DOES EMPLOYMENT MATTER TO THE LABOUR-MANAGED FIRM? SOME THEORY AND AN EMPIRICAL ILLUSTRATION*

Stephen C. SMITH**

INTRODUCTION

A theory of firm-local public goods in labor managed firms, which "nest" much of the previous literature in this field, is developed and applied here to the employment level and work hours decisions. Following up on the work of Law (1977), strong implications result for the comparative statics of the firm, and a wide class of models of labor managed firms are defined in which their supposed "backward bending labor demand curves" will not apply. An econometric methodology is developed whereby estimated coefficients yield information about this behavior. Two data sets are employed to illustrate this statistical methodology, from industrial cooperatives in Italy and labor managed plywood manufacturing firms in the U.S. Northwest. The illustrative results show that the null hypothesis that the labor demand curve is in fact vertical may not be refuted within this statistical treatment.

THE TRADITIONAL MODEL OF THE LABOUR-MANAGED FIRM (LMF)

The traditional approach to modelling the LMF has been to assume maximization of income per worker. Thus, in a very simple form, assuming perfect competition, a one-output production function with first increasing then decreasing returns to scale,1 and two factors, homogeneous labour and capital, the firm seeks to maximize,

\[ Y = (PQ - rK)/L, \]  

(1)

where, \( P \) = output price, \( Q \) = output, \( r \) = the rental price of capital, \( K \) = the capital stock and \( L \) = the labour force size. Maximization of (1) with respect to \( L \) and \( K \) implies,

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1 Derivations are given in Smith (1983) or may be obtained from the author.
\[ PQ_L = Y \]  \hspace{1cm} (2)
\[ PQ_K = r \]  \hspace{1cm} (3)

where \( Q_L \) and \( Q_K \) are the marginal products of labour and capital, respectively.

A controversy which has dogged the literature on the LMF is whether the firm would introduce lay-offs in response to an output price rise. A simple case of this "backward sloping supply curve" under a one-input, one-output regime (due to Ward (1958)), may be seen by considering non-labour costs as fixed at level \( F \), and rewriting (2) as:
\[ \frac{Q}{L} - Q_L = \frac{F}{PL} \]  \hspace{1cm} (2')

An increase in \( F \) implies in increase in \( \frac{Q}{L} - Q_L \) and obviously therefore an increase in \( Q \) under the assumptions made. Similarly an increase in \( P \) implies an decrease in \( \frac{Q}{L} - Q_L \) and therefore a decrease in \( Q \). These results become ambiguous as the assumptions on inputs and outputs are generalized, allowing for the complicating influence of substitutabilities and complementarities, but the potential for "perverse" behaviour remains.

Recently, Steinherr and Thisse (1979), Bonin (1981) and Brewer and Browning (1982) have shown (with different proofs) that the LMF (with either risk-neutral or risk-averse members) whose members have in some way to be compensated by the firm for losses due to its layoff decisions will not reduce employment at all in response to a price increase. Steinherr and Thisse and Brewer and Browning stress post-voting-random lay-offs, while Bonin stresses the right to voluntary exit of members. The result is dependent on initial employment being in the closed interval between income per worker maximizing and expected income maximizing levels. The authors differ in their interpretations, but a broad range of cases have been identified where the Ward result is decisively nullified, even for the one-input, one-output case.

The result really is straightforward. Intuitively, from the concavity of the net revenue product of labour, it follows that no lay-offs after a price rise can produce as high an aggregate labour income (including wages of both those who remain and those who are laid off to find employment elsewhere) as that generated by maintaining employment at current levels in the LMF. This, of course, assumes that alternative wages are no higher than initial wages in the firm.

WHEN EMPLOYMENT ITSELF MATTERS

Variations in the income-per-worker maximand itself can imply even stronger effects, lasting into the "long-run," and in some cases leading to a positive employment response with respect to price. The original criticisms of Ward-type results implicitly doubted this maximand, arguing that these firms would not immediately lay off worker-members to realize small income gains for the remaining workers as output price rose slightly (see, e.g., Robinson (1967)). This view implies some kind of group behaviour (or "solidarity") among
members, and is consistent with models in the union literature such as Fellner (1947) and Cartter (1959).

First consider an "externalities" approach, in which workers instruct management to consider employment as well as average incomes in making decisions. Suppose then that according to his or her estimate of worker preferences the manager seeks to maximize,

\[ U = U(Y, L) \]

\[ U_Y U_L > 0 \]

where,

\[ Y = (PQ - F)/L, \]

is the simple Vanek-Ward maximand for the LMF we began with. Given the same assumptions on the production function, we may represent the outcome as in Figure 1 below.

![Figure 1](image)

In the diagram \( L^* \) is the point which solves for,

\[ U_Y(\frac{\partial Y}{\partial L}) + U_L = 0, \]

and,

\[ U_Y(\frac{\partial^2 Y}{\partial L^2}) + [U_L^2 U_{YY} - 2U_L U_Y U_{LY} + U_Y^2 U_{LL}] < 0. \]

These results were first stated by Law (1977). Clearly, this equilibrium will lead to a higher level of employment than that given by
the Ward-Vanek maximand, where \( Y = \text{VMP}_L \), as illustrated in the diagram. It will not, however, necessarily eliminate the "pervasive" supply behaviour, as we see below.

Svejnar (1982) has examined the case where the relevant maximand is the expected income of a constituency of workers \( L \) such as a labour union or community broader than the number of workers who can find employment in the LMF. (The model is readily generalized to the case of several LMFs in the constituency). The workers are then modelled as maximizing expected income,

\[
I = (L/L)Y + ((L - L)/L) (W^a),
\]

where \( W^a \) represents a best alternative wage. The maximand leads to:

\[ PQ_L = W^a, \]

which is a testable implication.

It can be shown that from this analysis, an indifference curve may be derived in \( \{Y, L\} \) space with slope,

\[ -dY/dL = (Y - W^a)/L. \]

Most important, this result demonstrates that the existence of an "indifference curve" as examined in the previous model does not rely on the employment level entering the maximand per se. For purposes of comparison, the result is diagrammed in Figure 2.
A PUBLIC GOODS APPROACH

To the extent employment enters the utility function, it will have public good qualities. While a welfare model of the LMF (as well as other types of firms and labour unions) examining collective consumption is the subject of discussion in Smith (1983, Chapter VI), we discuss the two main results on employment here. The reason why employment should be seen as a public good is because the level of employment is observed by all workers at the same given level. The utility one worker gains from the level of employment cannot decrease the utility likewise gained by any other worker, nor is there any conceivable way to privatize this utility so that this utility is kept out of the public sphere. In the pure labour-managed firm each worker pays the same amount from extra employment, as incomes (PQ — rk)/L are equal. Suppose the utilities of a welfare reference group of N workers Ui enter a firm welfare function W, of any form except that for analytical simplicity it is twice continuously differentiable and responds positively to the utility of each worker. The arguments in those utilities are assumed to be income, Y = (PQ — rk)/L, and employment, L. The form and extent in which these enter may vary from worker to worker. The firm will then maximize,

\[ W = W \{ U_i(Y, L), \quad i = 1, \ldots, N \}, \]

subject to the transformation function,

\[ Y = \frac{(PQ - F)}{L}. \]

Setting the problem up as a Lagrangian, where m is a Lagrangian multiplier, the first order conditions for a maximum which result are:

\[(\partial W/\partial U_i) (\partial U_i / \partial Y_i) + m = 0, \quad \text{for all } i, \quad (1)\]

\[ \sum_{i=1}^{N} \left( \frac{\partial W}{\partial U_i} \right) (\partial U_i / \partial L) + m \left\{ \frac{PQ}{L} - \frac{(PQ - F)}{L^2} \right\} = 0, \quad (2)\]

\[ Y - \frac{(PQ - F)}{L} = 0. \quad (3)\]

Eliminating the Lagrangian multiplier and rearranging terms yields,

\[ -\sum_{i=1}^{N} (\partial U_i / \partial L) (\partial U_i / \partial Y_i) = \frac{PQ}{L} - \frac{(PQ - F)}{L^2}. \quad (4)\]

Thus the sum of marginal rates of substitution between income and employment should be equated to the marginal rate of transformation. The result is expressed graphically in the same fashion as in
Figure 1, except that the slope of the indifference curve is given a sum of the MRS interpretation.

The public goods approach also affords insights into such issues as the labour-leisure choice problem of the LMF. Suppose each (not necessarily identical) worker maximizes,

\[ U_i = U_i (Y_i, h_i), \]

where \( h_i \) represents work hours or effort levels (with the labour force size and other inputs held constant). This leads to:

\[ (1/L) P (\partial Q/\partial h_i) = \partial Y_i/\partial h_i. \]

This is not a public goods result. But to see that it has public goods implications, first sum this condition over all workers,

\[ \sum_{i=1}^{L} P (\partial Q/\partial h_i) = \sum_{i=1}^{L} (\partial Y_i/\partial h_i). \]

Since as is well-known the income-per-worker maximizing LMF pays workers their marginal products, the left-hand-side term represents the "wage bill," that is, income net of non-labour costs (\( F \) in the simple model short run). Dividing through by \( L \) yields,

\[ (1/L) (PQ - F) = (1/L) \sum_{i=1}^{L} (\partial Y_i/\partial h_i). \]

The left-hand-side, a marginal rate of transformation, is precisely the Vanek-Ward simple LMF maximand. Using this as a maximand constrains the marginal rate of substitution between income and leisure to be that of the mean worker (the condition for a privately provided public good). In this way we are not allowing each worker to maximize utility; this is equivalent to stating that like most such models, the Vanek-Ward model implicitly assumes identical workers.

It is important in all of this to note that it is not necessary to assume that the labour-managed firm has solved a preference revelation problem to adopt this framework. The level of employment and work hours have the characteristics of externalities as well as of public goods, and a manager may be instructed to take this into account without any specific description of each member's preferences. A social welfare function is just a mechanism to convert from individuals' utilities to organizational preferences. To the extent that this can be accomplished according to a public goods rule, the organization will behave Pareto efficiently. But it will move in the same direction to whatever degree it is capable of taking such externalities into account.
A COMPARISON OF APPROACHES

In summary, analyses of employment levels in LMFs may be placed in two categories, those which assume that only income matters and employment levels fall out as a residual, and those which assume that at least in specialized disequilibrium climates many LMFs will operate with both income and employment as their objectives. Expected income maximization as in the Svejnar model provides a sort of intermediate case in which we begin by assuming only expected income matters, and end up deriving an indifference curve in \( \{Y, L\} \) space.

We may conclude that the type of behaviour the LMF will exhibit is an empirical question, not resolvable \textit{a priori} by choosing from among the plethora of available models. Any of these models could be consistent with basic choice theoretic considerations.

COMPARATIVE STATICS WITH EMPLOYMENT A LOCAL PUBLIC GOOD

The results in this section develop those of Law (1977). If the LMF maximizes income per worker, and capital is variable as well as labour, it may be shown through comparative statics that:

\[
dL/dP = (Q_{KK} - LQ_L Q_{KL} + Q_L L Q_{KL}) + (PLQ_{KL} Q_{KK} - PLQ_{KL^2})
\]

Since we know the denominator of this expression is positive \((Q_{LL} Q_{KK} > Q_{KL^2})\), a sufficient condition for \(dL/dP > 0\) is that

\[
Q_K L Q_{KL} / Q Q_{KK} - LQ_L / Q > 1.
\]

This result was first stated by Ward (1958).

Thus technical conditions of production, and in particular the sign of \(Q_{KK}\), play a crucial role in output decisions of the LMF. (Note that to find \(dQ/dP\), we simply find \(dK/dP\) along the same lines and take \(dQ/dP = Q_L (dL/dP) + Q_K (dK/dP)\), but this would take us afield from the main object of study.) We want to ask how employment changes with respect to changes in output price with labour and capital variable but given that employment enters the LMF social welfare function \(W\). It may be demonstrated that if the LMF maximizes a welfare function \(W(Y, L)\) subject to a transformation function \(Y = f(L)\), the numerator of \(dL/dP\) will be:

\[
(Q(L) (Q_{KK}/L) \{ U_{YL} - (U_L/U_Y) (U_{YY}) - (U_L/K LQ_K/Q_{KK}) -
U_Y/L + U_{YY}/L \} \left[ (Q/Q) (L/Q) \right]
\]

and its denominator may be written,

\[-U_{LL} Q_{KK}/L - U_Y P L Q_{KL}/L + U_Y Q_{KL}/L +
U_{LY} U_L/U_Y + (U_L/U_Y) U_{YL} (Q_{KK}/L) - (U_L^2/U_Y^2) (Q_{KK}/L) U_{YY}.\]
The denominator is negative, derived as it is from the second order conditions in a $4 \times 4$ bordered Hessian, but the sign of the numerator is ambiguous. Substituting for the Cobb-Douglas utility function,

$$U = Y^a L^b,$$

however, the numerator may be shown to reduce to:

$$(Q/L)(Q_{KK}/L) \cdot [Y^{a-1} L^{b-1}] \cdot [b - a + (\partial Q/\partial L)(L/Q) a - a Q_{KL} L Q_K/Q Q_{KK}]$$

and, since $Q_{KK} < 0$,

$$\frac{dL}{dP} = 0 \text{ as } b/a = 1 - (\partial Q/\partial L)(L/Q) + Q_{KK} L Q_K/Q Q_{KL}.$$

Of course this reduces to Ward's result for $b = 0$, the implicit Ward assumption that employment does not matter. The case of the Stone-Geary welfare function may be shown to reduce to the same condition as above, with $Y$ and $L$ replaced by $Y - Wa$ and $L - L^*$, as defined earlier.

The results also reduce to those of Law (1977) for only labour variable: for the Cobb-Douglas case, $U = (Y^a L^b)$, positive supply responses to a decrease in fixed costs $F$, and vice-versa, require that $b > a$, while positive responses in output to an increase in output price require that $a/b < 1/(1 - e)$, where $e$ is the elasticity of output with respect to labour (for $0 < e < 1$).

An important point to note here is that comparative statics have been done with completely general utility functions $u(Y, L)$, which may include simple income maximization and so forth, subject to an equally general transformation function $y = f(L)$. It turns out that much of the major literature in the fields of labour unions and LMFs on income and employment determination may be analyzed as special cases of these comparative static results (Smith, 1983).

For the case of joint bargaining where the LMF has only partial bargaining power, $Y$, as in the case of Svejnar and Smith (1982), or as might apply in some union bargaining situations, precisely the original Stone-Geary conditions again obtain where the co-operative is maximizing income per worker.

The "larger constituency model," examined above, (Svejnar, 1982; Svejnar and Smith, 1982), where,

$$U(Y, L) = \exp(Y) = (L/L) [(PQ - rK)/L] + (L - L)/L Wa,$$

or,

$$[PQ - rK - WaL]/L + Wa,$$

also leads straightforwardly to the positive output elasticity condition for normal sloping supply curves.
A STATISTICAL METHODOLOGY FOR COMPARING HYPOTHESES
OF EMPLOYMENT DETERMINATION IN THE
LABOUR-MANAGED FIRM

Models relying on income maximization all imply that in the long
run, labour will be hired until the value of the marginal product of
labour equals income. On the other hand, if the labour force size
enters a firm utility or employee welfare function or if a constituency
larger than the number to be employed in the firm engages in ex-
pected income maximization, income will be greater than the value
of the marginal product of labour, even in the long run. (We also
require for this case that ANR_{PL} > W_a at the maximum point).

This leads us to intuitively suppose that at least where income
Y is greater than alternative wages W_a, the simple hypothesis test,

\[ Y - VMP_L = 0, \]

would be of interest.

We may place this test in a more rigorous context of interpreting
coefficients as follows. Suppose the firm utility or welfare function
assumes a Cobb-Douglas form,

\[ U = (Y^a)(L^b). \]

In Smith (1983) it is shown that maximizing this function with respect
to labour in the simple LMF case leads to:

\[ PQ_L = (PQ - F)/L - (b/a)(1/(L - Mm)) \pi. \]

For the case where capital as well as labour is variable, the equilibri-
um condition becomes,

\[ Q_L/Q_K = [(a - b)/a][Y/r], \]

where Y is defined as above. This condition is the obvious analogue
to that of the cost-minimizing firm, where b = 0, and Y is a fixed
wage. Since that the marginal product of capital should be equal
to capital cost is a part of this condition, the actual test is the same as
above.

These results suggest a regression of the value of the marginal
product of labour on income with no constant term. One regression
analysis is performed in two stages. In the first stage, a production
function analysis is performed to simultaneously provide consistent
estimates of output elasticities of labour and capital. In the second
stage, the above equation is estimated and the coefficient may be
interpreted as below.

Note then that if \((a - b)/a,\)

\[ = 1 \text{ then } b = 0, \]

\[ < 1 \text{ then } b > 0, \]
> 1 then \( b < 0 \),
= 0 then \( b = a \),
< 0 then \( b > a \).

Though we can recover \( a \) and \( b \) only up to one degree of freedom, this is sufficient to answer a number of questions (provided, of course, that the specification is correct). First, if the coefficient is equal to one, we may confirm the hypothesis that only income and not employment matters in these LMFs. If one minus this coefficient is equal to one minus the output elasticity with respect to labour, there should be no employment response to a price change; if greater, there should be a positive response. If the coefficient is less than zero, we have normal supply responses to changes in fixed costs, as discussed above. If the coefficient is equal to zero, we have a supply curve which does not respond at all to changes in fixed costs.

Additionally, if \( b > 0 \), we can test the Svejnar model with a t-test on \( VMP_L = Wa \), or we can assume the model to be correct and take the \( VMP_L \) as an estimate of the actual alternative wage.

Now consider the Stone-Geary case,

\[
U = [(Y - Wa)^\alpha] [(L - Lm)^\beta],
\]

where \( Wa \) is a best alternative wage, assumed to be the minimum wage, and \( Lm \) is some form of minimum employment level for the plant or constituency minimum. In Smith (1983) it is shown that this implies,

\[
PQ_L = (PQ - F)/L - (b/a) (L - Lm) \pi,
\]

where,

\[
\pi = PQ - F - WaL.
\]

which clearly reduces to the Cobb-Douglas result for \( Wa = 0 \), \( Lm = 0 \). Assuming that \( a > 0 \) (or that the marginal utility of income is positive).

if \( b/a \)
= 0, we have the null hypothesis,
> 0, there is positive marginal utility for employment,
\> 1. then \( b > a \),
< 0, negative marginal utility for employment.

In general, however, we can only identify the relative proportions of these coefficients.

STATISTICAL RESULTS ON EMPLOYMENT DETERMINATION IN LABOUR-MANAGED FIRMS

In this section we present results from econometric runs based on Cobb-Douglas welfare functions, in order to illustrate the methodology developed above.
We first consider data collected from four plywood manufacturing firms in the U. S. Northwest. As may be seen from an examination of the data in the table from the plywood firms below, the small data set size (26) led to large standard errors in the Cobb-Douglas production function, and may have contributed to the outcome that CES and Translog production functions did not yield meaningful coefficient estimates (and hence are not presented).

Table 1

*Results from Revealed Preference Income-Employment Welfare Functions for the Plywood Co-ops*

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Non-Linear Systems

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<td>(.2104)</td>
<td>(.7676)</td>
<td></td>
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<td>(.2104)</td>
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<tr>
<td>3s1s</td>
<td>.8020</td>
<td>.2680</td>
<td>1.5602</td>
<td>(yes)</td>
<td></td>
<td>2.05</td>
<td>1.70</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(.2260)</td>
<td>(.1803)</td>
<td>(.4261)</td>
<td></td>
<td></td>
<td>(.1803)</td>
<td>(.1803)</td>
<td>(.1803)</td>
<td>(.1803)</td>
</tr>
</tbody>
</table>

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2 Figures in parentheses are standard errors; dw refers to the Durbin-Watson statistic. The "1st equation" refers to the production function; the second equation refers to the "welfare" equation.
For these firms, the best available measure of capital flow was depreciation charges plus capital rental expenses. These values were deflated by the producer price index and used as a measure of capital input. Employment data was available in two forms, number of employees and annual hours worked by production workers. Each was used to derive a figure for annual labour input; results from the former are found in equations 1 and 3, the latter in equations 2 and 4. Value added was given by total output minus material inputs, deflated by an index constructed by the author which weighted equally the price level of four five-digit SIC products produced by these four firms (as reported by the Bureau of Labour Statistics). The measure for income was production workers’ wages. Care was taken to exclude anything which might be construed as income due to their position as capital owners rather than their position as workers (we have excluded not only dividends ("patronage shares") but also such categories as profit sharing, for which non-members are also eligible). Thus if anything we have understated accounting labour income, so as not to risk finding income greater than marginal product due to the inclusion of capital income, and thus bias the finding.

The results in Table 1 were obtained by first regressing the log of real value added on the logs of labour input and real capital flows, then using the estimated output elasticity of labour to regress the marginal product of labour on income. The theory predicted that the constant term in this regression would be zero; in each case this was borne out, with t-ratios of about .7. Thus runs without a constant term were done; results are presented for these as well. For the production function runs where dummies were added, runs with dummy variables were also done in the second stage. Two of the three dummies were significant with a t-test in this second stage.

As can be seen from the table, the estimates with this data were not particularly robust, which is the reason we have presented a wide range of runs. It is nonetheless interesting that in none of the runs could the null hypothesis of vertical labour demand be refuted, while in most of the runs in which firm dummies were used in the production function, the null hypothesis that the welfare elasticity of employment, b, was zero was refuted, in some cases overwhelmingly.

In equation 1a, dummy variables for firms were not used, and the large standard errors on both labour elasticity and the ratio b/a (which is just one minus the estimated coefficient (a−b)/a) make it impossible to refute either the null hypothesis of income per worker maximization or of sufficient welfare weight on employment to produce a vertical employment response to a price change.

In equation 1b, the intercept was constrained to be zero, in accordance with theory (in the first as in all other cases for these firms, the intercept was insignificantly different from zero). A much tighter standard error on the welfare ratio was obtained, but one is still not able to reject either null hypothesis.

In equation 2a, hours worked have been substituted for number of employees as our measure of labour input. Elasticity estimates and their standard errors are not drastically altered, but the estimated welfare ratio is lower. Still, in this as in case 2b, where the statistically
insignificant intercept term was dropped, neither hypothesis can be refuted at the 5% level.

In equation 3a, firm dummies have been added in the first stage, but only a general intercept term in the second stage. The t-test narrowly missed rejecting the null hypothesis of income per worker maximizing behaviour. In equation 3b, where the intercept term is dropped, and equation 3c, where firm dummies are added to the second stage of the estimating procedure as well, the null hypothesis of income per worker maximization is decisively rejected (within the structure of our technical assumptions), while the null hypothesis of vertical labour demand cannot be refuted.

In equations 4a through 4c, the same runs are repeated, this time with hours worked again replacing employees as the estimate of labour input. In each case, the null hypothesis of income per worker maximization is decisively rejected, but the large standard errors on output elasticity prevents us from establishing, within the assumptions, an increasing employment level with output price, despite the fact that point estimates of b/a are considerably higher than the point estimates for one minus the output elasticity of labour.

The non-linear estimates yielded higher standard errors than the non-linear approximations; however, convergence to these values was found beginning from a range of starting values. With this data, neither null hypothesis can be rejected.

Runs were also done on Italian industrial co-operative data gathered in a survey by the Lega Nazionale delle Cooperative (Lega), to which most Italian producer co-operatives belong. Results were as in Table 2.

<table>
<thead>
<tr>
<th>sq. dummy</th>
<th>elasticity</th>
<th>elasticity</th>
<th>(a—b)/a</th>
<th>dw1</th>
<th>dw2</th>
<th>rsq1</th>
<th>rsq2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. industry</td>
<td>.3648 (0.0460)</td>
<td>.6467 (0.0506)</td>
<td>.4273 (0.0075)</td>
<td>2.0348</td>
<td>1.7991</td>
<td>.8928</td>
<td>.9261</td>
</tr>
<tr>
<td>2. industry</td>
<td>.7937 (0.0643)</td>
<td>1.0042 (0.0643)</td>
<td>1.1279 (0.0197)</td>
<td>2.0381</td>
<td>1.9475</td>
<td>.8931</td>
<td>.9261</td>
</tr>
<tr>
<td>3. industry</td>
<td>.4356 (.1640)</td>
<td>.5743 (.1057)</td>
<td>.3429 (.0155)</td>
<td>2.0072</td>
<td>1.8895</td>
<td>.9012</td>
<td>.6512</td>
</tr>
<tr>
<td>4. firm</td>
<td>.4225 (.1158)</td>
<td>1.0867 (.0552)</td>
<td>.4949 (.0087)</td>
<td>1.4159</td>
<td>1.7991</td>
<td>.9146</td>
<td>.9261</td>
</tr>
</tbody>
</table>

3 Data were systematically collected for individual producer co-operatives over the years 1975–1980 for any firm with over US $1 million in sales for at least one year in that period. I would like to thank Alberto Zevi and Derek Jones for making this data available to me for use in the present study.

4 Table format is the same as for Table 1.
5. firm .5736 .6549 .6719 1.0210 1.7991 .8499 .9261 (.1596) (.0638) (.0118)
6. firm .1762 .6007 .2064 2.0808 1.7991 .9621 .9621 (.0785) (.0456) (.0036)
7. firm .1506 .3341 .1764 1.9425 1.7991 .9509 .9261 (.0899) (.0386) (.0031)
8. industry .7384 .3263 .8650 1.0546 1.7991 .7579 .9261 (.0543) (.0359) (.0151)

As explained below, alternative measures for capital produced different results; runs using both total and fixed assets are thus presented. Again, the most conservative estimate of worker income was used so as to go as far as possible to exclude any possibility of biasing results by considering capital income as income to labour proper.

The first three equations represent results for Cobb-Douglas, CES and Translog production functions, respectively, using total assets as a proxy for capital. Industry dummies were used in each case, which were highly significant. Note that the elasticity measures for the latter two functions are averages, for which standard deviations are presented. The Cobb-Douglas and Translog functions are consistent with a vertical labour demand curve but not income maximization, while in the case of the CES estimates, the reverse is true. One piece of guidance is provided by the fact that the Kmenta CES approximation and the Translog specification both nest the Cobb-Douglas specification. An F-test rejects both of these forms as improvements over the Cobb-Douglas, in this case as in all other specifications.

In equations 4 through 8, results from alternative specifications for the Cobb-Douglas production function are presented. In equation 4, the five industry dummies are replaced with 70 firm dummies (there were 317 observations over 71 firms); in equation 5 mixed assets are used in place of total assets as a capital measure, again using firm dummies. In equations 6 and 7, a time trend has been added along with firm dummies, with the total asset measure for capital used in equation 6 and the fixed asset measure in equation 7. In equation 8, we use industry dummies again along with a fixed asset measure for capital.

These alternative specifications do indeed lead to rather substantial differences in estimated output and welfare elasticities. Some of the output elasticities have large standard errors; some are rather tight estimates. But in each estimated run, one minus the labour output elasticity has fallen within a 95% confidence interval of the derived ratio of employment to income welfare elasticity, while at the same time the hypothesis of income maximizing behaviour has been overwhelmingly rejected. Though coefficients themselves were not stable, the implication of relatively vertical employment responses emerged consistently.
CONCLUSIONS

The labour-managed firm may act as if employment were a public good, so that the level of employment will be set where the sum of marginal rates of substitution between income and employment is equal to the marginal rate of transformation between income and employment, as derived in the text. This "public goods approach" has a number of implications for the LMF. In the paper, special attention is given to the implication that the sign of the labour demand or output supply curve will depend in a precise way on the output elasticities of labour and capital and the welfare elasticity of employment. A procedure for estimating these elasticities econometrically was developed and applied to two separate data sets on worker co-operatives. A vertical labour demand curve was implied by the results. This is a promising area of research, since as was argued in the paper the controversy over the slope of the LMF's labour demand curve can only be resolved empirically.

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DA LI JE ZAPOSLENOST ZNACAJNA ZA RADNIČKO PREDUZEĆE?
TEORIJSKO I EMPIRIJSKO OBJAŠNJENJE

Stephen C. SMITH

Režime

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