Table 2 and Figs. 3 and 5 of their paper) indicates that for many cases, especially those with flow rates less than about 600 $ml/s$, the flow resistance as defined in Eq. (5-1) is actually lower with the FVF than without. Beyond this flow rate, the flow undergoes a transition and the translaryngeal flow resistance with FVF suddenly surpasses that of the case without FVF.

As mentioned before, if it is assumed that anterior-posterior length of the glottal gap is 2 cm, (a value representative of that in human anatomy) then it can be estimated that the mean flow rate in the current simulations for the (-)FVF case is about 266 $ml/s$ whereas that for the (+)FVF case is about 322 $ml/s$. If the data in Fig. 3 of Alipour et al. (2006) is considered, it should be noted that for a pressure difference of 10 cm of water (which is similar to current simulation pressure difference), the flow rates for the (-)FVF and (+)FVF cases are about 330 $ml/s$ and 480 $ml/s$ respectively. At a lower pressure difference of about 7 cm of water, the values for the (-)FVF and (+)FVF cases are about 240 $ml/s$ and 360 $ml/s$ respectively in the study of Alipour et al. (2006). Thus, given all of the differences between current simulations and the *in-vitro* study of Alipour et al. (2006), the results regarding translaryngeal flow resistance are reasonably consistent with each other, thereby suggesting that the current simulations and modeling approach are capturing the key features of the laryngeal dynamics.

The unsteady flow rate through the glottal gap has a direct relation to the sound produced during phonation. In particular, the monopole sound strength is directly related to
the time-rate of change of the volume flow rate (Zhao et al, 2002), i.e. $\dot{Q} = dQ/dt$. In Figure 5-3, this quantity have been plotted for the two cases and it can be clearly seen that the (+)FVF case has a larger peak value of $\dot{Q}$. This implies that for a given pressure across the larynx, the presence of the FVF will result in a larger sound pressure. This observation is in line with the in-vitro study of Alipour et al. (2006), and also confirms the clinical observation that the removal of FVF results in a reduced ability to phonate (Maceri et al. 1985).

Figure 5-3  Comparison of the time-rate of change of the flow rate, $\dot{Q}$, for the two cases. This quantity is related to the monopole source strength of sound.

5.3.3 Dynamics of Glottal Jet and Mechanism for Impedance Reduction

The above results indicate that the presence of the false vocal folds increase flow rate as well as vocal fold vibration amplitude. However the physical mechanism that underlies this behavior needs to be understood. In this section a detailed analysis of the flow will be conducted to gain insight into this issue.
Figure 5-4 Contours of spanwise vorticity for the two cases over one vocal fold vibration cycle. The eight plots correspond to eight equispaced time intervals over the vibration cycle.

Figure 5-4 shows a sequence of spanwise vorticity plots over one vocal fold vibration cycle for the two cases. The sequence starts with the situation where the vocal folds are just about to open. At this time instant, the supraglottic regions contain vortices created in the previous cycle. In the (-)FVF case, there are some large vortices in the vicinity of the glottal exit whereas in the (+)FVF case, the large vortices are found further downstream between the FVF gap and beyond. As the vocal folds start to open, the formation of a vortex dipole has been observed for both the cases (Figure 5-4 (b)). However, immediately, the two flows start to exhibit a noticeable difference; for the (-)FVF case, the vortex dipole deflects significantly from the glottal centerline whereas for the (+)FVF case, the vortex dipole maintains a high level of symmetry. At a later time instance, the deflection of the shear layer emanating from the glottis is further accentuated for the (-)FVF case to the extent that the shear layer in Figure 5-4 (d) approaches the upper wall of the channel as it starts to roll up into a large counter-clockwise rotating vortex. In contrast, the shear layer from the (+)FVF case at the same time instant, deflects slightly towards the upper false-
vocal fold but otherwise maintains its streamwise orientation. At the later stages of the vibration cycle, the flow near the glottis for the (-)FVF case contains large vortices of both sign whereas in the (+)FVF case, the large vortices are limited to the supraglottic space between and downstream of the FVFs. These observations are inline with the experiments of Shadle et al. (1991) and Kucinschi et al. (2006) that found that the jet was straightened due to the presence of the FVFs.

The flow condition in the supraglottic region at the end of the cycle then sets the stage for the evolution of the flow in the next cycle. As the vocal folds open again, the vortices in FVF gap for the (+)FVF case are pushed out due to the incoming volume flux and are not able to influence the formation of the new glottal jet. In contrast, for the (-)FVF case, the new jet enters the supraglottic space and is immediately influenced by the large vortices created in the previous cycle. Depending on the arrangement of these vortices at the instant the jet begins to emerge, the jet can deflect upwards or downwards.

Jet deflection is a key feature of this flow and we examine this behavior further for the two cases. In order to track the deflection of the jet, the \( y \)-coordinate of the maximum jet velocity over time is determined at a streamwise location corresponding to \( x=3.2 \text{cm} \) which is just downstream of the glottis. Figure 5-5(a) shows the time averaged streamwise velocity at this streamwise plane for the two cases. As can be observed, the peak glottal jet velocity is in the 15 to 20 m/s range and the peak velocity for the (+)FVF case is higher than the (-)FVF case. In Figure 5-5(b), the \( y \)-coordinate corresponding to the peak velocity has plotted for the two jets over ten vibration cycles and this plot clearly shows the effect of the FVF on the glottal jet deflection. Whereas in the (+)FVF, the jet deflection about the centerline is small (< 0.2 cm), the jet deflection in the (-)FVF case is large, reaching up to
about 0.8cm. Thus, the presence of the FVF significantly reduces the jet deflection immediately downstream of the glottis.

![Streamwise velocity profile](image1)

![Temporal variation in jet deflection](image2)

**Figure 5-5** (a) Streamwise velocity profile immediately downstream of the glottis at one time instance for both cases. (b) Temporal variation in jet deflection for both cases

It is also noted that the deflection in both cases has a stochastic character and the jet deflects randomly in one direction or the other from cycle-to-cycle. This behavior is directly associated with the vortex structures present in the supraglottic region. These vortices undergo complex non-linear interactions and result in flow configurations which are different for each cycle. As the glottal jet emerges into the supraglottic region, it immediately encounters these vortical structures and deflects according to the particular vortex configuration present at that instant. The presence of the FVF moves these vortices further downstream (into the FVF gap and beyond) and the glottal jet therefore does not encounter strong vortices until it reaches the FVF gap. Furthermore, the relatively narrow FVF gap also does not allow the jet to deflect too much from the centerline. Thus both these factors result in a significantly reduced jet deflection in this case.
The above observations about the glottal aerodynamics point to a mechanism for the reduction in flow impedance with the addition of the FVF. In order to understand this mechanism it is useful to revisit the pressure loss mechanisms in pipe and channel flows. If fully developed flow in a pipe or channel with no area variation is considered, loss in pressure is exclusively due to viscous losses and this is usually called the "major" loss (White, 1992). In addition, there are also the so called "minor" losses which are attributed to sharp bends, expansions, contractions etc. Such features produce flow separation and unsteadiness and sometimes even turbulence. All of this increases the mixing and this tends to increase the viscous losses in the flow (White, 1992). Thus, features that enhance the mixing in the flow also tend to increase the resistance to the flow. Comparing the supraglottic flow for the (-)FVF and (+)FVF cases, it seems apparent that the suppression of the large-scale random jet deflection in the (+)FVF will reduce mixing in the supraglottal flow and in doing so, reduce the losses associated with this phenomenon. In order to confirm this, contours of the fluctuation shear stress $\tau'_{xy}$ which is defined in Eq(5-2), have bee plotted for both cases,

$$\tau'_{xy} = \left\langle \frac{v'_1 v'_2}{U_G^2} \right\rangle$$

(5-2)

where $v'_1$ and $v'_2$ are the velocity fluctuations in the $x$ and $y$ velocity components, $U_G$ is the time averaged velocity through the glottis (estimated as $U_G = \bar{Q} / \bar{G}_FV$) and $\langle \cdot \rangle$ denotes a time-average. From Figure 5-6, it can be seen that the (-)FVF case shows a peak magnitude of the fluctuation shear stress of 0.07, whereas that of the (+)FVF case is about 0.055 which represents a 21% reduction. In addition, the region of large fluctuation shear stress is also
much larger for the (-)FVF case. This clearly shows that the false vocal folds reduce the fluctuation shear stress and hence the viscous losses in the flow, and provides a clear mechanism for the observed reduction in flow impedance. This phenomenon also has been reported by PIV study (Drechsel and Thomson, 2008). This study investigated the influence of supraglottal structures on the glottal jet created in a two-layer synthetic, self-oscillating vocal fold model. They found that the magnitude of the velocity fluctuation decreases with the presence of the false vocal folds. However, data on flow rate and vibration amplitudes has not been provided in this study.

Figure 5-6 Contours of fluctuation shear stress in the glottal jet. (a) (-)FVF case (b) (+)FVF case.
5.3.4 Vocal Fold Vibration

As discussed in the previous chapter, the asymmetric flow produces asymmetric driving force on the true vocal folds, which leads to an asymmetric vibration. Also as shown in 5.3.3, the presence of false vocal folds straightens the jet flow and reduces the asymmetry of flow. Thus, a smaller asymmetric force should be expected for (+) FVF than (-) FVF and it will lead to more symmetric vibration for (+) FVF case. In order to examine these conclusions, a comparison of correlation of the superior vocal fold surface point pair (shown in Figure 5-7) has been made between (+) FVF and (-) FVF.

![Figure 5-7 Phase plane plots of the displacement of the points on the superior side of the two vocal folds.](image-url)

(a) $x$ displacement (inferior-superior)  
(b) $y$ displacement (abduction-adduction) from centerline.
Figure 5-7 shows a comparison of the $x$- and $y$- displacements of the vocal folds for the two cases. As can be seen, there is some degree of asymmetry in the vocal fold vibration for both cases. Whereas the $x$-displacement in the (-)FVF case seems to be less symmetric than the (+)FVF case, the converse seems to be true for the $y$-displacement. A similar behavior is seen for the inferior points (not shown here)

The asymmetry can be quantified by computing the correlation coefficient for the above displacements. For instance for the $x$-displacement, the correlation coefficient $R_x$ can be computed as

\[
R_x = \frac{\sum x'_b x'_t}{\left( \sum x'_b x'_b \times \sum x'_t x'_t \right)^{1/2}}
\]

where $x'_b$ and $x'_t$ are the displacements of the points on bottom and top vocal folds about their mean position and the summation is over the time-steps in the simulation. A similar definition is used for $R_y$ and furthermore, the summation is carried out over ten vibration cycles.

<table>
<thead>
<tr>
<th></th>
<th>Inferior Points</th>
<th>Superior Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-)FVF Case</td>
<td>$R_x$ 0.652</td>
<td>$R_y$ 0.951</td>
</tr>
<tr>
<td></td>
<td>$R_x$ 0.653</td>
<td>$R_y$ 0.954</td>
</tr>
<tr>
<td>(+)FVF Case</td>
<td>$R_x$ 0.698</td>
<td>$R_y$ 0.859</td>
</tr>
<tr>
<td></td>
<td>$R_x$ 0.725</td>
<td>$R_y$ 0.898</td>
</tr>
<tr>
<td>% difference</td>
<td>+7%</td>
<td>-10%</td>
</tr>
<tr>
<td></td>
<td>+11%</td>
<td>-6%</td>
</tr>
</tbody>
</table>

Table 5-2 shows the computed correlations coefficient for the two sets of points. Interestingly the simulations reveal that whereas the FVFs increase symmetry in the $x$-
motion (inferior-superior motion), they decrease symmetry in the y-motion (abduction-adduction motion). The increase in symmetry in the inferior-superior motion is due to the increased symmetry in the supraglottic flow for the (+)FVF case which results in a more symmetric pressure force on the two vocal folds. The increase in asymmetry in the abduction-adduction motion for the (+)FVF case is associated with the increased glottal flow and vibration amplitude for this case. In general, as the intensity of the glottal flow and vibration amplitude increases, the non-linearities in the glottal flow are enhanced and this can lead to more complex and asymmetric abduction-adduction motion in the vocal folds.
CHAPTER 6 LOCAL GRID REFINEMENT

As stated in Chapter 4, high-fidelity 3D models of phonation require CPU times and memory that is about two orders of magnitude larger than those for 2D models. Thus in order to conduct comprehensive 3D modeling studies of phonation substantial advances is needed in the computational and numerical methodology. An obvious computational approach to this problem is use parallel computing to reduce the turn around time and this approach is currently being developed by other members of our research group. A second approach is to increase the computational efficiency of the simulations approach. Noting that although the Cartesian grid based immersed boundary approach simplifies the task of grid generation for complex geometries, the single-block Cartesian grid does not allow for the optimal control of grid resolution in the flow domain. Take for example the flow through the 3D glottal model shown in Figure 6-1 and the typical Cartesian grid that is employed in the study. The regions that require high resolution are the glottal gap as well as the supraglottal region where the glottal jet is expelled. The region upstream of the glottis does not require significant resolution in either direction and neither does the region beyond the false-vocal folds. Thus, an optimal grid would be one that allows high resolution only in the critical areas. However, the use of a single-block Cartesian grid leads to grids that are non-optimal. For instance, the high vertical resolution provided in the glottal gap is automatically extended throughout the computational domain leading to wasted resolution. In 3D, the situation becomes even more non-optimal.
Thus, if methods can be developed in conjunction with the IBM which allow us to place resolution more precisely where it is needed, it could significantly speed up the solution process. The potential for decreasing the turnaround time for simulations can be understood by noting that CPU time for typical Navier-Stokes solvers scales as $O(M^\beta)$ where $M$ is the total number of grid points and the exponent $\beta$ varies from 1 for an ideal multigrid method to 2 for a typical stationary iterative method (such as Gauss-Siedel line successive over-relaxation). For the current solver, $\beta$ has been empirically found to be about 1.2 (see Figure 2-4) for canonical flows and might go up to 1.5 for more complex configurations. Now assuming that for a 3D flow, optimal resolution placement can reduce the grid size in each direction by 50%, then for $\beta=1.5$, it would reduce the CPU time to less than 5% of the original time. Even for a 25% reduction in grid in each direction and an efficient multigrid with $\beta=1.2$, CPU time would be reduced to 35% of the original time. Thus the potential for speeding up the simulations with such a method is immense and this is the motivation for the development of the local mesh refinement (LMR) scheme that is described in the current chapter.
6.1 Background

Figure 6-2 shows the topology of a locally refined mesh that could be used in the flow model. The essential feature of LMR is that adjacent cells can have significantly different solutions – usually the resolution in adjacent cell different by a factor of upto 2, 4 and 8 for 1D, 2D and 3D grids respectively. Note that with this locally refined mesh, high resolution is provided only in regions of high gradient and regions that do not have high flow gradients retain coarse grids.

![Typial topology of a locally refined mesh that could be used in modeling phonation.](image)

Local mesh refinement in general is playing an ever increasing role in the efficient solution of industrial and scientific problems. In recent decades, extensive research and efforts have been undertaken to develop local refinement procedures (Berger, 1984, Khokhlov, 1998, Zhu, 1988, Pember, 1995, Minion, 1996, Almgren, 1998, Gerritsen, 1998, Cao, 1999, Li, 2001, Roma, 1999). Although there are a number of variations, all these methods allow for the use of multi-resolution approximation in the simulation. If the local refinement is done based on some automated criteria (based on truncation error estimates etc), then the method is also called adaptive mesh refinement (AMR).

AMR methods fall into two categories based on the way the refinement strategy. The first category includes those that involve mesh/stencil refinement. In these algorithms, either the existing mesh is split into several smaller cells or additional nodes are inserted.
locally, the latter leading to the so called \( h \)-refinement. This group of adaptive algorithm can be further categorized by the mesh type, i.e., hierarchical structured grid approach and unstructured mesh refinement approach. One representative of structured grid approaches is the adaptive Cartesian mesh refinement proposed by Berger and Oliger (1984). Their approach is established on regular Cartesian meshes, but arranged hierarchically with different resolutions. At the fine-coarse cell interfaces, special treatment is required for the communications between the meshes at different levels. With regard to the unstructured mesh refinement approach, Zienkiewicz and Zhu (1988) reviewed the state-of-the-art of AMR strategies in the finite-element community and discussed the important role of error estimation and automatic adaptation in the finite element analysis. These types of locally adaptive refinement approaches have been widely applied to the numerical simulations in many areas, such as the compressible flow (Pember, 2002), incompressible flow (Chacon, 2006, Ding, 2006, Almgren, 1998), and flow-structure interaction (Roma, 1999).

Unlike the local refinement algorithm, the second category of adaptive algorithms involves global mesh-redistribution. These methods move the mesh points inside the domain in order to better capture the dynamic changes of solution. Therefore, such techniques are usually referred as moving mesh method or \( r \)-refinement. Applications of the moving mesh method have been extended to many challenging problems, such as the thin flame propagating, drop formation, non-breaking free surface wave (Ding, 2006, Cao, 1999, Li, 2001). However, the current effort is towards local refinement strategies.

Recently unstructured \( h \)-refinement has been employed in conjunction with the immersed boundary method (Tullio, 2007, Singh, 2007) since it is relatively easy to develop highly complex local refinement topologies. However, the approach here results in
loss of “structure” in the grid and consequently, one cannot take advantage of powerful block-iterative and geometric multi-grid schemes for solving the discretized equations. For the structured $h$-refinement, it is relatively difficult to work with complex refinement topologies but since the structured nature of the grid topology is maintained, one has access to all of the powerful solution methodologies that available for such grids.

In the current study, in order to take advantage of the highly efficient multi-grid method available in the existing solver (shown in Figure 2-4), a new nested grid refinement (NGR) procedure has been developed that provides a high level of flexibility in local refinement while retaining the advantages of a structured grid topology. In order to implement this method, a new data structure called the “Eulerian Global Map (EGM)” has been introduced to manage the connectivity between the nested refined grid blocks. The use of this data structure simplifies the local refinement approach and proves to be robust even for highly complex refinement topologies. In addition, a level-set type of approach is used to determine the criteria for refinement and cell blanking used to construct nested grid with highly complex topologies.

6.2 Nested Grid Refinement (NGR) Methodology

6.2.1 Data Structure and Mesh Adaptation

In the current approach, local refinement is achieved through a multi-level grid nesting procedure which consists of individual structured grid blocks inserted into a background mesh. In the past, the difficulty of tracking the connectivity for structured block grid refinement inhibited the wide-spread use of this method, especially for complex moving boundary problem. When the blocks are moving or the blocks do not have a rectangular
shape, the topological relations between blocks are not straightforward and it requires maintaining the connectivity for each single grid cell. Traditional tree structure treats the mesh as an unstructured mesh and has the same disadvantages. In the current effort the grid refinement is implemented by equal division of a cell into 8 finer cells (4 for 2D case). Thus refinement has an explicit layered structure and a map is created for each refinement layer and covers the whole domain.

Figure 6-3(a) shows a three layer block mesh as an example. In this mesh, the background block is rectangular and two finer refined blocks are used at the right lower corner and each refinement block has a unique ID. It should be noted that each layer might have multiple blocks. Here for the illustration purpose, each layer only has a single block. The first layer is always only composed of background block (block 1) and finer layer has a double resolution of previous layer. For each layer, a Global Eulerian Map is created with same resolution. Figure 6-3 (b), (c), (d) show the corresponding maps for each layer. Each cell in the map is used to store the corresponding block ID and since only an integer or short type variable is stored, the overall memory usage of maps is not very large, even when the whole domain stores the map for each nested grid. For the simplicity of methodology, only one layer difference is allowed between the adjacent blocks and the map only shows one-layer-deep information from both coarser and finer layers. Thus the neighboring nodes of one cell can be tracked through the corresponding map. For example, for the cell of block2 at left lower corner (shown in Figure 6-3(c)), eastern neighbor nodal value can be interpolated from block3, western neighbor value can be interpolated from block1, northern neighbor nodal value can be gotten directly from block2 and southern value can be computed through boundary condition. Further more, the location of each
block is tracked using the global map index of lower-left corner cell. For example, lower corner of block2, which has local index of (1, 1), has a global map index of (3, 1) of map 2. By adding the offset to the left-lower corner, the local index of each cell can be transferred to the global map index uniquely. Thus with this map, connectivity and location of blocks are easy to track and very robust even for very complex geometric shape.

Figure 6-3 (a) Three layer mesh structure which refines the grid at right lower corner. (b) Block ID contours of first layer map, (c) Block ID contours of second layer map, (d) Block ID contours of third layer map.

Mesh refinement in this study is implemented in two ways, specified region refinement and level-set refinement. For the first type, the refinement region is given before the
simulation, and the refined block is usually rectangular or cuboidal (in 3D). This type of refinement is mainly used to details of flow in regions whose extend is know ahead of time (such as wakes).

For the second type of refinement, a level-set method (Osher and Sethianm, 1988) is used to determine the region of refinement. Level-set refers to a filed variable that correspond to a distance from a specified curve/surface in space. Thus, we if want to create a refined mesh which tightly surrounds an immersed body, then level-set based refinement allows us to accomplish this. The underlying nested mesh might still have a simple rectangual/cuboidal shape but “cell blanking” coupled with level-sets can be used to create nested grid that are effectively curvilinear in shape. Figure 6-4 shows a level-set refinement block, in which only the cells, which distance is less 0.2 from the cylinder surface, are active through cell blanking. Thus the cells are divided into active cells and inactive cells and the Navier-Stokes equations are only solved on the active cells. With this type of
refinement, it is very easy to produce a refinement for very complex shape body and to create meshes that provide fine-resolution to boundary layers.

6.2.2 Solution Procedure and Block Interface Matching Scheme

In this study, the same finite difference scheme (equation 2-3 – 2-11) is employed as the background immersed boundary solver. As per the fractional step, first the advection-diffusion equation is solved simultaneously for all block. Once the convergence has been achieved for all of blocks, the pressure Poisson equation is solved for all of block simultaneously until the convergence is reached for all of the blocks. It should be noted that here the pressure Poisson equation is solved by line-SOR scheme for time being. Mulitgrid method could be implemented based on line-SOR method but has not been accomplished in the current effort.

The key aspect of any refinement procedure is the approach taken to pass information at the boundary of two blocks with different resolutions. In the current method, a ghost cell methodology combined with flux conservation is used at the block interfaces. A schematic of the typical block interface is shown in Figure 6-5. For simplicity, a two-dimensional (2D) flow with a two-block domain is employed to illustrate this method. It is worth noting that the current solver is 3D and 3D cases employ the same methodology as with the 2D case. As shown in Figure 6-5, the coarse (left) and fine (right) grid domains are denoted by $D_1$ and $D_2$ respectively. The cell, which is outside the block and has an adjacent neighbor inside the block, is called ghost cell, such as NE, E, SE for $D_1$ and 1w, 2w for $D_2$. The information exchange between blocks is implemented through the ghost cells and in order to have a seamless connection between blocks, the fluxes, such as mass fluxes, velocity
gradient flux and pressure gradient are conserved at the block interfaces. Due to the explicit
treatment of momentum term, $\partial U_j \mu_j / \partial x_i$, momentum flux $U_j \mu_j$ is only matched once at
all of block interfaces at next time step.

For the interpolation from fine-to-coarse grid, for example, the ghost value of cell-
centered variable $\Phi$ at cell E of $D_1$ can be computed by linear interpolation of surrounding
fine nodes (1, 2, 3, 4). This leads to the following relationship shown in Equation (6-1)

$$\Phi = \sum_{i=1}^{4} \beta_i \phi_i$$  \hspace{1cm} (6-1)

where $\beta_i$ is interpolation weight, $\phi_i$ is the generic flow variable at fine block.
The coarse-to-fine interpolation for the ghost cells of the fine-grid is somewhat more complicated. Robustness of the procedure is enhanced significantly by ensuring conservation of fluxes across the block interface. However, the fluxes at the interface usually are not the same between the coarse grid and fine grid. In order to ensure that fluxes at block interfaces have unique values, the fluxes are enforced to be equal to the fluxes computed from the fine grid. Thus, for the coarse block, instead of ghost values, the fluxes are given at block interfaces as the boundary conditions.

In order to ensure conservation of mass flux, the face-normal velocity $U_i$ can be computed in terms of mass flux at finer level. For example, face velocity $U_e$ can be computed based on the face velocity $U_{1w}$ and $U_{2w}$,

$$U_e \times dy = U_{1w} \times dy_1 + U_{2w} \times dy_2$$

(6-2)

where $dy$ is the $y$ grid size for coarse grid and $dy_1$ and $dy_2$ are the $y$ grid size for fine grid cell 1 and 2 respectively.

Momentum flux, which is computed as $U_i u_j$ for any cell face can also be conserved on the block interface in a similar manner. For example, the x-momentum flux per unit mass for the east face of the coarse cell, which would normally be computed as $U_e u_e$ (where $U_e$ is the volume flux per unit area on the east-face and $u_e$ is a measure of the x-momentum on the east face), is replaced by $\pi_e$ where:

$$\pi_e \times dy = \pi_{1w} \times dy_1 + \pi_{2w} \times dy_2$$

(6-3)

where $\pi_{1w} = U_{1w} u_{1w}$ and $\pi_{2w} = U_{2w} u_{2w}$ are the momentum flux per unit mass on the west faces of the fine mesh cells 1 and 2 computed based on the solution on the fine mesh. Note
here that $U_{2w}$ are the face velocities for the two fine mesh faces that constitute the coarse mesh face. The values $u_{1w}$ and $u_{2w}$ can be interpolated from $u_1$ and $u_{1h}$ and $u_2$ and $u_{2h}$ respectively.

Fluxes that are proportional to gradients such as the diffusive flux and pressure gradient can also be conserved at the block interface. Consider for example the gradient flux $\partial \Phi / \partial x$ on the east-face of the coarse mesh cell under consideration. This would generally be estimated as $(\partial \Phi / \partial x)_e \approx (\Phi_E - \Phi_p) / (x_E - w_p)$. However, for the case where the east-face is on the interface block, the gradient at this face is replaced by $\gamma_e$ wherein:

$$\gamma_e \times dy = \frac{\phi_1 - \phi_{1h}}{x_1 - x_{1h}} \times dy_1 + \frac{\phi_2 - \phi_{2h}}{x_2 - x_{2h}} \times dy_2$$

which essentially feeds the fine-mesh based gradient to the coarse cell. Eq (6-4) is also used ghost values of fine block and the ghost value $\phi$ can be interpolated from the surrounding coarse mesh node values. For example, ghost value $\phi_{1w}$ can be computed by interpolation from $\Phi_N$, $\Phi_{NE}$, $\Phi_E$ and $\Phi_p$ and ghost value $\phi_{2w}$ can be computed by interpolation from $\Phi_S$, $\Phi_{SE}$, $\Phi_E$ and $\Phi_p$.

6.2.3 Flow Chart of NGR Solver

The flow chart describes exactly how the simulations with NGR work. In the current solver, a block iterative method is used for solving the field (advection-diffusion and pressure Poisson) equations. The outer iteration in this method consists of a sequential
solution of the various blocks (starting with the coarsest and proceeding to the finest) and for each block, line-SOR method is used to solve the governing discrete equation.

1. Start Simulation.

2. Specify initial conditions, $u^n$, $U^n$, $p^n$, and physical boundary conditions.

3. Setup “Eulerian global map” according to the block topologies.

4. Identify the ghost-cells for all blocks and store the connectivity for interpolation.

5. $t^{n+1} = t^n + \Delta t$.


   6.1. Compute ghost-cell velocity $u^n$, face velocity $U^n$, convective flux $U^n u^n$, diffusive flux $\partial u^n / \partial x$ at block interfaces. (Eq. 6-1~6-4)

   6.2. Compute $N_i^n$, $N_i^{n-1}$ and $D_i^n$ for each block. (Eq. 2-3)

   6.3. Solve advection-diffusion equation for each block. (Eq. 2-3)

   6.4. Compute ghost-cell velocity $\ast u^n$, diffusive flux $\partial u^n / \partial x$ at block interfaces. (Eq. 6-1, 6-4)

   6.5. If $\ast u^n$ are converged for all of blocks, then proceed to next step, or return to 6.3.

7. Compute $U^*$ and recompute $U^*$ at conservative interfaces for each block. (Eq. 2-4~2-8)

8. The pressure Poisson step.

   8.1. Solve Poisson equation for each block. (Eq. 2-10)

   8.2. Compute ghost-cell pressure $p^*$, and pressure gradient $\partial p^* / \partial x$ at block interfaces for each block. (Eq.6-1, Eq.6-4)

   8.3. If $p^*$ is converged for all of the blocks then proceed to next step, or return to 8.1.
9. Correction step

9.1. Compute $p^{n+1}$ for each block. (Eq. 2-11)

9.2. Compute $u^{n+1}$ and $U^{n+1}$ for each block. (Eq.2-12, Eq.2-13)

9.3. Recompute $u^{n+1}$ and $U^{n+1}$ at interfaces based on conservative pressure gradient and pressure ghost value for each block (Eq.6-1, Eq.6-2)

10. If stationary boundary problem go to 5, or proceed to next step.

11. If moving boundary problem, move the boundaries and surrounding blocks; go to 3.

6.3 Validation

6.3.1 2D Lid-driven cavity flow

The first test case is lid-driven cavity flow. In this test, a steady incompressible lid-driven flow problem in a square cavity is solved by NMR. A two-dimensional lid-driven cavity flow at Reynolds number of $Re = 1000$ is considered. For this case, two-layer refinement is used. The background coarse mesh is $64 \times 64$ uniform mesh and the region which is $0.16d$ away from the four boundaries is refined by four fine blocks (shown in Figure 6-6(a)). The vorticity contour is shown in Figure 6-6(b) and it is smooth across the block interfaces. Figure 6-6 (c) and (d) show the vertical centerline $u$-velocity and horizontal center line $v$ velocity. Both of them are smooth cross the interfaces and match the pervious study (Ghia, 1982). The performance of NGR method is also compared with single block grid. Two single block cases with a uniform grid, $64 \times 64$ and $128 \times 128$ respectively, are employed to simulate the same flow. Vertical centerline $u$-velocity and horizontal center line $v$ velocity for these two cases are also plotted and compared with
Figure 6-6 2D lid-driven cavity flow at Re = 1000, (a) mesh configuration, (b) vorticity contour, (c) comparison of $u$ at vertical center line, (d) comparison of $v$ at horizontal center line.

NGR solution in Figure 6-6 (c) and (d) respectively. For the $64 \times 64$ single block, solution does not match the NGR solution which indicates higher resolution is required. For the higher resolution case, $128 \times 128$, the grid independent solution is reached. The CPU time
is 574 s for the 128×128 single block case and is 399 s for NGR method on a 1.86Ghz Intel Dual Core Desktop PC. Thus the NGR method is about 30% faster than single block method. The advantage of the NGR method will increase with increasing grid.

6.3.2 Two-dimensional Flow Past a Circular Cylinder

The second test case is 2D flow past a circular cylinder at Reynolds number based on the diameter of 100. The 2D cylinder of normalized diameter one is placed at the center of a 30×30 square domain and a 32×32 coarse background mesh is used to cover the whole domain. On the top of this background mesh, a 49×49 rectangular refinement block (the green block shown in Figure 6-7) has been employed to refine the mesh within a region between 3.75 ≤ x ≤ 26.25 and 3.75 ≤ y ≤ 26.25. On the top of this green block, another rectangular refinement block (the blue block shown in Figure 6-7) further refines the mesh. These two rectangular blocks are designed to capture the wake and another three layer refinement blocks have been placed on the top of the blue block to provide enough resolution for the boundary layer. These three blocks are level-set type blocks and refinement region for these three blocks are 0.2d, 1d and 2d away from the surface of cylinder respectively. The drag coefficient and lift coefficient, defined as $C_D = F_D / \left( \frac{1}{2} \rho U_\infty^2 d \right)$ and $C_L = F_L / \left( \frac{1}{2} \rho U_\infty^2 d \right)$ respectively, where $F_D$ and $F_L$ are drag force and lift force respectively, have been computed and plotted with non-dimensional time $t U_\infty / d$ in Figure 6-7 (b). The mean drag coefficient, maximum lift coefficient and Strouhal number, defined as $St = f U_\infty / d$, where $f$ is the vortex shedding frequency, have been compared with other studies (shown in Table 6-1). The comparison shows that current results match previous studies very well. Figure 6-7 (c) and (d) show the velocity
contour and vorticity contour respectively in the vicinity of cylinder. It shows that boundary layer has been resolved and both velocity and vorticity are smooth cross the block interfaces.

Figure 6-7 flow past 2d cylinder at Re = 100, (a) mesh configuration (b) computed temporal variation of drag and lift coefficients. (c) u velocity contour at vicinity of cylinder (d) vorticity contour at vicinity of cylinder.
### Table 6-1: Comparison of computed mean drag coefficient, maximum lift coefficient, and Strouhal number with results from previous 2D cylinder simulations.

<table>
<thead>
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<th>$C_D$</th>
<th>$C_L$</th>
<th>$St$</th>
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<tr>
<td>Current Simulation</td>
<td>1.331</td>
<td>0.298</td>
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<tr>
<td>Dias and Majumadar</td>
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<td>0.283</td>
<td>0.165</td>
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<tr>
<td>Lai and Peskin (2000)</td>
<td>1.33</td>
<td>0.330</td>
<td>0.165</td>
</tr>
<tr>
<td>Tseng and Ferziger (2003)</td>
<td>1.42</td>
<td>0.29</td>
<td>0.164</td>
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</table>

#### 6.3.3 3D Flow Past a Sphere

The third case is flow 3D flow past a sphere at a Reynolds number (based on the diameter) of 350. The 3D sphere of diameter 1 is placed at location of (5, 7.5, 7.5) inside a $15 \times 15 \times 15$ rectangular domain (shown in Figure 6-8(a)). A $65 \times 65 \times 65$ background mesh is employed to cover the whole domain and the rectangular region, $1.875 \leq x \leq 12.1875$ by $5.625 \leq y \leq 9.375$ by $5.625 \leq z \leq 9.375$, is refinement by a rectangular block (green block shown in Figure 6-8(b)). The level-set type blocks, 1.0, 0.5 from the sphere surface respectively is employed to resolve the boundary layers. Figure 6-8(c) shows the $u$ velocity at the center-plane corresponding to $y = 7.5$ and Figure 6-8(d) shows the wake vortex structure. The planar symmetric vortex topology observed in the wake has also been observed before in the simulations of Mittal (1999) and the current NMR methodology therefore results in a qualitative match with previous results.
Flow Past a Complex 3D Body: Human Swimmer

In this case, flow past a complex geometric shape - that corresponding to a human swimmer, has been tested to show the ability of the current solver to handle highly complex geometrical shapes. The geometric shape of the swimmer has been taken from von Loebbecke (2008) and has been located at coordinate (2, 2, 2) inside a $8 \times 4 \times 4$ rectangular
domain. A $65 \times 33 \times 33$ background mesh (shown in Figure 6-9(a)) covers the whole domain and three level refinement with blocks, 1 unit, 0.5 unit and 0.2 unit from body surface, respectively, are employed (shown in Figure 6-9(b), (c), (d)). The grids generated show that the mesh adaptation algorithm can deal with complex shape body and produce reasonable grid refinement. Figure 6-10(a) shows the u-velocity contour at the center plane of $z = 2$. From this figure, it can be found that velocity is smooth across the block interfaces and the boundary layer has been well resolved.

Figure 6-9 (a) Computational domain and swimmer body shape (b) side view of refinement mesh in the vicinity of body (c) top view of refinement mesh in the vicinity of body (d) front view of refinement mesh in the vicinity of body.
6.3.5 Flow Past 2D Cylinder with Moving Blocks

In this case, the dynamic grid refinement capability, i.e. the ability to perform simulations with time-dependent refined blocks has been tested with the case of flow past a 2D stationary cylinder. A 2D cylinder of radius 1 has been placed at a location of (5, 5) inside a 10×20 rectangular domain and a 128×64 background mesh has been employed to cover the whole domain. The cylinder has been surrounded by two concentric elliptic shape refinement blocks, which are initially defined by \((x - 5)^2/9 + (y - 5)^2/4 = 1\), \((x - 5)^2/2.25 + (y - 5)^2/1 = 1\) respectively. During the simulation, the two blocks are subjected to a specified sinusoidal rational motion, defined as \(\theta = (\pi/6)\sin(0.2\pi)\). Figure 6-11 shows that vorticity is smooth across the block interfaces different instant time when the refinement block is moving.

Thus, it has been shown that a nested mesh refinement algorithm can be used successfully to simulate flow with the current IBM in a manner that employs a grid with more precise control of resolution. The method developed here represents a significant
advance in the ability to employ Cartesian grid based IBM for complex flows. The method should significantly reduce the computational cost of all simulations, especially those associated with modeling of phonation in complex/realistic geometries. The application of this method to biomechanical modeling of phonation is left as a future task.

Figure 6-11 Vorticity contours at different time instant for flow past 2d stationary cylinder with two concentric refinement blocks with specified sinusoidal motion.
CHAPTER 7 SUMMARY AND CONCLUSION

Medialization Laryngoplasty is the common surgical procedure to treat vocal fold paralysis. However, this procedure has a high revision rate (up to 24%) due to the high sensitivity of the surgical outcome on the location and shape of implant. Any tool that could help guide the surgeon in choosing the right shape and location of the surgical implant could significantly advance the treatment of vocal fold paralysis/paresis. The approach, which forms the motivations of the current work, is to develop patient-specific biophysical models of the larynx and use these to assist with planning this surgery. However, phonation is a very complex biological phenomenon resulting from highly nonlinear biomechanical coupling between glottal aerodynamics and vocal fold tissue. Thus, developing a biophysical model of this organ is a non-trivial proposition. In the past, due to the complexity and nonlinearity of flow-tissue interaction, reduced order modeling has been employed (lumped mass vocal folds model or inviscid glottal flow model) for this type of study. Such models could not provide enough fidelity for direct clinical diagnosis and surgical guidance. In the current work, a high fidelity flow-tissue interaction computational tool has been developed which couples the full Navier-Stokes, fluid dynamics governing equations and the Navier equation for continuum solid mechanics. The tool developed here is subjected to qualitative, as well as quantitative validation, and used to examine the biophysics of phonations. The limits of the current modeling approach are also examined and ways to overcome these limits explored. In the following, we summarize the key aspects of the current study and highlight the primary accomplishments.
7.1 Phonation Modeling

In order to provide details of flow field and high fidelity of driven force on vocal folds, an existing immersed boundary method solver (VICar3D) has been employed to solve the Navier-Stoke equations. At the same time, in order to provide the correct vibration pattern, a new three-dimensional finite-element solid mechanics solver has been developed to solve the Navier equation for continuum, viscoelastic material. The vocal fold model also includes a penalty-coefficient based model vocal folds collision. The fluid and solid solvers are coupled explicitly to form the flow-tissue interaction tool to investigate the phonation problem. The geometric shape of vocal folds has been based on a high resolution CT scans and a three-layer inner anatomical structure has been adopted for the vocal folds. The vocal fold properties are assumed to be transversal isotropic and mechanical properties are taken from previous experimental measurements. Thus, high-fidelity modeling is used both for the flow as well as the vocal fold dynamics. This is a significant advance over what has been the norm in this field.

7.2 Biophysics of Phonation

The fluid-structure interactions associated with phonation have been successfully reproduced in a 2D laryngeal model. Self-sustained vocal fold vibrations with vibratory models that correspond to physiological observations have been captured (Zemlin, 1988) and the vibration frequency is in the correct phonatory frequency range (Titze, 1994). The glottal flow has been shown to form a pulsatile turbulent jet and this jet flow is highly asymmetric. The subglottal pressure effects on fundamental frequency have been investigated. The fundamental frequency shows a nonlinear increase at lower end of
phonatory pressure range and a constant value at or above the normal frequency due to the material linear assumption. This behavior matches previous observations (Ishizaka, 1981).

An original contribution to the biophysics of phonation is a detailed study of the effect of false vocal folds on phonation. Through a systematic comparison of two models, one with false vocal folds and one without, we show that that the presence of the false vocal folds decreases translaryngeal flow impedance and increases the mean flow rate through the glottis. This also results in a larger amplitude of vibration in the true vocal folds. Thus, false-vocal folds tend to aid phonation by reducing the effort required to phonate and by increasing the sound intensity for a given effort.

A key contribution of this investigation is the identification of a physical mechanism for the reduction in translaryngeal impedance due to the false vocal folds. The simulations show that the presence of the false vocal folds reduces the stochastic deflection in the glottal jet. This reduces the mixing in the flow as evidenced by a reduced magnitude of the fluctuation shear stress. This in turn reduces the viscous losses and thereby the translaryngeal flow resistance.

Three-dimensional simulation has also been performed on a simple laryngeal model. The simulation has successfully captured the transient self–sustained vocal fold vibration pattern and the jet flow also shows a strong asymmetric effect. This indicates that the 3D solver can be employed to simulate a three-dimensional phonation case. However, due to large disparity between the glottal gap width and channel width and the larger transient time, these simulations are very time-consuming and are only feasible after significant speedup in solver.
7.3 **Local Mesh Refinement**

In order to achieve significant improvement in computational speeds, a new local-grid refinement approach that employs a hierarchical nested grid approach has been developed and applied to a sharp interface immersed boundary solver. The key feature of the methodology is that the structured grid approach is retained at all the refinement levels and this allows use of powerful line-SOR schemes and a geometric multigrid method. Simulations of canonical flows have been conducted and these indicate that the solver accurately reproduces the key features of the flows.

7.4 **Further Work**

Although the current code shows the capability to simulate complex flow-tissue interaction for phonation, several key improvements are still needed for clinical usage. First, even though the local refinement methodology has been integrated in the solver, further work is needed to couple it with geometric multi-grid method. Second, more sophisticated refinement criteria, such as “velocity gradient based” are needed to incorporate to further reduce the computational cost. Finally, a more realistic 3D larynx shape is needed which also demands a parallel capability.
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APPENDIX I

Numerical Integration table (Wang, 2003)

Gauss Integration

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2D Harmer Integration

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*because of the integration field of triangular and variables, the sum of weight coefficients should be 1/2, so the weight coefficients in the table should time 1/2.

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<td>( a )</td>
<td>( \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} )</td>
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<td></td>
<td>( a )</td>
<td>( \alpha, \beta, \beta, \beta )</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( b )</td>
<td>( \beta, \alpha, \beta, \beta )</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>Second</td>
<td></td>
<td>( R = O(h^3) )</td>
<td>( c )</td>
<td>( \beta, \beta, \alpha, \beta )</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( d )</td>
<td>( \beta, \beta, \beta, \alpha )</td>
<td>( \frac{1}{4} )</td>
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</table>
\[
\text{Where} \\
\alpha_1 = 0.0597159 \\
\beta_1 = 0.4701421 \\
\alpha_2 = 0.7974270 \\
\beta_2 = 0.1012865 
\]
*because of the integration field of tetrahedron and variable itself, the sum of weight coefficients should be 1/6, so the weight coefficients in the table should time 1/6.*

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
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<td>( \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} )</td>
<td>( 9 )</td>
<td>( 20 )</td>
<td>( 9 )</td>
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<tr>
<td>R = O(h^4)</td>
<td>( \frac{4}{5} )</td>
<td>( 20 )</td>
<td>( 20 )</td>
<td>( 9 )</td>
<td>( 20 )</td>
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Third
Nodal force for distributed surface load

2D Formulation

Assume on single segment pressure is perpendicular to the segment

\[ P(x, y) = \sum_{i=1}^{2} N_{i}P_{i} \]

Where \( N_{i} = \frac{L_{i}}{L} \).

The equivalent node load

\[ P_{e,i} = \int_{L_{i}} N_{i}P_{i}dL \]

So

\[ P_{e,i} = \int_{L_{i}} N_{i}(N_{i}P_{i} + N_{j}P_{j})dL \]
\[ = \int_{L_{i}} (N_{i}N_{i}P_{i} + N_{i}N_{j}P_{j})dL \]
\[ = \int_{L_{i}} N_{i}N_{i}P_{i}dL + \int_{L_{i}} N_{i}N_{j}P_{j}dL \]

Since

\[ \int_{L_{i}} N_{i}^{2}dL = \int_{L_{i}} \left( \frac{L_{i}}{L} \right)^{2} \frac{1}{L^{2}} \int_{L_{i}} L_{i}^{2}dL = \frac{1}{3}L , \]
\[ \int_{L_{i}} N_{i}N_{j}dL = \int_{L_{i}} \frac{L_{i}}{L} \frac{L_{i} - L_{j}}{L} \frac{1}{L^{2}} \int_{L_{i}} L_{i}(L - L_{j})dL = \frac{1}{6}L , \]

So

\[ P_{e,i} = \int_{L_{i}} N_{i}N_{i}P_{i}dL + \int_{L_{i}} N_{i}N_{j}P_{j}dL = \frac{L}{3}P_{i} + \frac{L}{6}P_{j} \]
3D Formulation

Assume on single element surface pressure is perpendicular to the surface

$$P(x, y, z) = \sum_{i=1}^{3} N_i P_i$$

Where \( N_i = A_i / A \), \( A = \frac{1}{2} \left| \begin{array}{ccc} x_i & y_i & z_i \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{array} \right| \)

The equivalent node load

$$P_{ij} = \iint_A N_i P dA$$

So

$$P_{ij} = \iint_A N_i (N_i P_i + N_j P_j + N_k P_k) dA$$

$$= \iint_A (N_i N_i P_i + N_j N_i P_j + N_k N_i P_k) dA$$

$$= \iint_A N_i N_i P_i dA + \iint_A N_j N_i P_j dA + \iint_A N_k N_i P_k dA$$

Since

$$\iint_A N_i^a N_j^b N_k^c dA = \frac{a!b!c!}{(a+b+c+2)!} 2A$$

So

$$\iint_A N_i dA = \frac{1!0!0!}{(1+0+0+2)!} 2A = \frac{A}{3}$$

$$\iint_A N_i^2 dA = \frac{2!0!0!}{(2+0+0+2)!} 2A = \frac{A}{6}$$

$$\iint_A N_i N_j dA = \frac{1!1!0!}{(1+1+0+2)!} 2A = \frac{A}{12}$$

So

$$P_{ij} = \iint_A N_i^2 P_i dA + \iint_A N_i N_j P_j dA + \iint_A N_i N_k P_k dA$$

$$= \frac{A}{6} P_i + \frac{A}{12} P_j + \frac{A}{12} P_k$$