System Design Under Uncertain Time-Varying Requirements: Nuclear Weapons Case Study

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Dedication

This praxis is dedicated to my family.

- To my wife Hillary for her unwavering love, support, and encouragement for my academic pursuits for more than twenty years now: meus amor tibi decurrit ad astra et retro
- To my boys, CJ and Grant, never forget that learning is growth and “nam et ipsa scientia potestas est”
- To my parents, Richard and Violet, for ensuring that when I was growing up there was always a book in my field of view. minha gratidão a você é profunda

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Abstract of Praxis

System Design Under Uncertain Time-Varying Requirements: Nuclear Weapons Case Study

Many military systems like ships, planes, and missiles have service lifetimes measured in decades. As their costs rise and numbers shrink, it is essential that they remain useful over that timeframe as well. Complicating this, the utility of such systems is contingent upon the choices made by potential adversaries, which are beyond the control of system designers. This points towards the possibility that the demands made upon them, and hence their requirements, may change more rapidly than they can adapt or be replaced. This is an especially consequential issue for two strategic nuclear weapons systems, the Intercontinental Ballistic Missile (ICBM) and the Ballistic Missile Submarine (SSBN), which are the subjects of this praxis. Defense acquisition currently has no process with which to account for uncertainty of this type over spans of time this long.

This praxis presents an approach to quantitatively address system design where requirements are directly coupled to uncertain, exogenous factors. Acquisition of ICBMs and SSBNs is modeled with an inventory management construct, where the challenge is to deliver requisite capability in sufficient quantity at the appropriate time. Robust optimization techniques are used to generate solutions for different design variants that are evaluated for their Pareto optimality under a range of potential adversary-choice scenarios or 'futures'.

The praxis demonstrated that system engineering parameters can be evaluated under severe uncertainty with robust optimization. Results from this type of analysis can inform major system design decisions, especially when performed in the early stages of the acquisition process. The broad implication of this work is that future uncertainty can be quantitatively accommodated within systems design processes in a manner that does not incur the risks of using stochastic forecasts beyond their practical limits. It also demonstrates that the Intelligence Community and the Department of Defense can make better use of existing information for long range planning and analysis.

Table of Contents

Dedication ..... **iv**

Acknowledgements ..... **v**

Abstract of Praxis ..... **vi**

List of Figures ..... **xi**

List of Tables ..... **xiii**

List of Acronyms ..... **xv**

Chapter 1 — Introduction 1

1.1 Background 1

1.2 Research Motivation 3

1.3 Problem Statement 5

1.4 Thesis Statement 6

1.5 Research Objectives 6

1.6 Research Questions and Hypotheses 7

1.7 Scope of Research 8

1.8 Research Limitations 9

1.9 Organization of Praxis 10

Chapter 2 — Literature Review 11

2.1 Introduction 11

2.2 Requirements within Defense Acquisition 11

2.3 Characterizing Uncertainty 14

2.5 Inventory Management Optimization 25

2.6 Summary 28

Chapter 3 — Methodology 30
3.1 Introduction 30
3.2 Base Data 30
3.2.1 US Arsenal: maxima and minima 32
3.2.2 Russian Arsenal 33
3.2.2.1 Total Russian Warheads 33
3.2.2.2 Russian warheads as robust variables 38
3.3. Optimization Design 42
3.3.1 Design Alternatives and Decision Variables 42
3.3.2 Optimization Variables 44
3.3.3 Objective Function Construction 47
3.3.3.2 Blue Ceiling – Nuclear and Conventional 55
3.3.3.3 Blue Lower Bound 58
3.3.3.4 Objective Function 60
3.3.3.5 Constraints 62
3.3.4 Optimization Execution 63
3.4 Policy Analysis and Hypothesis Evaluation 63
3.4.1 Scenario Selection 63
3.4.2 Development of Data 67
3.4.2.1 Legacy Blue weapon systems 68
3.4.2.2 New Weapon Systems 69
3.4.3 Hypothesis Testing 73
3.4.3.1 Hypothesis 1 73

3.4.3.2 Hypothesis 2 74
3.4.3.3 Hypothesis 3 75
Chapter 4 —Results 76
4.1 Introduction 76
4.2 Hypothesis 1 77
4.3 Hypothesis 2 82
Chapter 5 - Discussion and Conclusions 91
5.1 Discussion 91
5.2 Conclusions 93
5.3 Contributions to Body of Knowledge 94
5.4 Recommendations for Future Research 95
References 97

List of Figures

Figure 2-1 Probability Mass Function for Soviet Nuclear Weapons in 1995 16

Figure 2-2 Soviet/Russian Nuclear Weapons 1986-1995: Predicted vs Actual 17

Figure 3-1 Methodology Elements 30

Figure 3-2. US SSBNs and ICBMs 32

Figure 3-3. SSBN, ICBM Warhead Loading 33

Figure 3-4. Soviet/Russian strategic nuclear warhead count 34

Figure 3-5. Annual change in Soviet/Russian nuclear inventory 35

Figure 3-6. CDF of Soviet/Russian annual changes to nuclear inventory 35

Figure 3-7. Future Russian Nuclear Warhead Inventory 38

Figure 3-8. Russian Nuclear Warhead Inventory: Random Walks and Extrema 39

Figure 3-9. Relative Distribution of Soviet/Russian nuclear weapons 41

Figure 3-10. Notional relationship between min/max and floor/ceiling 48

Figure 3-11. Ceiling and Floor for Blue weapons 49

Figure 3-14. Components of Objective Function 60

Figure 3-13. Scenario 1: mean Red warhead growth with Blue ceiling and floor 64

Figure 3-14. Scenarios 2 and 3 (25% and 75% percentile) 65

Figure 3-15. Futures 4-7 65

Figure 3-16. Historically Informed Scenarios 66

Figure 3-17. Scenarios 8-10 66

Figure 3-18. Example Scenario 68

Figure 3-19. Legacy Blue Warheads 69

Figure 3-21. Blue warheads: Legacy and Optimized 70

Figure 3-20. Robustly Optimized Ordering Quantities 70

Figure 3-22. Comparison of Optimized Acquisition and Objective Function 71

Figure 3-23. Determination of Regret 72

Figure 4-1. Hypothesis 1 Results: Nuclear-only Scenarios 78

Figure 4-2. Hypothesis 1 Results: Nuclear/Conventional Scenarios 79

Figure 4-3. Hypothesis 2 Results 83

Figure 4-4. Hypothesis 3 Results 87

List of Tables

Table 1-1. US Strategic Nuclear Forces (NPR 2018) 2

Table 2-1. Quantitative Intelligence Probabilities (DNI, 2015) 15

Table 2-2. Strategic Nuclear Weapon Acquisition as Inventory Management 29

Table 3-1. Design alternatives for optimization 44

Table 3-2. Single Item Acquisition 45

Table 3-3. Multi-item Acquisition 45

Table 3-4. Summary of Blue variables and constants 46

Table 3-5. Summary of Key Quantities 47

Table 3-6. Targeting Russian weapons 51

Table 3-7. Exchange Ratio: US → Russia 52

Table 3-9 Blue ceiling: Blue variables and constants 53

Table 3-8. Blue ceiling: Red variables and constants 53

Table 3-10. Red warhead distribution and Blue warhead count 54

Table 3-11. Threat to Russian Strategic Nuclear Forces (Volkov, 2006) 56

Table 3-12. Russian losses from American non-nuclear weapons 57

Table 3-14. Blue floor parameters 60

Table 3-15. Industrial Base parameters 62

Table 3-16. Data Example 72

Table 3-17. Hypothesis 2 74

Table 3-18. Hypothesis 3 75

Table 4-1. Design 'Abbreviations' 76

Table 4-3. Design Alternative Dominance in Nuclear and Conventional Scenarios 80

Table 4-2. Design Alternative Dominance in Nuclear-Only Scenarios 80

Table 4-4. Hypothesis 1 - Summary Statistics 81

Table 4-5. Production Rate Variations 82

Table 4-6. Design Alternative Dominance over Acquisition Rates 84

Table 4-7. Hypothesis 2 - Summary Statistics 85

Table 4-8. Russian Force Design Variants 86

Table 4-9. Design Alternative Dominance over Russian Force Design Variations 88

Table 4-10 Hypothesis 3 Statistics 89

Table 4-11. Sensitivity to Russian Force Design 90

List of Acronyms

ALCM	Air-Launched Cruise Missile
AoA	Analysis of Alternatives
CBO	Congressional Budget Office
CIA	Central Intelligence Agency
CRS	Congressional Research Service
DoD	Department of Defense
DoN	Department of the Navy
GAO	Government Accountability Office
ICBM	Intercontinental Ballistic Missile
JCIDS	Joint Capabilities Integration and Development System
NIC	National Intelligence Council
NIE	National Intelligence Estimate
RO	Robust Optimization
SLBM	Submarine-Launched Ballistic Missile
SSBN	Nuclear-Powered Ballistic Missile Submarine

Chapter 1 — Introduction

1.1 Background

Since 1945, the United States has developed and maintained an arsenal of strategic nuclear weapons that can directly attack an adversary's homeland. In the decades since, the public narratives regarding their ostensible purpose and requisite capabilities have ebbed and flowed with the geopolitical tides. Central to all of them up until 1991 has been the intent for those forces to deter Soviet attack against the United States and our allies. After the dissolution of the Soviet Union and the subsequent collapse of previous justifications for nuclear weapons, there were widespread expectations that the American narrative would adapt to the changed realities of the post-Cold War world (Global Zero, 2010). There was indeed a significant drop in the number of weapons in the US arsenal. The inventory went from more than 10,000 in 1991 to 1,420 in 2022 (Department of State, 2022) but the narrative remained largely intact.

Every presidential administration since Clinton has issued a Nuclear Posture Review that reflects their view of the purpose for nuclear weapons and describes the nominal composition of strategic nuclear forces. They have successively affirmed a basic construct that includes submarine-launched ballistic missiles (SLBMs) carried aboard purpose-built submarines (SSBNs), intercontinental ballistic missiles (ICBMs) deployed in hardened silos, and air-launched cruise missiles (ALCMs) and gravity bombs delivered by strategic bombers (Nuclear Posture Review (NPR) 2010, 2018, 2022). The strategic nuclear weapons platform for the United States are listed in Table 1-1. All the submarines, missiles, and aircraft deployed today are Cold War-era designs, and most are

actual veterans of that time. They are rapidly reaching the end of their useful lifespan and will soon begin leaving the operational inventory.

Table 1-1. US Strategic Nuclear Forces (NPR 2018)

Domain	Platform	Weapons
Sea	Ohio-class SSBN	Trident II SLBM
Land	Minuteman III ICBM	Minuteman III ICBM
Air	B-52H, B-2A bombers	ALCM, B61/B83 gravity bomb

The United States is in the early stages of an extended period of force recapitalization. Systems being fielded include a new submarine, the COLUMBIA-class SSBN, a follow-on SLBM, the Sentinel ICBM, the B-21 Raider bomber, and a new ALCM. The cost is expected to be immense, amounting to hundreds of billions of dollars over the coming decades. Early estimates from the Congressional Budget Office posited that the total costs will likely exceed $380 billion in then-year dollars (Congressional Budget Office, 2013). The systems are expected to be part of the inventory of strategic nuclear weapons for decades, e.g., the COLUMBIA class will be in use through 2085 (Department of the Navy, 2021)

Reflecting the intrinsic significance of nuclear weapons and the immense resources required to build, operate, and maintain them, the US Government and its supporting contractors in industry and academia are expending considerable effort in support of acquisition. They are collectively working to ensure that the systems being fielded are being built to appropriate requirements in a cost-effective manner on a timeline that meets defined military specifications. However, despite those efforts, the Government Accountability Office has noted that the Navy and Air Force consistently underestimate the costs of major weapon systems (e.g., Government Accountability Office (GAO),

2019), implying that the CBO number is optimistic. In addition, to the consternation of operational commanders, schedule delays continue to accumulate, forcing delivery timelines to shift to the right.

While these are not insignificant concerns, they can and are being addressed within the well-defined and mature processes of government acquisition and construction engineering. However, they occlude a more basic question of sufficiency. Are the forces being fielded going to serve the national need over their lifespan?

1.2 Research Motivation

Federal Law requires the President to inform Congress of any changes to the employment strategy of nuclear weapons. Those documents are known as 'Section 491 Reports' and provide limited but updated views on the broad outlines of US nuclear strategy. A salient excerpt provides the motivation for this praxis.

> "The new guidance requires the United States to maintain significant counterforce capabilities against potential adversaries. The new guidance does not rely on a "counter-value" or "minimum deterrence" strategy. (10 U.S. Code § 491, 2012)

This is an explicit disavowal of the strategic bombing strategy employed by the Unites States in World War II in which German and Japanese cites were subject to devastating direct attack. In 'counterforce' employment strategies, consistent with the Laws of Armed Conflict, nuclear weapons are only used against other weapons directly and supporting systems, e.g., ICBM or military airfields. Often missed in this formulation is the crucial consequence that it explicitly links the requirements of

American forces to those fielded by countries that are 'potential adversaries' such as Russia or China.

The question of what our forces need to do is therefore influenced in no small part by decisions and investments made in Moscow and Beijing. These decisions may be forecast by the intelligence community but typically only out ten years. Worse, those forecasts can and do change on even shorter timescales. In some commercial contexts, ten years may be a long time relative to business or technology cycles, but it is exceptional short when dealing with strategic nuclear weapons.

This problem was highlighted in 2022 when in congressional testimony, the Commander of US Strategic Command noted that China "likely intends to have at least 1,000 warheads by 2030, greatly exceeding previous DoD estimates" (Defense Authorization Request, 2022). The long design and acquisition timelines for strategic weapons cannot easily respond to a change of that magnitude over such a short timeframe.

Complicating this picture is the notion of 'mutual vulnerability'. The strategic deterrence framework jointly adopted by the US and Russia is stable because arms reduction treaties seek to "remove incentives for a nuclear first strike" (United States, 1990). In practice this means that the United States agrees that Russia should retain the ability to strike the US with nuclear weapons under all circumstances (Charap et al., 2022). This further implies that US strategic nuclear weapons should not be able to threaten that capability.

This research is motivated by the fact that requirements for American nuclear weapon systems are sensitive to the actual numbers of comparable weapons deployed by potential adversaries. The uncertainty this introduces into design requirements can be managed but only if it is addressed. This research intends to provide a means to explicitly account for intrinsic uncertainty of exogenous requirements in the design of long-serving systems such as our strategic nuclear forces.

1.3 Problem Statement

The problem statement for this praxis is as follows:

> *Strategic nuclear weapon systems requirements change faster than designs can adapt, risking late delivery of inadequate weapons system capability*

This problem is analogous to the more familiar engineering management challenge of identifying customer requirements early in the design process. As noted by Wang and Tseng (2014), this can be a fraught exercise as it involves elicitation/elucidation of desired form and function with a clarity from customers that simply may not exist. While there are methodologies to deal with this problem in commercial application, they are of limited utility when forced to deal with the uncertainties inherent to the problem of this praxis, where the requirements are beyond the control of the 'customer', the US Government. As described in more than a decade of reports from the Congressional Research Service (O'Rourke, 2022), this contributes to cascading effects that risk late delivery of inadequate weapons systems, potentially resulting in the failure to meet US Strategic Command operational requirements.

1.4 Thesis Statement

The fundamental premise of this praxis is that:

> *A robust optimization model is required to produce weapon systems designs against uncertain requirements to reduce schedule risk and improve system capabilities*

In current approaches to military acquisition, weapon system designs are developed from well-defined requirements, including projections of adversary capabilities. By forcing specificity to the future trajectory of the threat, as-designed weapon systems capabilities may be poorly matched to unanticipated needs. Robust optimization allows uncertain but bounded scenarios to be integrated into weapon system design processes.

1.5 Research Objectives

This research is intended to use robust optimization techniques to:

- Demonstrate the feasibility of using robust optimization and forecasting to help define or constrain system requirements before major attributes have been specified
- Evaluate the potential schedule and performance consequences of exogenous design drivers in the early stages of the design process
- Create a tool with which requirements analysts within the Pentagon, in the Office of the Secretary of Defense (Cost Assessment and Program Evaluation), the Navy Staff, or the Air Staff, can use to incorporate more realistic intelligence projections into the acquisition process

1.6 Research Questions and Hypotheses

The Research Questions explore the relationships between exogenous design drivers and the implications for both weapon system design and acquisition timelines. The Hypotheses provide sharp focus on the details of implicit tradeoffs reconciled through optimization.

RQ1: Can an optimization model be built to select a system design and acquisition plan that minimizes risks due to uncertain requirements?

RQ2: Which weapon system attributes are the objective function's binding constraint?

RQ3: Which adversary weapon system attribute is the most significant factor in objective function optimization?

H1: The optimal weapon design combination for nuclear-only and nuclear/conventional constraints is the heavier ICBM (10 warheads) and largest SSBN (24 missile tubes)

H2: More rapid missile and submarine production rates will improve design optimality for all scenarios

H3: The optimum designs are insensitive to Russian force design philosophy

1.7 Scope of Research

This praxis will examine the question of engineering design under severe uncertainty for two of the three major nuclear weapons systems, the SSBNs (and implicitly the missile that they carry, the SLBM), and the ICBMs. The heavy bomber force is excluded because their role in the American arsenal has changed dramatically over time and are no longer part of the core political/military calculus from which the Russian-derived optimization objective function is constructed.

Only the principal attributes of the weapon systems will be subject to optimization. They are limited to the inventory (or Order of Battle) of ships, missiles, and nuclear warheads. The details of other important properties, e.g., the range of a missile or the yield and accuracy of a warhead), are subsumed within the assumptions of the objective function. The weapon system attribute subject to optimization are the warhead carrying capacity of the ICBM (3 or 10) and the SLBM carrying capacity of the SSBN (16, 20, or 24).

The research is forward looking from today but will include consideration of previous epochs of nuclear weapon acquisition. The history of Soviet/Russian nuclear forces are used as a point of reference to stress future demand. The adversary futures are limited to those that are functionally agnostic of the American order of battle. Entangled futures could be evaluated through game theoretic approaches to optimization in follow-on work. This constrain is imposed for research tractability.

1.8 Research Limitations

A major limitation of this research is the source information from which the analysis is built. Not surprisingly, the actual design criteria for new strategic weapons systems are classified and therefore not in the public domain. Similarly, informed projections for the future size of Russian and Chinese nuclear arsenals are closely held by the Intelligence Community and not available for public scrutiny. This necessarily constrains the definitiveness of the analysis performed under this praxis. The truly authoritative data for contemporaneous engineering design analysis will only be available to those involved in the process today.

Fortunately, none of this precludes useful exploration of the topic. Because nuclear weapons are the product of massive investments of public resources, there are any number of organizations who collect and analyze relevant information. Much of it enters the public discourse through the engagement between the Department of Defense (DoD) and Congress regarding the resourcing of military systems. A residual of appropriations process is availability of the amplifying information and justifications put forth by the DoD to get Congress to fund the development, construction, and operations of strategic weapon systems. These residuals in the forms of budgetary analysis and congressional testimony are collated by government watchdog and research organizations, e.g., the Government Accountability Office (GAO) and Congressional Research Service (CRS), and non-government organizations (the Federation of American Scientists (FAS)), alike. These organizations publish detailed evaluations of design intentions and the decision space in which they are adjudicated. These in turn provide the base data that underwrite this praxis.

The only major implications of this data constraint are that the analysis and conclusions should be considered as an illustrative rather than definitive example of engineering design trade-off analysis for strategic weapons. The process and tool are extensible to the domains where definitive data reside, i.e., a fully informed decision-space.

1.9 Organization of Praxis

This praxis is organized to set context for the central problem. It then establishes a process and methodology with which requirements uncertainty can be incorporated into system design processes. The principal content of the praxis is organized as follows:

- Chapter 2 – Literature Review: discussion of the military requirements processes and its limitations, instantiated requirements, robust optimization, and the design framework appropriate for the problem
- Chapter 3 – Methodology: detailed description of how weapon system attributes, requirements, and future uncertainties are modeled and incorporated into a robust optimization schema to test the research hypotheses
- Chapter 4 – Results: presents the optimization outputs
- Chapter 5 – Discussion and Conclusion: analysis of the optimization outputs and their implications for weapon system design

Chapter 2 — Literature Review

2.1 Introduction

This chapter is divided into four sections, each covering a different aspect of the praxis. The first two sections highlight relevant shortcomings of the current state of practice for requirements generation within the Department of Defense and forecasts of adversary force levels within the Intelligence Community. Current practices are rigorous and well-exercised but do not yet account for the span of content that informs the development of strategic weapon systems. The third and fourth sections provide contextual review of approaches to optimization and inventory management as they apply to the particulars of this praxis' problem statement. They show how the problem resembles well-understood challenges and is amenable to existing tradecraft. The chapter concludes with a summary of how this disparate content aligns to the research questions and hypotheses.

2.2 Requirements within Defense Acquisition

Acquisition within the Department of Defense is managed through a complex regulatory and budgeting process. Requirements definition and validation is managed by the Joint Requirements Oversight Council (JROC) reporting to the Chairman of the Joint Chiefs of Staff (Chairman of the Joint Chiefs of Staff, 2021). It operates through the execution of a process known as the Joint Capabilities Integration and Development System (JCIDS). The formal design process for military systems begins under the auspices of JCIDS.

There are several types of precursors to system design within JCIDS, all of which can speak to needs and requirements. They include emergent identification and description of needs from lessons learned from exercises or real-world operations, capability demonstrations, and research innovations. Most important are the formal Capabilities Based Assessments. They map "between strategic guidance, operational missions… capability requirements, and capability solutions" (JCIDS Manual, 2021). They inform the development of the Initial Capabilities Document, a preliminary articulation of system functions, which in turn leads into a formal Analysis of Alternatives (AoA) in which major design parameters are formally considered. (Joint Chiefs of Staff (JCS), 2021)

By current guidance, the selection of the number of missile tubes on a submarine or the warheads on ICBM would be determined through an AoA. The Navy completed its Sea-Based Strategic Deterrent Analysis of Alternatives in 2009 (O'Rourke, 2010). The Air Force completed its Ground-Based Strategic Deterrent Analysis of Alternatives in 2014 (Congressional Research Service (CRS), 2022). Neither AoA has been released to the public but reporting by the Congressional Research Service is illustrative. The Navy AoA appears to have been motivated by concerns about affordability of the new submarine and the potential impact its procurement would have on the rest of the Navy shipbuilding budget. A focus on cost reduction at the expense of the implicit requirements of deterrence points toward system options that included shrinking the size of the ballistic missile submarine (SSBN) fleet, reducing the number of missile tubes, carrying a smaller submarine-launched ballistic missile (SLBM), and stretching out

procurement (O'Rourke, 2010). Pointedly, two of those options, the number of missile tubes and procurement timelines, are explicitly addressed by this praxis.

While there is a clear rationale for this approach, the last time the United States designed an SSBN the selection of missile tubes was apparently more political than anything else. Navy studies showed that the cost to put an SLBM at sea decreases as the number of missile tubes per submarine increases. An initial requirement for twenty missiles per ship was dressed in the language of cost-effectiveness but the ultimate decision for the Ohio-class to have 24 tubes was 'arbitrarily made'. Anecdotes from the time suggest that the Secretary of Defense publicly misspoke (24 vice 20 tubes) and the Navy didn't challenge it (Spinardi, 1994). This suggests that a slip of the tongue led to a 20% increase in the striking power of the Ohio-class. Whatever the truth, there is no public evidence for a substantive basis for such as critical design parameter and current processes do not provide a readily available answer to the problem of this praxis.

The JCIDS process and its antecedents are far from perfect. They are often derided for the penalties in time and cost that they impose upon military acquisition (e.g., F-35 program) and subject to periodic reform efforts (e.g., Section 809 Panel, 2017). Despite its purpose in minimizing acquisition risk and process uncertainty, exercising the process still leads to shortcoming traceable to the system design stage for relatively straightforward systems such refueling tankers (Government Accountability Office (GAO), 2017). Strategic nuclear weapons are subject to JCIDS processes but have unique attributes for which there are no provisions to address them within existing formalisms. These weapons simultaneously satisfice the needs of the 'warfighter' and national strategic deterrence.

In JCIDS, systems are treated as discreet entities. It allows for interaction with other systems through interface specifications but cannot account for interdependence. The intercontinental ballistic missiles (ICBMs) and SSBNs covered by this praxis have no technical interface. Their functional independence means that they are developed in isolation from each other. Also missing from the JCIDS process is the ability to account for variability in requirements, and in this case, a requirement derived explicitly from an adversary, i.e., the size of their future strategic nuclear forces.

Adversary information is part of the process though. The Defense Intelligence Agency generates products called Validated On-Line Lifecyle Threats (VOLTs) that are the "authoritative threat assessment tailored for one specific program" (Office of the Undersecretary of Defense for Acquisition and Sustainment, 2021). They are used for the design and development of system survivability and risk management requirements. VOLTs do speak to prospective threat systems but only to their capabilities. They are treated as static and not subject to dynamic evolution. There is no guidance on developing requirements under uncertainty or in circumstances where the design and total procurement are interrelated.

2.3 Characterizing Uncertainty

The major analytic organs of the Intelligence Community produce projections of adversary forces as a matter of routine. Many are generated by the miliary intelligence agencies, (e.g., Office of Naval Intelligence, 2015) but systems like strategic nuclear weapons are typically addressed by the Central Intelligence Agency (CIA) or National Intelligence Council (NIC). In contrast to VOLTs and other intelligence support to acquisition, major assessments of future adversary forces explicitly acknowledge and

account for uncertainty. Analytic documents couch forecasts and assessments with adjectives such as 'probably' and 'unlikely'. Historically those characterizations were unquantified, and it was left to the reader, often a senior decision-maker, to determine how best to interpret the language of these documents. In 2015 the Director of National Intelligence (DNI) issued guidance to help connect the language of assessment to quantitative measures. Intelligence Community Directive 203 provides a standardized mapping of language to probabilities.

Table 2-1. Quantitative Intelligence Probabilities (DNI, 2015)

Set 1	Almost no chance	Very unlikely	Unlikely	Roughly even chance	Likely	Very likely	Almost certain
Set 2	Remote	Highly improbable	Improbable	Roughly even odds	Probable	Highly probable	Nealy certain
Set 3	1-5%	5-20%	20-45%	45-55%	55-80%	80-95%	95-99%

Even with this guidance, it can still be a difficult to translate prose into probabilities. Consider for example the now-declassified National Intelligence Estimate (NIE) from 1985 titled "Soviet Capabilities for Nuclear Conflict Through the Mid-1990s". It provides a detailed forecast of Soviet weapons and includes an assessment of overall numbers.

> "While the Soviets would not necessarily expand their intercontinental attack forces beyond some 12,000 to 13,000 warheads in the absence of arms control constraints, they clearly have the capability for significant further expansion, to between 16,000 and 21,000 deployed warheads by the mid 1990s…" (CIA, 1985)

The document also includes a contingent projection, in this case a warhead levels that would be accountable under a future arms control treaty. Using these data, we can construct the notional probability mass function (PMF) in Figure 2-1.

Figure 2-1 Probability Mass Function for Soviet Nuclear Weapons in 1995

The PMF that characterizes possible scenarios in this manner can serve as an input to a Markov chain approach or for a stochastic optimization. These two approaches are viable in principle but are inadequate in practice. The principal reason is that the forecast horizon of ten years is short compared to the decades-long expected operational lifespan of ICBMs and SSBNs. Where the Intelligence Community does look beyond that timeframe, the quantitative elements are typically no more granular than demographics or economic trends (e.g. NIC, 2021). When designing weapon systems that will not reach operational status within the ten-year projection of these types of assessments, the projections are of limited utility.

Even if the NIEs were used as the basis of short-term projections to minimize near-term uncertainty, they are not without their shortcomings. The 1986 NIE on Soviet nuclear capabilities stated, "This year our projections of the force levels the Soviets could achieve…is a few thousand warheads lower than last year's" (CIA, 1986). Those projections are compared to the actual trajectory of Soviet/Russian forces in Figure 2-2. Note that this timeframe includes the collapse of the Soviet Union. While that event anticipated by a few, consideration of that possibility evidently never made it into this type of analysis. This projection represents the best information available at the time but is demonstrates significant error and is inadequate for optimization.

Figure 2-2 Soviet/Russian Nuclear Weapons 1986-1995: Predicted vs Actual

Unfortunately, this type of error persists to the present day. The most recent example of which are the official projections for the size of China's nuclear forces. In 2019, the

head of the Defense Intelligence Agency stated in that he assessed that China's nuclear arsenal "is likely to at least double in size" by 2029 to ~ 600 weapons (Kristensen, 2019). The Department of Defense's 2021 estimate for China's 2030 warhead inventory was "at least 1,000" and they acknowledge that it was a substantial change from previous years (Office of the Secretary of Defense, 2021). It remains to be seen how accurate any of the projections turn out to be, but they further highlight the difficulty and limitations of using even the best available information to make quantitative predictions about the future. There is no known Intelligence Community-approved process for generalizing projections in a manner amenable to rigorous quantitative analysis.

2.4 Robust Optimization

The problem statement for this praxis is:

> *Nuclear weapon systems requirements change faster than designs can adapt risking late delivery of inadequate weapon system capability*

The problem statement points toward three basic approaches to find solutions. The first and most obvious approach would be to use the methodologies of Multi-Criteria Decision Making (MCDM) already in use in support of JCIDS. The basic question of system design can be addressed in several different ways. Quality Function Deployment, the Analytic Hierarchy Process, and other MCDM techniques have a long track record of success. They allow for the effects and relative importance of different influences on design and overall effectiveness to be dealt with in systematic and traceable manner, without avoiding the inevitable injects of subjective valuations and requirements uncertainty. Robust Decision Making (RDM) is an emerging field within this discipline that confront the perils of relying on static predictions for decision support (Lempert,

2019). RDM is evolving in the direction that can accommodate future uncertainty but to date is constrained by the subjectivity of 'plausibility' and the cognitive biases that define it. However useful they are, they all generally unsuited for this praxis for the same reason. They cannot easily address either requirements that change over time in an indeterminate manner or the temporal dynamic between system design and subsequent acquisition.

The second approach is through the use of 'least worst' analysis. As this praxis deals with nuclear weapons, minimization of the worst-case outcome is an eminently reasonable way to frame the problem. This points to the family of techniques known as 'minimax' or 'minimizing the maximum loss' (Wald, 1945). Commonly used in game-theoretic constructs, these have origins in decision theory but apply to circumstances where the interaction need not be zero sum. As with MCDM, it can deal with uncertainty, even time-varying systems (Guo et al., 2002) but a noted limitation in the approach is that the algorithms are conservative in that they are worst-case avoidant. Solutions can be robust in that are minimally sensitive to the influence of "uncontrollable factors" but may have undesirable residual properties. For example, the most robust solutions may incur excess in 'costs' when compared to those that have worse 'worst cases'.

Bertsimas and Sim (2004) introduced the concept of treating the algorithmic conservatism as a parameter that can be adjusted. The varying conservatism is encoded as a measured relaxation of constraints. Viable solutions are not required to be precisely bound by them. This method made it possible to identify solutions that are close to the 'least worst' solution but with 'costs' that are smaller, often substantially so, thereby

allowing for a broader exploration of problems like portfolio optimization. This opening came with the caution that critical evaluation is still required of solutions that violate constraints, no matter the degree. This is particularly important for engineering applications where many constraints reflect physical properties. The concept behind this 'relaxation' carries forward into other methodologies.

The final approach to solving the problem statement is through the use of optimization. It is important to note here the optimization context differs from traditional engineering optimization, which also deals with design choices. The principal distinction is that engineering optimization is typically measured against performance measures of a machine or a subcomponent, e.g., train underframe (Peng et al., 2019) or box girders (Wu et al., 2018). Common to all of these is that they are a closed system with definable stressors (Kang et al., 2012). There are scarce examples within the literature about optimization of the gross attributes of an engineered system. Even a good example of this for aircraft design (Prigent et al., 2016) is optimized against objectives self-contained within the system – cost, fuel consumption – and lack external dependencies.

Stochastic optimization (or programming) is the most widely used type of optimization, where uncertainty is modeled with random variables with known (or at least partially known) distributions. It is well suited for problems where the distributions are defined or can be surmised with confidence (Gorissen et al., 2015) and are time-varying. These techniques can be very powerful in settings where the future may be variable but predictable, e.g., seasonal product demand (Asmaa et al., 2019). However, in many real-world settings the requisite level of information may neither be available nor even extant. In the latter case, underlying processes may be non-anticipative or

adapted, where knowledge of future is intrinsically unknowable. These circumstances are addressed by robust optimization (RO), a field with strong roots within and similarities to stochastic optimization.

Facilitating the growth of optimization from stochastic programming and optimization and into robust frameworks were some subtle but critical changes to how uncertainty is treated analytically. In a paper on robust dynamic programming, Iyengar (2005) makes an important distinction between 'uncertainty' and 'ambiguity'. He distinguishes between uncertainty quantities with defined distributions and ambiguous ones with "unknown probability measures". In helping to bridge between stochastic and robust techniques, he showed that dynamic programming can be adapted to cases with a Markovian construct but where there is limited information about the probabilities, an essential early step toward a more considered view of uncertainty.

As the use of RO matured, it was recognized that early approaches required that all decisions be made prior to the removal of uncertainty (Ben-Tal et al, 2004). This forestalls the use of emerging information for decision making. New techniques were developed in order to deal with cases where information is revealed over time (Goh and Sim, 2011) (Chen and Zhang, 2009). They have the attribute of being 'distributionally robust' in that they find solutions that are feasible for the worst case under the parameters of an ambiguous distribution.

An additional means of bridging between stochastic and robust models is the use of new statistical characteristics of distributions known as directional (forward and backward) deviations. Defined by Chen et al. (2007), they help to better characterize asymmetric distributions and their basic ambiguity. In the same year Chen and Sim

(2009) showed that the "mean, covariance, direction deviations, and support" describe a distribution with sufficient specificity for robust approaches to identify better solutions than purely stochastic approaches. The utility of these deviations is evident in their broad use in a variety of RO applications (Leyffer et al., 2020).

Robust Optimization refers to a collection of methods that aim to "to [protect] the decision-maker against parameter ambiguity and stochastic uncertainty." (Gabrel et al, 2013). This is done by finding "a design point at which the output is least sensitive to the variable of the input noise variables" (Nejadseyfi et al, 2019). To reiterate the contrast with stochastic optimization, robust optimization does not require probability distributions to be known even while still constructed on assumptions. Formulated in the framework of linear optimization, the basic problem takes the following form (Gorrisson et al, 2015):

$$\min_{x}\{c^T x : Ax \leq d\}_{(c,A,d)\in U} \quad (1)$$

Where $\{c, A, d\}$ are the uncertain (real) coefficients and U is the uncertainty set.

The construct of Equation 1 is based upon several assumptions but the most significant one is that decision variables represent "here and now decisions". This means that decisions are made all at once at the outset without the benefit of information available later (Ben-Tal and Nemirovski, 2008). The limitation of the 'here and now' assumption has driven the development of a robust optimization variant known as Adjustable Robust Optimization (ARO). In this construct, the decision variables are no longer 'here and now' but 'wait and see' or two-stage. They can be made sequentially, in stride with the revelation of information, i.e., the resolution of uncertainty or the collapse of ambiguity. It is distinguished from 'static' RO in that it allows decision variables to be

changed as uncertain data is made available (Yanıkoğlu et al., 2019). It has been shown that ARO solutions are at least as good as those developed through 'static' RO (Gorrison et al, 2015). Because of the intrinsic flexibility in solution exploration, they are often better.

The key technique that underwrites robust optimization is the process by which the specific characterization of uncertainty is transformed into a deterministic construct that can be solved with existing formalisms (Ben-Ta et al, 2004) (Bertsimas et al, 2011). This is called a robust counterpart and it takes different forms depending upon the type of uncertainty. While the forms are principally aligned to the constraints, a substantial body of the literature relates to the problem of computational tractability, because it is not uncommon for this optimization to be NP hard (Iyengar 2005) (Calafiore and Campi, 2006) (Goh and Sim, 2011).

Given tractability and the ability to find solutions, at the heart of adjustable robust optimization are Linear Decision Rules (LDRs) (also known as affine policies) that capture the optimization's prescriptive results. LDRs ingest the robust uncertainty as it is manifest through revelation and output the optimized decision(s). LDRs are a tractable approximation to the 'adjustable robust counterparts' of multi-stage decision making (See and Sim, 2010). They are not functions of the immediate robust uncertainty but account for the totality of 'history' up to that a decision point. In this they stand in contrast to static RO (and stochastic optimization) decisions and are non-anticipative (Goh and Sim, 2011). They are a set of rules and the principal output of a robust optimization. They are used to determine optimal decisions under a given trajectory of uncertainty, i.e., a

specific scenario where the ambiguous variables are sequentially revealed. As scenarios represent a projection in time, they are also known as 'futures.'

Research has also confronted another fundamental but unstated assumption about robust optimization related to scenarios. The assumption is that the 'revealed' data is accurate and available in a timely manner. This is especially problematic for this praxis because the scenarios are determined by the decisions an adversary makes about their strategic nuclear weapons arsenals. At the present time when relevant information is shared through treaty verification mechanisms, the revealed data may be accurate. Were those information sharing pathways to be eliminated, i.e., in a post-treaty environment, any data subsequently revealed might not but just inaccurate but suspect. Fortunately, it has been shown (de Ruiter et al, 2017) that it is possible to treat "past data as an uncertain model parameter" while retaining computational tractability.

Many potential beneficiaries of RO are practitioners, not theoreticians, who may lack the technical expertise or inclination to construct RO models entirely from scratch. Over the years, several toolboxes have been introduced into the community that seek to reduce the barriers for use. They each create an interface vernacular that allows for optimization problems to be described in practitioner-intelligible terms, e.g., objective functions and constraints. These are then broken down and transformed into components that interface with commonly available solvers, e.g., CPLEX, Gurobi, and MOSEK. Different toolboxes also make use of different languages such as MATLAB, e.g., ROME (Goh and Sim, 2011) and RSOME(Chen et al, 2020), and Python, e.g., ROmodel (Weibe and Misner,2021).

2.5 Inventory Management Optimization

The basic challenge of inventory management is balancing demand with ordering quantity in order to minimize the penalty of either insufficient or excess stock. The objective functions for these types of problems take the form of cost minimization. The primary cost of unit acquisition is readily accounted as the ordering quantity at a particular point in time multiplied by the per unit cost. For any amount of inventory on hand there is a cost associated with storage that is typically calculated on a per unit basis as well (See and Sim, 2010). There is an additional cost associated with having insufficient inventory, known as the backlog cost. This cost is calculated on a per unit basis as well and represents a combination of acquisition, storage, and sales costs. The decision variables in inventory management are the ordering quantities as a function of time.

The inventory management problem is straightforward to state but the process of finding solutions is complex. Fortunately, it is a well-studied topic in the field of Operations Research (e.g., single period management (Miller 1986)), motivated in no small measure by the importance of inventory management to profitability (Baldenius and Reichelsten, 2006). Inventory optimization is being addressed through a variety of familiar algorithms (Perez et al, 2021). Agent-based Monte Carlo simulations (Chu et al., 2015) avoid the tractability problems of dynamic programming the reduce real-world complexity to simulation 'black boxes' the output of which can be evaluated for optimality through evaluation of KKT criteria. Blood supply management is considered an inventory management problem with uniquely difficult attributes: uncertain demand, literally life-or-death time sensitivities, and the variety of 'supply', i.e., blood types, and

non-manufacturing origin of the product. Blood inventory optimization has been evaluated using stochastic linear programming and multi-objective evolutionary algorithms with varying results. Evolutionary algorithms can effectively deal with model complexity (Zahiri et al., 2018) while the structure of stochastic linear programming allows it to generalize problems, making them more adaptable to local conditions (Dillon et al., 2017). Machine learning is now being applied to this problem as well. Reinforcement learning with simulation has shown promise in describing the behavior of supply chains driven by uncertain (and non-stationary) demand (Mortazavi et al., 2015).

The work of Mortazavi et al. (2015) highlighted the importance of having a good model of product demand. Unsurprisingly, constructing such a model is problematic, not only upon successive moments of decision, but over protracted periods. Forecast models that build upon prior demand are often useful, especially when demand is correlated (Baykal-Gürsoy & Erkip, 2011). However, even those models suffer from the limitation of their utility being tied to the manner in which uncertainty is characterized. For example, Bertsimas and Thiele (2006) provided examples where different distributions with the same statistical properties will yield different results when used for inventory management. As the field has matured, robust treatments of inventory management embrace ambiguity rather than uncertainty (Ben-Tal & Nemirovski, 1998) (Bertsimas & Sim, 2004).

A clean distinction between different approaches to inventory management was summarized by Govindan and Cheng (2018). Their rubric for discrimination between uncertainty context had three divisions. The first includes problems with random variables with known distributions, amenable to stochastic approaches. The second

group covers fuzzy decision making. The final one includes circumstances where the distribution of random parameters is poorly characterized, if at all, and the domain of robust optimization. The authors note that robust approaches have received less attention than the other options but suggest that it may be more applicable for real-world problems.

Under ideal circumstances, i.e., when future demand can be predicted with accuracy and precision, a question of how much inventory to have on hand and how much to order and when to order it is a straightforward problem. This is true even the case where there are there is demand variability, such as that exhibited for products that have seasonal demands. These ordering quantities are variable but anticipatory. In a case where customer demand is highly uncertain the inventory management challenge becomes much more difficult but is tractable with robust optimization techniques (Goh & Sim, 2011).

Despite the promise, robust optimization has received limited attention in the inventory management literature. Foundational work by Berstimas and Thiele (2006) addressed the deficiencies of dynamic programming (tractability) and model 'demand' in manner that does not require full knowledge of the distribution. The basic construct for robustly optimized inventory management was described in See and Sim (2010). In addition to laying out the basic relationships between demand, inventory, and cost, they implemented a factor model for demand that allows for accounting of "business factors as well as time series forecast effects of trend, seasonality, and cyclic variations". These were critical steps in connecting the abstractions to practical issues. More recent applications have refined the tradecraft (Gourtni et al., 2020) and tackled more complex inventory problems, e.g., multi-product demand and bimodal inventory storage, and built

bridges to other disciplines, e.g., control theory (Wagner, 2018) and game theory (Bertsekas, 2021).

2.6 Summary

The thesis statement for this praxis is:

> *A robust optimization model is required to produce weapon systems designs against uncertain requirements to reduce schedule risk and improve system capabilities*

The modelling approach adopted for this praxis is to abstract the process of buying and fielding strategic weapons systems as an inventory management problem solvable through robust optimization. The acquisition of nuclear weapon systems is analogous to an inventory demand problem where there is a demand signal and there are costs associated with having too many or insufficient quantities on hand. 'Demand' here is an exogenous input and is manifest in the Russian strategic nuclear force order of battle. It serves as the basis for one of two primary measures of American strategic nuclear forces, the upper bound for oversufficiency, or 'too much'. The other measure – the lower bound – is a minimum requirement as specified by the United States. The 'ordering quantity' is the measure of what systems to acquire (i.e., system design), in what quantity and in what sequence to satisfy 'demand'. The remaining element, 'inventory', represents an excess or deficit relative to a requirement (informed by 'demand').

The objective function aligns demand with inventory, subject to the appropriate constraints. An inventory deficit represents accumulating unmet demand that, through

the objective function, will influence optimal ordering quantities over time. They are measured as the difference between the maximum American nuclear warhead count relative to the Russian-informed upper bound and the minimum American nuclear warhead count relative to the American-defined lower bound. The alignment of classic inventory management with the organization of the praxis is summarized in Table 2-2.

Table 2-2. Strategic Nuclear Weapon Acquisition as Inventory Management

Inventory Management Framework	Nuclear Weapon Acquisition Analogue	Rationale
Customer Demand – time-varying and uncertain	Russian strategic weapon arsenal over time	Time varying and beyond US control
Inventory – dynamic measure of supply vs demand	Difference between available US strategic forces and those needed to satisfy requirements	Measure of the extent to which the nuclear warhead inventory meets requirements
Objective Function – minimize total inventory costs over time	Minimize excess or deficit relative to requirements	Geopolitical costs are incurred when forces are oversufficient - 'too big' - or insufficient - 'too small'
Ordering Quantity – meet net demand to date while anticipate future	Different ICBM and SSBN options	Basis for assessing the relative utility of different system configurations, i.e., designs, to meet requirements

Chapter 3 details the methodology used to bring together the design attributes of ICBMs and SSBNs and the future of Russian nuclear forces into a robust optimization framework modeled on an inventory management approach. This framework is then used to explore the research questions by generating optimal acquisition policies and critically evaluating the recommendations against the hypotheses.

Chapter 3 — Methodology

3.1 Introduction

This chapter describes the praxis processes and how they are organized and integrated to investigate the research hypotheses outlined in Chapter 1. The three major components of the methodology involve assembling the base data, formulating and running the optimization, and analysis of acquisition policy and hypotheses.

Note that throughout this chapter, the term 'Blue' is used interchangeably with 'United States' or 'American' and 'Red' is likewise used for 'Soviet Union' and 'Russia'.

Figure 3-1 Methodology Elements

3.2 Base Data

Like any complex system, nuclear forces can be characterized under a number of rubrics. For this analysis, the salient attributes are limited to those necessary to bridge between design decisions, system acquisition, and the measures of utility captured in the objective function. The units of measure for the objective function are cardinal counts of strategic nuclear weapons over time. However, in the design decision space, the count of weapons is a derivative quantity. The decision criteria under consideration are counts of

ballistic missile submarines (SSBNs) and intercontinental ballistic missiles (ICBMs) that carry these weapons. These then are the primary data.

These data highlight an essential aspect of the praxis design problem. When a missile or a ship is acquired, the capability it brings is not fixed because they do not have to be loaded to maximum capacity. With the ability to vary warhead loadouts, an ICBM represents a range of potential capability. Similarly, we see that all available missile tubes on a submarine need not be used. Flexibility of this nature is an important attribute of strategic nuclear weapons. It allows them to meet nuclear weapon employment guidance over an expansive range of political circumstances. However, ue to the entangled nature of strategic weapons, what is 'flexible' to us becomes 'uncertainty' to an adversary. In the absence of trust about intentions, uncertainty skews toward the most threatening possibility. Under the nuclear weapons treaties in effects since 1991, the uncertainty has been mitigation through a regime of intrusive inspections.

Despite the inspections, Russia's deep skepticism about American strategic intent persists. When American capabilities are explicitly accounted in Russian strategic calculations, the size of the American arsenal at a specific time, e.g., the 'benign' period of US-Russian relations in 2013, is not used as a foundation for their strategic planning over the long term. In a relationship with an overt trust deficit, e.g., US-Russia beginning with the Russian annexation of Crimea in 2014, a more common approach is to evaluate an adversary from a 'worst case' perspective. Leaders often note that intentions can change rapidly but capabilities generally cannot. The major implication of this for the praxis methodology is that strategic weapon design and acquisition optimization must account for both a minimum and a maximum capability. The minimum represents a

capability 'floor' for evaluation against US requirements. The maximum represents a capability 'ceiling' to be measured against Russian tolerance thresholds and their ability to attack the United States with nuclear weapons in any circumstance.

3.2.1 US Arsenal: maxima and minima

Although this praxis focuses on future systems, it must account for the transitional period where existing (legacy) systems are retired. This allows us to narrow the scope of the American data to the recent history of US strategic nuclear weapons. Because this is an investigation of design rather than a nuclear weapon analysis per se, the scope is further narrowed to the capability of American weapons rather than their actual warhead load and operational status. The 21st century inventory of ICBMs and SSBNs is show in the figures below. The actual count is shown through 2022. In 2023 and beyond, the numbers reflect their planned removal from the active arsenal.

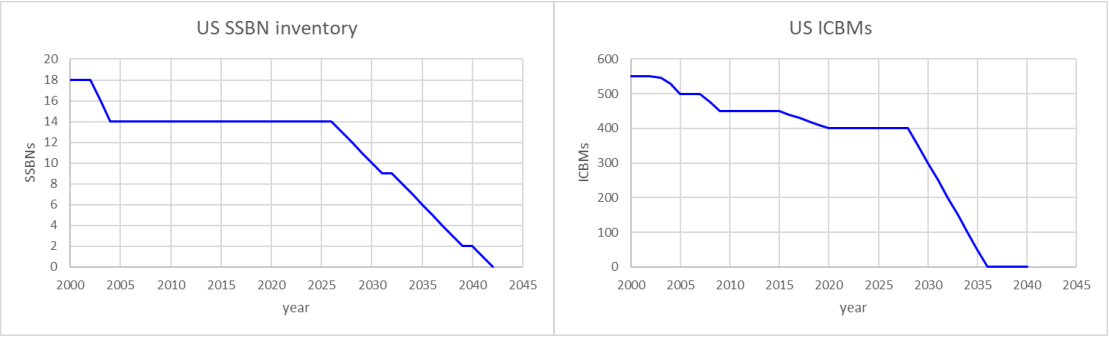

Figure 3-2. US SSBNs and ICBMs

We are next concerned with the range of warheads that these can carry by the following:

$$\#\,SSBNs \times \frac{missile\ tubes}{SSBN} \times \frac{max/\min\ warheads}{missile} = max/min\ SSBN\ warhead\ count \quad (2)$$

$$\#ICBMs \times \frac{max/\min\ warheads}{missile} = max/min\ ICBM\ warhead\ count \quad (3)$$

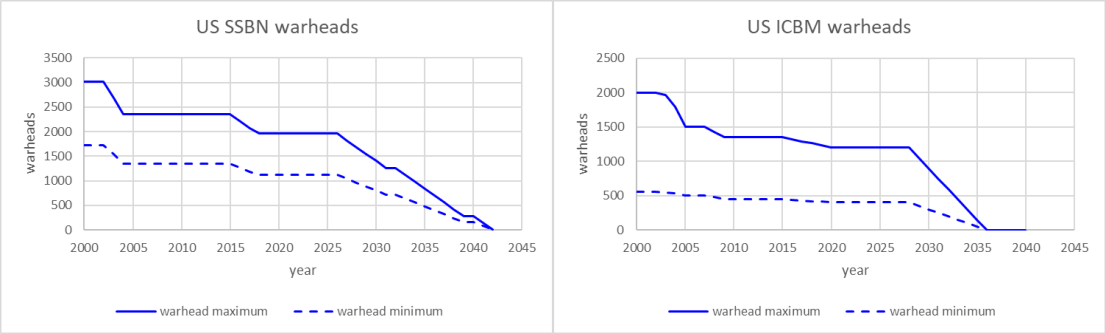

Figure 3-3. SSBN, ICBM Warhead Loading

The numbers from Figure 3-3 represent legacy Blue capability. The missile and submarine designs evaluated by this praxis will backfill this capability drop in the outyears. When combined with the output of the optimal acquisition policy for new systems, together they represent the complete range of threat possibilities calculable by an adversary.

3.2.2 Russian Arsenal

3.2.2.1 Total Russian Warheads

Central to the praxis is the total count of Russian strategic nuclear weapons over time. Figure 3-4 shows the Soviet/Russian arsenal of strategic nuclear weapons from 1958 to 2022. It shows slow build up through the 1960s and early 1970s followed by the explosive expansion through the 1980s and the end of the Cold War. The numbers saw an equally precipitous drop following the collapse of the Soviet Union and the subsequent dismantling of substantial portions of its arsenal. In the 21st-century, the trend shows the impact of arms control treaties with United States where the number of nuclear warheads has steadily dropped down to a treaty-compliant level of about 1985 warheads today (Kristensen & Korda, 2022).

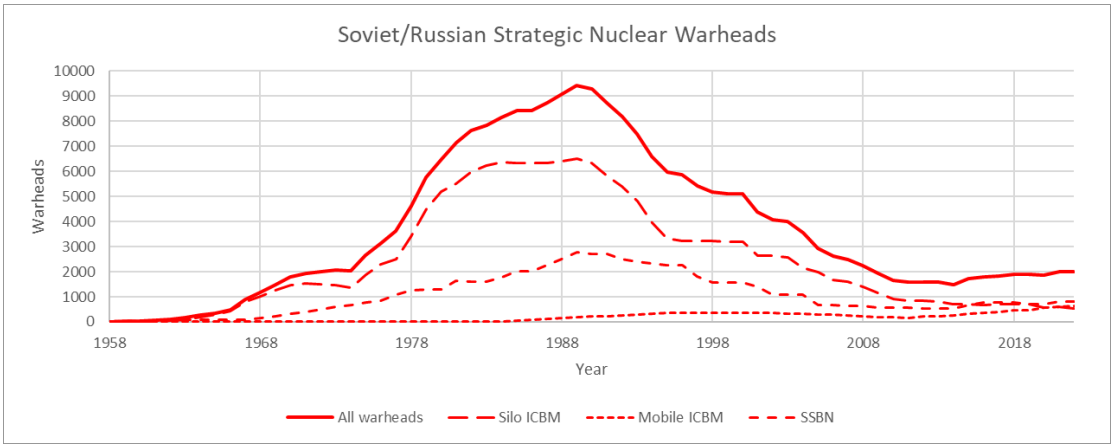

Figure 3-4. Soviet/Russian strategic nuclear warhead count

(Data from Suh (2022) and Kristensen and Korda (2002-2022))

As noted in Chapter 2, the Intelligence Community makes projections about future forces and have done so with varying degrees of success. The analytic challenge of forecasting endures, and the community has invested substantial resources into improvement of their tradecraft. They will always be subject to the inherent limitations of making predictions about the behaviors of people or bureaucracies.

Figure 3-4 represents the totality of extant empirical information about the choices that the Soviet and Russian governments have made about their arsenal. It is therefore the source of information from which the statistical model of the future is developed. As the decision variables describe annual acquisition, the representation of uncertainty must be framed in the annual rate of change of the Russian inventory. The historic year-to-year change is shown in Figure 3-5.

Figure 3-5. Annual change in Soviet/Russian nuclear inventory

This is recast as a cumulative distribution function (CDF) in Figure 3-6.

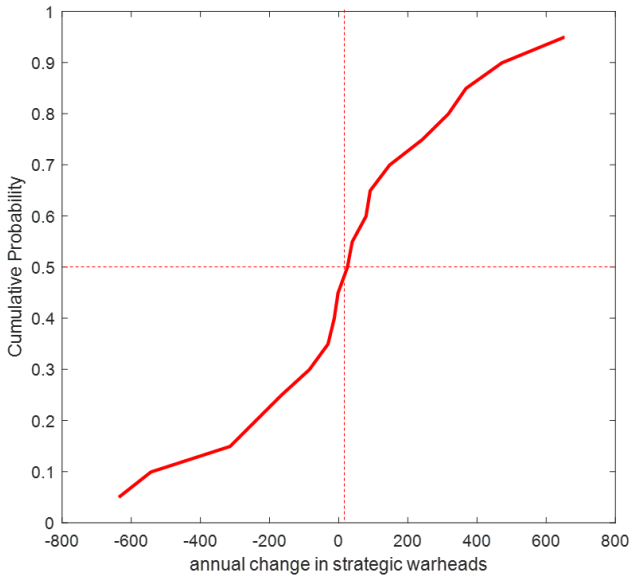

Figure 3-6. CDF of Soviet/Russian annual changes to nuclear inventory

With a mean of +26 warheads/year, this distribution is established as the model that will inform the future trajectory of Russian inventory. Represented by the robust variable *z(t)*, it is encoded with the distributional properties of the CDF. They include the main standard deviation and covariance, and important for asymmetric temporal variables, the forward and backward deviations. No further specifications of potential futures is required at this point. Scenarios behaving under the distribution of properties are captured within the robust algorithms.

It is worth emphasizing the distinction between robust and stochastic optimization here. A stochastic model requires the scenario be fully described by a probability distribution or mass function (e.g., Figure 2-1). As we have seen, the Intelligence Community has difficulty generating an accurate one for strategic nuclear weapons, which makes a stochastic approach problematic here. In the robust formulation, a complete characterization of the distribution is not required. As little as the distribution's support is sufficient to conduct a robust optimization. Additional properties, e.g., mean or deviations, if available, can be accommodated and improve the accuracy of the optimized policy. As the distribution's description gets more complete, it approaches a stochastic model. Here, the robust variable z(t) captures what is known about the distribution of Figure 3-7.

In the robust framework, where Red (z, t) is the total inventory of Russian weapons over time is calculated by:

$$Red(z(t), t) = Red(z(t), t-1) + z_2(t) \quad (4)$$

Where z_2 is

$$z_2(t) = (1 - \beta)\, z(t) + \beta\, z_2(t - 1) \quad (5)$$

This variable incorporates both the uncertain choice based on z(t) and a fraction (β) of the weapon inventory change in the previous timestep (Graves, 1999). This second term is an exponential smoothing that moderates what would otherwise be random walks by introducing a degree of year-to-year correlation of Russian decision-making (see Figure 3-8).

An autocorrelation test shows that this distribution is non-stationary but robust optimization does not require that it be. Nevertheless, the basic attributes of the distribution are unlikely to change significantly, even with another 50 years of data. The basic quasi-symmetric shape should remain because the very large changes on the margins are likely to be historical anomalies. The weapons added to the inventory in the 70s and 80s were products of a command economy operating under a Soviet industrial policy that emphasized quantity over quality (Bukharin et al., 2000). It was those weapons that were eliminated in the 1990s, and account for the largest annual decreases. Russia's economy is smaller than that of the Soviet Union and its industrial production capacity is limited, even before the strict sanctions imposed after the invasion of Ukraine. Contemporary Russian emphasis is on highly capable and therefore expensive nuclear weapons (Putin, 2018). For the foreseeable future, Russian has no ability to mimic the extreme production rates of their Soviet predecessors.

One assumption that needs to be made though is that the outer margins of the CDF are sufficiently scoped to bound feasible scenarios, regardless of their likelihood.

Within the 66 years span of data are three major types of events that affect weapon inventories: an arms race, the collapse of competition, and an arms control regime. They collectively span the historical margins of great power competition (JCS, 2019) but not actual historical extrema. The CDF does not include the cases of wartime weapons production or the complete dissolution of the Russian state. Both those are circumstances for which the American nuclear arsenal is not designed and are therefore be excluded from the optimization.

3.2.2.2 Russian warheads as robust variables

The robust variable Red (z, t) describes not a particular Russian future but all ones possible under the given z (t) because any particular year-to-year change is essentially impossible to predict. Over the space of decades, Red (z, t) represents an extraordinarily

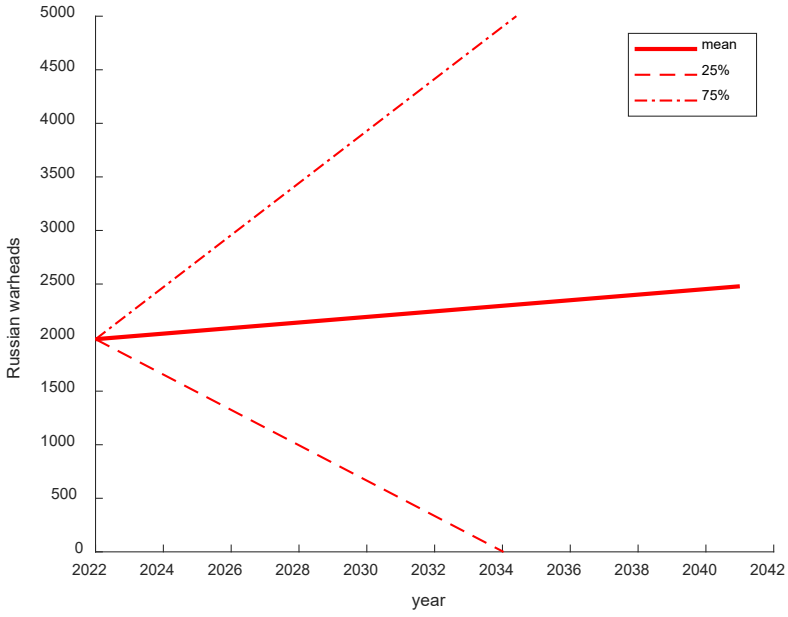

Figure 3-7. Future Russian Nuclear Warhead Inventory (25th, 50th, 75th percentiles)

large number of possible scenarios, three variations of which are show in Figure 3-7. The center line shows a scenario where the annual change is a constant: the mean or an increase of 26 warheads per year. The bottom dashed line represents the 25th percentile, or the functional evaporation of the Russian arsenal. The top dashed line shows twenty years of unabated aggressive expansion of their nuclear arsenal back into levels not seen in decades. These scenarios are mathematically feasible but ahistorical.

Feasible trajectories include the β-moderated (Equations 4 and 5) random walks shown in Figure 8a. The more distant any point is from the mean, the less probable it is as actual future. The trajectory of Red warheads will be determined by a complex mix of factors including geopolitical, economic, military, domestic political, and more but their net effect will be contained in Figure 8b. This figure further emphasizes the utility of the robust approach because it accounts for possible but improbable scenarios in the determination of the acquisition policy (e.g., CIA, 1986) that would otherwise be excluded by a purely stochastic approach to optimization.

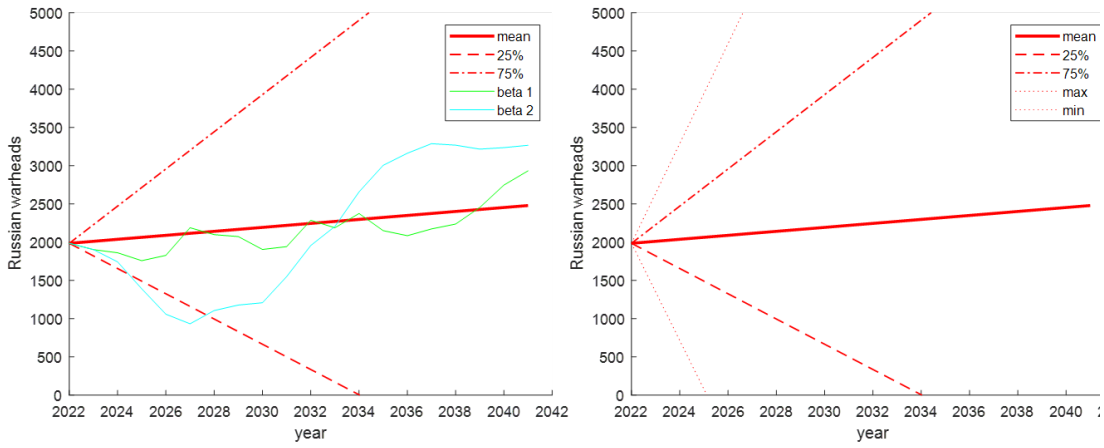

Figure 3-8. Russian Nuclear Warhead Inventory: Random Walks and Extrema

3.2.2.3 Russian Intra-Arsenal Distribution

While the overall number of Russian warheads is the principal measure that informs the optimization, the nuances of the interrelationships between Russian and American weapon systems require additional granularity. The total warhead count is the sum of the three principal types of nuclear strategic weapons in the Russian arsenal, silo- and mobile-ICBMs and submarine launched ballistic missiles. As shown in Figure 3-9a these have their own distributions. Ideally, they could be modelled as independent robust variables as well but the optimization problem of three independent robust variables remains computationally intractable. The single robust variable $z(t)$ remains the overall warhead count to be parametrically apportioned into the three weapon categories.

Figure 3-9b shows the change in the relative distribution of those three types of weapons over time. The changes in evidence are a clear reflection of the technological maturation of the weapon systems and evolution of doctrinal thinking. The long preponderance of silo-based ICBMs was a function of the dominance of Red Army thinking (Bukharin et al., 2000) and the dual use nature of ICBMs (they shared technological foundations with the Soviet space program). The subsequent relative growth of both mobile ICBMs and submarine-based in the post-Cold War Russian arsenal reflects improvements in missile accuracy and deepening concerns about the possibility of an American first strike (Bukharin et al., 2000).

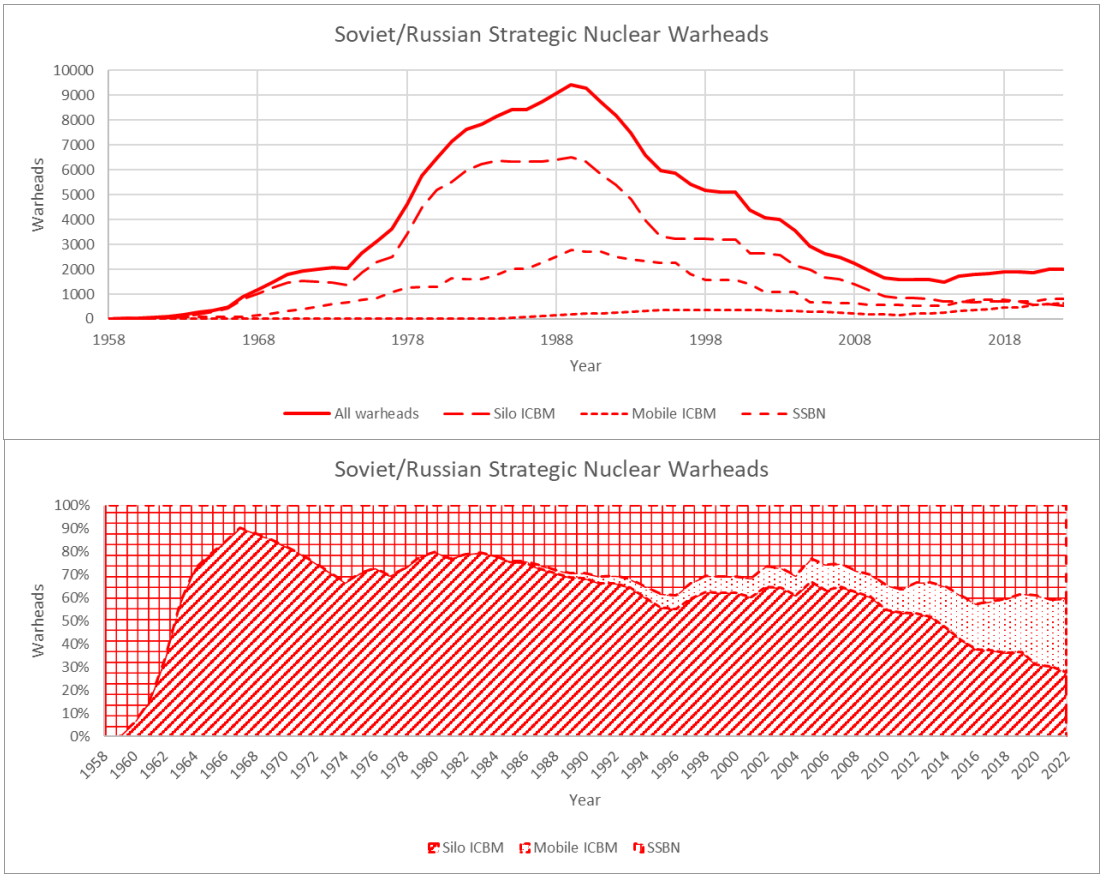

Figure 3-9. Relative Distribution of Soviet/Russian nuclear weapons

The distribution of forces in 2022 are ~ 40% SSBN-based weapons, ~30% mobile ICBM-based weapons, and ~30% silo ICBM-based weapons. Based on what is known of Russian intentions for their force, these ratios represent a balance of capability that is consistent with contemporary Russian doctrine, i.e., the ability to inflict unacceptable damage against the United States after absorbing a first strike (Putin, 2020). According to the Russians they are well along the process of recapitalizing their forces (Woolf, 2022) so we will take this fractionalization as the Russian baseline for the foreseeable

future. For the purposes of the optimization:

$$Russian\ warheads\ on\ SSBNs = 0.4\ Red(z,t) \quad (6)$$

$$Russian\ warheads\ on\ mobile\ ICBMs = 0.3\ Red(z,t) \quad (7)$$

$$Russian\ warheads\ on\ siloed\ ICBMs = 0.3\ Red(z,t) \quad (8)$$

Based on a nominal weapon load for each, this can be further devolved into a count of submarines and ICBM.

3.3. Optimization Design

As described in Chapter 2, the approach to optimization adopted for here is to abstract the process of buying and fielding strategic weapons systems as an inventory management problem. Inventory management is principally concerned with three variables: customer demand, inventory, and ordering quantity. Their relationship is managed through an objective function.

3.3.1 Design Alternatives and Decision Variables

The optimization of this praxis is intended to help discriminate between design options. The options themselves would be an expected output from an Analysis of Alternatives performed in accordance with Department of Defense directives. While strategic missiles and nuclear submarines are immensely complex machines, their geopolitical measure of utility is defined largely by one parameter: how many nuclear weapons can they carry. The design alternatives for the optimization are based on the different warhead carrying capacity of ICBMs and SSBNs.

There are two design variants for the ICBM. The baseline variant resembles what is currently in use, the Minuteman III, and its planned replacement, the Sentinel. The

maximum loadout of the Sentinel is not known but the missile it is replacing can carry between 1-3 warheads each (Missile Defense Project (MDP), 2016a). The second alternative is based on the largest ICBM America has fielded to date, the MX or Peacekeeper missile. Deployed from the 1980s through 2005, each missile could carry up to 10 warheads (MDP, 2017).

The submarine design space is more complex because the principal design decision is the number of missiles to carry rather than a specific warhead capacity. Simplifying this analysis, the Navy made the decision that the next submarine will carry the Trident II/D5 missile and its descendants, each carrying up to 8 warheads (MDP, 2016b). The submarine design variants are based on the number of missiles that the ship can carry, measured by the number of missile tubes per ship. The first variant is a 16-missile tube submarine, which is a proxy for the 16-tube COLUMBIA-class being built today and its predecessors, the "41 for Freedom" SSBNs built in the 1960s. For the optimization, it can carry 16 missiles with 4 – 8 warheads per missile (Brookings Institute, 1998). The warhead capacity of the ship is therefore 64– 128 warheads. The second variant mirrors today's SSBN fleet. These are OHIO-class submarines that have 20 operational missile tubes. They are treaty modified versions of the as-built design ('New START OHIOs'). The third variant is that of the original OHIO-class submarine design with 24 missile tubes, with a fully loaded capability of 192 warheads. (MDP, 2016b)

The design optimization will discriminate between the six ICBM/SSBN combinations of the options in Table 3-1.

Table 3-1. Design alternatives for optimization

ICBM		
Option	Warhead capacity	Reference
1	1 - 3	Minuteman III
2	1 -10	MX/Peacekeeper

SSBN		
Option	Warhead Capacity	Reference
1	64 - 128	"41 for Freedom" Columbia
2	80 - 160	"New START" OHIO
3	96 – 192	OHIO

3.3.2 Optimization Variables

The optimization has two decision variables: the quantity of ICBMs and SSBNs to order as a function of time. The decision variables are not the optimization variables though. They are instead the number of weapons in inventory over time. To get from ordering quantity to inventory we must account for production lag, modeled through a parameter that imposes a delay between an order and its entry into inventory. Table 3-2 shows how a decision variable flows into a change in inventory for a single item with a build time of three years. The 'Buy' row represents orders placed. The 'Build' row tallies the number of items under construction. The 'In inventory' row shows the total number of orders that have transitioned into inventory.

Table 3-2. Single Item Acquisition

	Year									
	1	2	3	4	5	6	7	8	9	10
Buy	1									
Build		1	1	1						
In inventory					1	1	1	1	1	1

The flow of a more complex acquisition sequence is shown in Table 3-3 (also with a three-year lag).

Table 3-3. Multi-item Acquisition

	Year									
	1	2	3	4	5	6	7	8	9	10
Buy	2	1	0	1	0	1	0	0	1	0
Build		2	3	3	2	1	2	1	1	1
In inventory					2	3	3	4	4	5

As noted in Section 3.2.1, the total inventory is the sum of legacy systems and new orders. As the designs are for systems to replace existing ones that are aging out, the ordering quantity/decision variables will only add to inventory. Inventory is only decremented by the retirement of weapon systems currently in the arsenal.

The accounting of the warhead total over time is captured by summing the number of missiles in the arsenal and their nominal warhead load. The inputs are listed in Table 3-4.

Table 3-4. Summary of Blue variables and constants

Variable or Constant	Description
Blue_maximum	Count of the maximum warhead count for all systems (legacy and new), robust variable
Blue_minimum	Count of the minimum warhead count for all systems (legacy and new), robust variable
IC_old	Count of legacy ICBMs (missiles only)
IC_new	Count of new ICBMs (missiles only), robust variable
IC_old_warheads	Maximum warhead load of legacy ICBMs
IC_new_warheads	**Maximum warhead load of new ICBMs – Design parameter**
BN_old	Count of legacy SSBNs
BN_new	Count of new SSBNs, robust variable
BN_old_tubes	Missile tubes of legacy SSBNs
BN_new_tubes	**Missile tubes of new SSBNs – Design parameter**
BN_missile_load_max	Maximum warhead load of submarine-launched ballistic missiles
BN_missile_load_min	Minimum warhead load of submarine-launched ballistic missiles

For a given year, the maximum Blue warhead loadout is defined by:

$$Blue_{maximum} = IC_{old} * IC_{old_warheads} + IC_{new} * \boldsymbol{IC_{new_warheads}} + (BN_{old} * BN_{old_tubes} + BN_{new} * \boldsymbol{BN_{new_tubes}}) * BN_{missile_load_max} \quad (9)$$

Likewise, the minimum Blue warhead loadout is defined by:

$$Blue_{minimum} = IC_{old} + IC_{new} + (BN_{old} * BN_{old_tubes} + BN_{new} * \boldsymbol{BN_{new_tubes}}) * BN_{missile_load_min} \quad (10)$$

Note that there is no constant for the minimum warhead load for ICBMs because it equal to 1. Legacy and new SSBN missile loadouts are the same because the Navy plans to continue using the same missile – the Trident II/D5.

The flow from decision variables to optimization variables is shown in Table 3-5

Table 3-5. Summary of Key Quantities

Variables/Quantities	Description
Decision Variable	Ordering quantity for ICBM and SSBN variants
Transitional Quantities	New ICBMs and SSBNs in production
	New ICBMs and SSBNs in inventory
	Warheads carried by new missiles and submarines
Optimization Variables	Blue_maximum and Blue_minimum

3.3.3 Objective Function Construction

US nuclear counterforce doctrine posits that American forces are intended to hold-at-risk Russian weapons. But implicit in the last several decades of arms control is the influence of Russian views on the size on the American force. This means that there are two different value constructs being stressed simultaneously. The first is the American requirement to have sufficient weapons to achieve nuclear weapon employment policy objectives – a minimum. The second is to not have so many weapons that Russian security is unduly threatened – a maximum. The design optimization will be evaluated with a single objective function that combines two distinct but coupled components. The objective function must account for the following:

- With a specified quantity of ICBMs and SSBNs in the American arsenal:
    - the maximum number of Blue warheads that could be fielded ($Blue_{max}$)
    - the minimum number of Blue warheads that could be fielded ($Blue_{min}$)
- The level below which the Blue warhead minimum should not drop ($Blue_{floor}$)
- The level above which the Blue warhead maximum should not rise ($Blue_{ceiling}$)

As shown in Figure 3-10, the ceiling and floor define a region of acceptability, a 'Goldilocks' zone, where the maximum Blue for a given number of missiles and submarines is not 'too much' and the minimum Blue is not 'too few'. As described in the following section, the 'ceiling' and 'floor' are defined through the paradigm of 'reciprocal first strike', where both the US and Russia evaluate their forces against a worst-case scenario of a surprise all-out attack by the other against their strategic nuclear force, i.e., the stressor of strategic stability (United States, 1990). The 'ceiling' is evaluated against two cases: when only nuclear weapons are used and when nuclear and conventional (non-nuclear) forces are included. There is only a single case for evaluating the 'floor'.

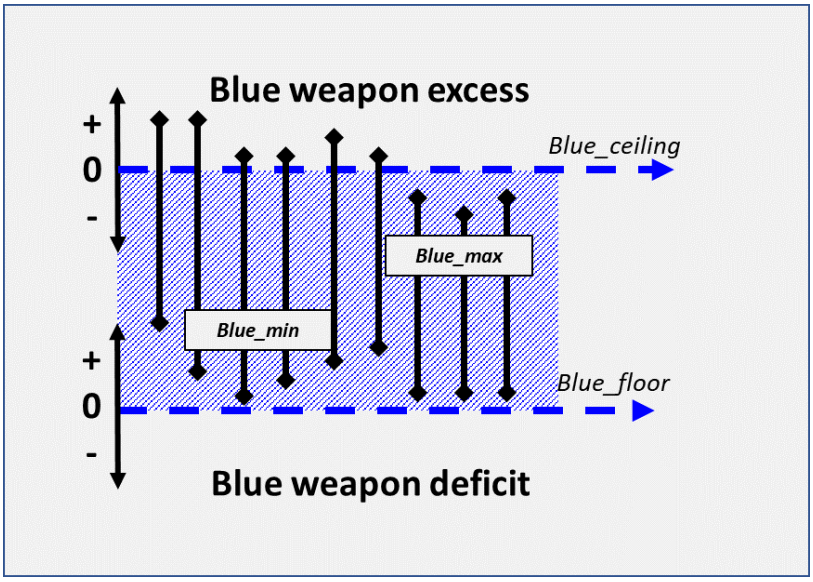

Figure 3-10. Notional relationship between min/max and floor/ceiling

The objective function for this construct will seek to minimize the difference between the Blue maximum and Blue minimum and the Blue ceiling and floor respectively. While any Blue force structure that has extrema within the Goldilocks

zone would satisfy the minimum and maximum criteria, US nuclear strategists and planners put a high premium on the intrinsic flexibility of the force (NPR 2010, 2018, 2022). That is manifest here by maximizing the span between the Blue minimum and maximum by adopting a dual parity model as shown in Equation 11. The first two terms penalize differences between the Blue_max and Blue_ceiling. The last two do the same for Blue_min and Blue_floor. The notation $|x|^+$ refers to the positive part only.

$$min \begin{cases} \left|\left(Blue_{max}(t) - Blue_{ceiling}(t)\right)\right|^+ + \left|\left(Blue_{ceiling}(t) - Blue_{max}(t)\right)\right|^+ \\ \left|\left(Blue_{min}(t) - Blue_{floor}(t)\right)\right|^+ + \left|\left(Blue_{floor}(t) - Blue_{min}(t)\right)\right|^+ \end{cases} \quad (11)$$

The ceiling for Blue weapons is derived from a calculation of how many weapons Russia will have left after an American first strike. The threshold of over-sufficiency is that number of Blue weapons that, if used, would leave Russia with fewer than its required minimum. The floor for Blue weapons is calculated in a parallel manner but with the Russians attacking the US. The minimum acceptable value for Blue is therefore the absolute minimum plus additional forces that are held at risk by Russia. These are illustrated in Figure 3-11.

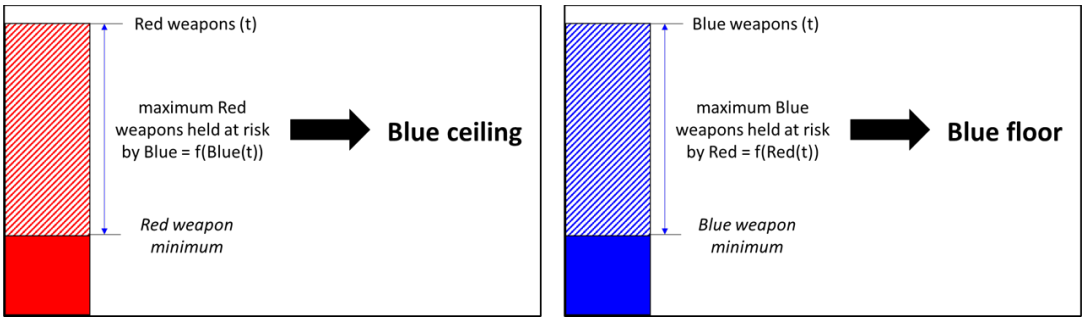

Figure 3-11. Ceiling and Floor for Blue weapons

3.3.3.1 Blue Upper Bound – Nuclear Only

The ceiling for optimal Blue warhead count is closely linked to Russian force structure. The Blue upper bound is a threshold that ensures that there aren't so many forces that they can eliminate the Russian arsenal below a minimum acceptable level, thereby setting conditions for strategic instability (Institute for the World Economy and International Relations (IMEMO), 2010). This minimum is assessed as being ~ 100 weapons (Kofman & Fink, 2022). The optimization must determine how many Russian weapons would survive an American first strike. This topic can of course be evaluated with highly detailed modeling and simulation but as we are concerned here with design optimization, not nuclear warfare, such an exercise is unnecessary. The survivability of Russian nuclear forces will instead by evaluated by the algebraic construct wherein the count of Russian weapons is decremented by different American capabilities.

When considered as potential targets of American attack, the Russian ICBM and SSBN forces have five different postures. Siloed ICBMs are in a fixed location, as are the garrisons for mobile ICBMs and SSBN home ports. Mobile missiles can operate dispersed from their garrisons in a manner that makes them much more difficult to find and target. Similarly, submarines underway at sea are essentially immune from attack by American nuclear weapons.

Calculating how American weapons decrement Russian ones requires us to understand how the different types of weapon systems can interact, because it is not simply the case of one weapon can shoot it can shoot another. It is made more complex by the different operational postures the weapons systems can take. The basic relational mapping between American weapons used against Russian weapons is shown in

. Nuclear weapons delivered from American ICBMs or SLBMS can hold at risk all Russian nuclear weapons except those carried on submarines at sea.

Table 3-6. Targeting Russian weapons

	US ICBM warheads	US SSBN warheads
Siloed ICBMs	Can hold at risk	Can hold at risk
Mobile ICBMs in garrison	Can hold at risk	Can hold at risk
Mobile ICBMs dispersed	Can hold at risk	Can hold at risk
SSBNs in port	Can hold at risk	Can hold at risk
SSBNs at sea	Cannot hold at risk	Cannot hold at risk

Table 3-6 is basis of a weapon assignment matrix (Xia et al, 2018) that allows us to understand how the size of the American arsenal threatens that of Russia. However, it is not simply a one-to-one relationship either because of the way in which nuclear weapons are concentrated and distributed across the Russian forces. Table 3-7 shows how this is so. A single American nuclear warhead can destroy one submarine in port, but that submarine, fully loaded, can carry up to 96 warheads (Kristensen & Korda, 2022). This represents an incredibly unbalanced exchange ratio in favor of the US. Similarly, one warhead can destroy all the mobile missiles in garrison, estimated be on the order of 68 warheads (Kristensen & Korda, 2022).[1] The estimate of two warheads to destroy a silo reflects the fact that they are specifically hardened against nuclear attack. Whatever

[1] This calculation is based on the nominal number of ICBMs assigned to a garrison, their warhead loadout, and the fraction that are likely to be dispersed at any given time.

the actual number is, it is with silos that the exchange ratio shifts in favor of the Russians. In the highly unlikely event that mobile ICBMs could be found and attacked by ballistic

Table 3-7. Exchange Ratio: US → Russia

US warheads		Russia loses on average	
1	Holds at risk	1 submarine in port	96 warheads
1	Holds at risk	1 mobile missile garrison	68 warheads
2	Holds at risk	1 siloed ICBM	5 warheads
10	Holds at risk	1 deployed mobile ICBM	4 warheads
Null	Holds at risk	1 submarine at sea	0 warheads

missiles, the value of 10 warheads is chosen to represent a barrage around a missile in transit, a highly conservative Russian scenario.

The variables and constants used to formulate the nuclear-only $Blue_{ceiling}$ are listed in Tables 3-8 and 3-9. Robust variables, i.e., those that have dependence upon the Russian robust distribution parameter z(t) are italicized.

Table 3-9. Blue ceiling: Red variables and constants

Variable or Constant	Description
Red	Total inventory of Red nuclear warheads
Red_min	Minimum number of Russian warheads required to survive an American attack
Red_IC_silo	Count of Russian ICBMs in silos
Red_IC_silo_warheads	Average warhead load of an ICBMs in silo
Red_IC_mobile	Count of Russian mobile ICBMs
Red_IC_mobile_warheads	Average warhead load of a mobile ICBM
Red_IC_mobile_readiness	Fraction of mobile ICBMs dispersed from garrison
Red_IC_mobile_garrison	Count of Russian mobile ICBM garrisons
Red_IC_mobile_garrison_warheads	Average warheads total of mobile ICBMs in garrison
Red_BN	Count of Russian SSBNs
Red_BN_warheads	Average warhead load of an SSBN
Red_BN_readiness	Fraction of SSBNs at sea
Red_BN_ports	Count of Russian SSBN home ports

Table 3-8 Blue ceiling: Blue variables and constants

Variable or Constant	Description
$Blue_{max}$	Total inventory of Blue nuclear warheads (maximum load)
Blue_Red_IC_silo	Blue warheads against Red ICBMs in silos
Blue_Red_BN	Blue warheads against Red SSBNs in port
Blue_Red_IC_mobile	Blue warheads against Red mobile ICBMs that are dispersed
Blue_Red_IC_mobile_garrison	Blue warheads against Red mobile ICBM garrisons
Blue_slack	Blue warheads unallocated

For clarity, the relationship between the variables in Tables 3-8 and 3-9 are illustrated in the series of equations that follow. They show how the primary uncertain variable Red (z, t) is structurally decomposed into a form that aligns with Table 3-7. The overall counts of Blue and Red warheads are as follows.

Overall count of US warheads:

$$Blue_{max}(z,t) = Blue_{Red_IC_silo} + Blue_{Red_BN} + Blue_{Red_IC_mobile} + Blue_{"Red_IC_mobile_garrison} + Blue_{slack} \quad (12)$$

Overall count of Russian warheads:

$$Red(z,t) = Red_{IC_silo_warheads} * Red_{IC_silo} + Red_{IC_mobile_warheads} * Red_{IC_mobile} + Red_{BN_warheads} * Red_{BN} \quad (13)$$

These function as constraints for the actual objective function, components of which are listed in Table 3-10..

Table 3-10. Red warhead distribution and Blue warhead count

	Red warhead count	Blue warheads to hold at risk
Siloed ICBMs	Red_IC_silo_warheads * Red_IC_silo (z, t)	2 * Red_IC_silo
Mobile ICBMs in garrison	Red_IC_mobile_garrison_warheads (z,t) * Red_IC_mobile_garrison	1 * Red_IC_mobile_garrison
Mobile ICBMs dispersed	Red_IC_mobile_readiness * Red_IC_mobile_warheads * Red_IC_mobile (z, t)	10 * Red_IC_mobile_readiness * Red_IC_mobile
SSBNs in port	Red_BN_warheads * (1 – Red_BN_readiness) * Red_BN (z, t)	1 * Red_BN_ports
SSBNs at sea	Red_BN_warheads * Red_BN_readiness * Red_BN (z, t)	null

These are then combined as summarized in the equation below.

$$Red(z,t) - (attrition\ from\ Blue_{max}) \geq Red_{minimum} \quad (14)$$

Where the attrition from maximum Blue is determined by comparing the total Blue inventory (Table 3-9)) with the second column in Table 3-10. If there are an excess of Blue warheads available according the counting rule, then that element of Red is decremented in its entirety. If there is a deficit, then the residual Red warheads are tallied and compared to Red_min. Calculated at every timestep, the ceiling for Blue weapons is the smallest value for which this inequality does not hold.

For implementation purposes, several simplifying assumptions are made. The first is that all of the SSBN bases and mobile ICBM garrisons will be targeted first, due to the favorable exchange ratio. This allows for a fixed decrement of the available Blue warheads. Target next in the sequence are the ICBM silos. In practice, even the smallest Blue arsenal will have sufficient numbers to target the credible range of potential Russian silos. The implication of this is that the balance of sufficiency for Russian weapon is principally determined by the number of dispersed mobile ICBMs and SSBNs at sea.

3.3.3.2 Blue Ceiling – Nuclear and Conventional

A nuclear-only representation of the balance between Russian and American forces reflects only the American view of the relationship. The Russians take a broader look at the threat to their nuclear forces that includes American non-nuclear or conventional forces. The former head of the Russian Strategic Rocket Force wrote an article in 2006 in which he described in general terms the nature of the American threat

as Russia perceived it. As indicated in Table 3-11, he gives Americans credit for both the capability and intent to use a variety of conventional weapons against their nuclear forces. This view, simply expressed, presents a framework with which the Blue ceiling can be constructed.

Table 3-11. Threat to Russian Strategic Nuclear Forces (Volkov, 2006)

N	Pre-attack weapon count
ΔN_{tech}	Losses to mechanical issues[1]
ΔN_{CW}	Losses due to conventional weapons
ΔN_{NMS}	Loss due to nuclear missile systems
ΔN_{ABM}	Losses due to anti-ballistic missile defenses
N_{tgt}	Weapons that will reach the United States $= N - \Delta N_{tech} - \Delta N_{CW} - \Delta N_{NMS} - \Delta N_{ABM}$

The Russians have expressed particular concern with classes of weapons that have the combination of accuracy and explosive yield to create effects previous reserved for nuclear weapons. Specifically called out are the Tomahawk cruise missile (Kapitanets, 2003) and the yet-to-be-operational hypersonic Conventional Prompt Strike missile (Garayev, 2009). The commonly articulated Russian view of American capability and intent diverges wildly from anything that looks plausible or sensible to the United States, it is a persistent feature in Russian quantitative analysis of vulnerability. Similarly, the Russians also are deeply concerned. about the latent capability of American ballistic missile defenses. This fear has roots in the earliest days of the Cold War and peaked in the heyday of Ronald Reagan's Strategic Defense Initiative. Dormant after the fall of the

Soviet Union, they were immediately inflamed by America's withdrawal from the Anti-Ballistic Missile Treaty in 2003. The variety of capabilities being developed are frequently cited by the Russians as affirmation of our continued hostile intent toward them (Gareyev, 2009) (Gerasimov, 2014). Finally, the Russians also account for prospective losses to their SSBNs at sea from the combat actions of American submarine force (IMEMO, 2010).

In this broader model of engagement, the Russians anticipate that the 'inevitable' American attack begins with thousands of conventional weapons (Kapitanets, 2003), followed by strategic nuclear strikes, attrition of submarines at sea, and concluding with the destruction of missiles and warheads in flight by what is by other measures a modestly scaled architecture for American ballistic missile defenses (Missile Defense Agency, 2022). Each of these non-nuclear factors represent additional losses to Russian strategic nuclear forces. Detailed analysis of the magnitude of those potential losses, as understood from a Russian frame of reference, is beyond the scope of this praxis, but will be accounted for in the form of decrements to the Russian strategic nuclear forces available for retaliation against the United States with the values show in Table 3-12.[2]

Table 3-12. Russian losses from American non-nuclear weapons

American capability	Russian loss
Precision guided weapons	25% of dispersed mobile ICBMs
Anti-submarine Warfare	1 SSBN at sea
Ballistic Missile Defenses	50 warheads

[2] Note that the values in the table are arbitrary but illustrative. An informed replication of this praxis can and should include supporting research and analysis to better define the potential range of these parameters.

These terms are then included in the modified equation for the Blue ceiling below:

$$Red(z,t) - (attrition\ from\ maximum\ Blue) - (attrition\ from\ conventional\ Blue) \geq Red_{minimum} \quad (15)$$

As with the nuclear-only case, the value for Blue_ceiling is calculated at every timestep.

Regardless of the actual potential loss, the principal effect on the optimization is a reduction of the upper bound to American force structure. Given the losses of survivable Russian mobile ICBMs and SSBN to non-nuclear means, the Blue ceiling is more sensitive to the losses inflicted on those same forces by American nuclear weapons. The addition of conventional weapons to the calculation further limits Russia's ability to inflict unacceptable retaliatory damage. This optimization constraint will further reduce the margin between the upper and lower limits of US nuclear capabilities. The addition of conventional forces is only evaluated for Hypothesis 1.

3.3.3.3 Blue Lower Bound

The lower bound for Blue forces, or Blue_floor is determined in a similar manner as the previous section but where the roles are reversed and Russia attacks an unalerted American force. The weapon alignment matrix for Russian weapons targeting American weapons is similar to Table 3-6 and shown in Table 3-13.

The US doesn't have mobile ICBMs and the warhead loading is a parameter not an uncertainty. Like Russian submarines, American submarines at sea are non-targetable but can be held-at-risk when in port. American silos are hardened against nuclear attack and require more than one warhead to ensure destruction. Because Russia lacks a conventional weapons capability in comparable numbers to the United States, the

Blue_floor calculation can neglect the possibility of attrition by non-nuclear forces. And although Russia does have an operational ballistic missile defense capability, it is only deployed around Moscow (Bukharin et al., 2000) and can be excluded from the calculation as well.

Table 3-13. Exchange Ratio: Russia → US

Russian warheads		US loses
1	Holds at risk	1 submarine in port
2	Holds at risk	1 siloed ICBM
Null	Holds at risk	1 submarine at sea

In an analogous construct to equation 13, the Blue floor is determined as the threshold Blue order of battle for which this inequality is not satisfied.

$$(Minimum\ Blue) - (attrition\ from\ Red) \geq Blue_{minimum} \quad (16)$$

The relative simplicity of the Russian targeting paradigm has the practical consequence of confining the interaction of Red and Blue variables in the optimization to US ICBM silos. The relevant parameters are listed in Table 3-16. Robust variables are italicized

Table 3-14. Blue floor parameters

Variable or Constant	Description
$Blue_{min}$	Total inventory of Blue nuclear warheads (minimum load)
Blue_min	Minimum number of American warheads required to survive a Russian attack
Blue_IC	Count of American ICBMs
Blue_IC_warheads	**ICBM warhead load: Design parameter**
Blue_BN	Count of American SSBNs
Blue_BN_readiness	Fraction of total SSBN count at sea
Blue_BN_tubes	**Count of missile tubes per ship: Design parameter**
Blue_BN_missile_warheads	SLBM warhead load

The only constant to be defined is Blue_min, or the minimize level of survivable American warheads. As the US has never formally promulgated a requirement for 'how much enough' (National Security Archive, 2021), we will select a value of 100 warheads based on published analysis (National Academy of Sciences, 1997).

3.3.3.4 Objective Function

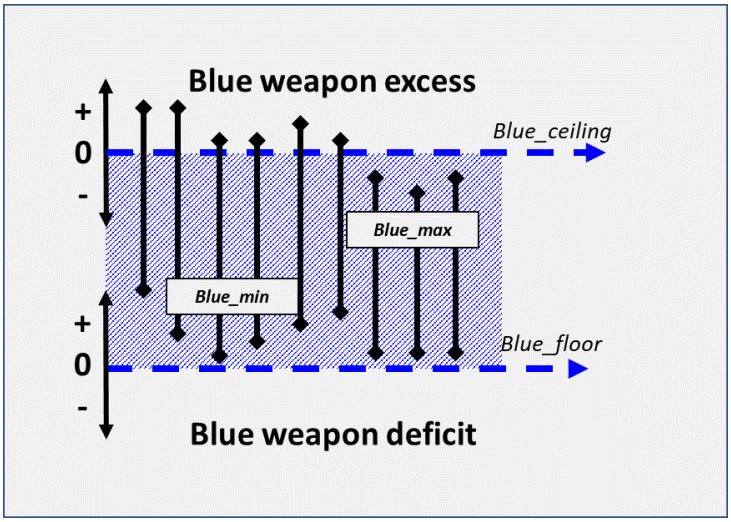

Figure 3-12. Components of Objective Function

As shown in Figure 3-14, there are two objectives being optimized for this praxis, one each relative to Blue floor and Blue ceiling. The optimization variables are the vectors representing the Blue_maximum and Blue_minimum over time. In order for the solver to work within the code, the objective function must result in a scalar, which requires the objectives to be scalarized (Gunantara, 2018). The scalarization here is informed by See and Sim (2010) and Kang et al. (2012). The objective function minimizes the mean differences between the time-series optimization variables and the objective of Blue_floor and Blue_ceiling. They are scalarized with a vector whose values are set to '1' but in a classic inventory management construct are typically associated with cost. The final objective function is:

$$\min\{ W_1 * \text{mean } |A|^+ + W_2 * \text{mean } |B|^+ + W_3 * \text{mean } |C|^+ + W_4 * \text{mean } |D|^+ \quad (17)$$

Where:

$A = Blue_{max}(t) - Blue_{ceiling}(t)$	maximum warheads above maximum
$B = Blue_{ceiling}(t) - Blue_{max}(t)$	maximum warheads below maximum
$C = Blue_{min}(t) - Blue_{floor}$	minimum warheads above minimum
$D = Blue_{floor} - Blue_{min}(t)$	minimum warheads below minimum
W_n	scalarization vectors

3.3.3.5 Constraints

In addition to the constants and variables described in the preceding sections, there is an additional set of parameters that inform the optimization (Table 3-15). These parameters affect and constrain the production rate and production numbers of the ICBMs and submarines. As a baseline for the optimization, the initial values are determined in a manner consistent with the recapitalization occurring within the existing operational and industrial footprint.

Table 3-15. Industrial Base parameters

Parameter	Value	Comment
ICBM production rate	50/year	Limitation of industrial base
ICBM production lag	2 years	Manufacturing constraint
ICBM total count	450	Limited by count of ICBM silos. Expansion of this is possible but politically infeasible
SSBN production rate	2/year	Shipyard capacity
SSBN production lag	7 years	Assumption for steady-state production. First ship is ~9 years.
SSBN total count	24	Ship-count expected when the two SSBN homeports were being built in the 1970s and 1980s.

Even though this is an extremely coarse representation of missile and ship production, it serves as critical constraints on the ordering policy. Unlike the more rapid production rate described in Wang & Tseng (2014) the production lag intrinsic to these types of systems strongly inhibit the ability of any algorithm to determine an optimal solution. That is because the rate of change of the demand may be much faster than the rate at which acquisition can respond. Given this, it is possible that the production rate is actual a binding constraint. For that reason, the sensitivity of the optimization to production rate is evaluated by Hypothesis 2.

For completeness, note that two otherwise important parameters are excluded from this analysis. The first is the availability of American warheads. There are significant pit production issues at the weapon labs (Woolf, 2020). Modeling the manner in which they could constrain the optimization is beyond the scope of this praxis. However, the Russians are likely to assume we could solve the pit production problem regardless (Kapitanets, 2003). Also, we will neglect the production of SLBMs. They are still in production and will remain in inventory even as older SSBNs are retired. For the purposes of optimization, warheads and SLBMs are available in whatever quantity necessary and are therefore not a constraint.

3.3.4 Optimization Execution

This problem is solved with established robust optimization algorithms. With the selection of design parameters, the number of years to run, the algorithm generates an optimized acquisition policy in the form of Linear Decision Rules. These are the bases for hypothesis evaluation (Goh & Sim, 2011).

3.4 Policy Analysis and Hypothesis Evaluation

The final step in the optimization process is to use the acquisition policies, developed by the application of robust optimization, and to show what they reveal about acquisition to possible scenarios, which in turn are used to generate the data for hypothesis evaluation.

3.4.1 Scenario Selection

The key determination of a scenario here are the choices that Red has made as captured by the robust variable Red (z,t) as it evolves over time. Here, a scenario is instantiated as a time series of the Red order of battle derived from a specific trajectory

through the space defined by Red (z, t). To illustrate this, consider a scenario where the rate of change of the Russian order of battle every year is the mean of the CDF in Figure 3-6, or + 26 warheads/year. The twenty-year projection of this is illustrated in Figure 3-13. The Russian order of battle starts at 1985 warheads in 2022 (Kristensen & Korda, 2022) increases to 2479 by 2041. The dashed lines underneath represent the derived Blue_ceiling and Blue_floor respectively.

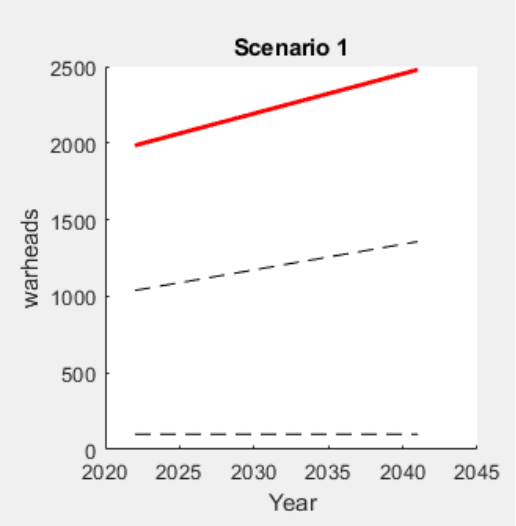

Figure 3-13. Scenario 1: mean Red warhead growth with Blue ceiling and floor

This scenario is ahistorical – no epoch in the history of nuclear weapons has seen a growth pattern that resembles this, but it is established as a point of reference. As companions to the mean are the two more extreme cases one where the rate of change is equal to the 75th percentile (+ 243 warheads/year) in the z distribution the other which is a 25th percentile of the distribution (- 165 warheads/year). As with the mean, both are ahistorical as well but are taken to represent upper and lower bounds of Red inventory trajectories.

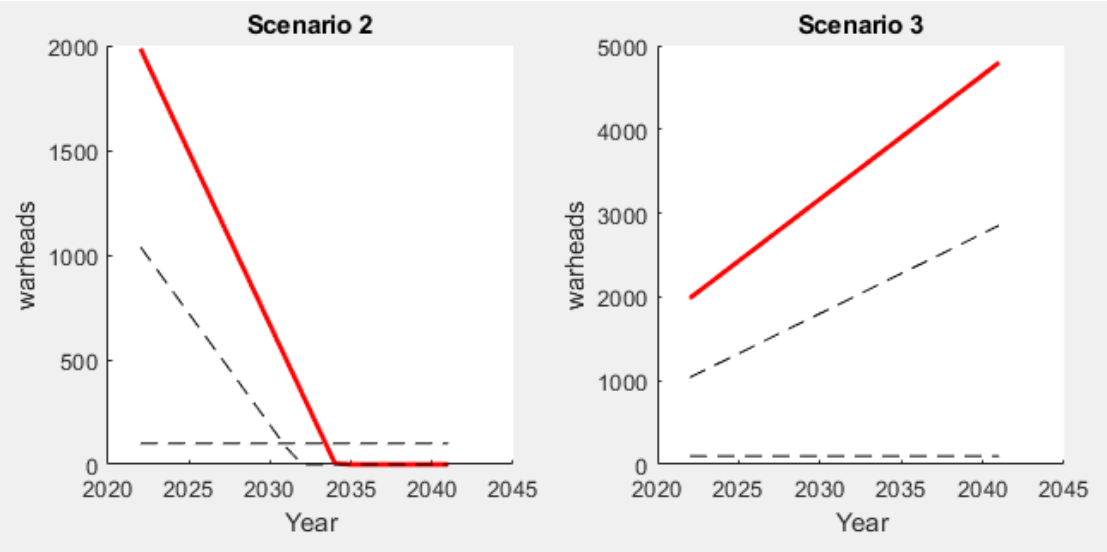

Figure 3-14. Scenarios 2 and 3 (25% and 75% percentile)

The next four scenarios represent circumstances that stress acquisition plans with significant production lags. The first (Scenario 4) is a case where the Red rate of change begins negative and transitions to positive. In the next (Scenario 5), the Red rate of change starts positive and transitions to negative. The Red rates of change in Scenarios 6

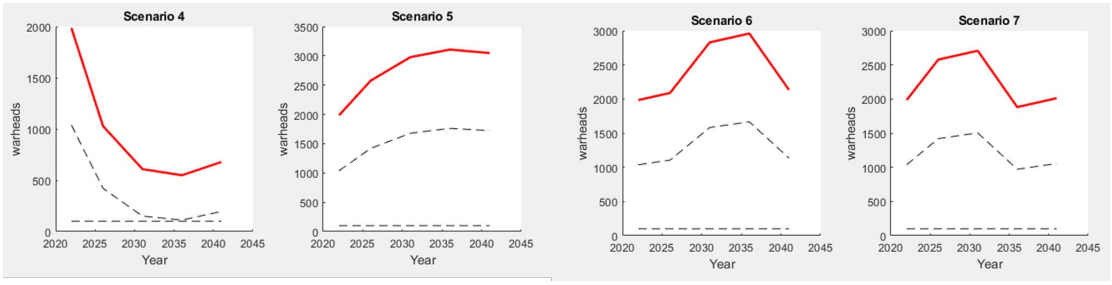

Figure 3-15. Futures 4-7

and 7 are oscillatory. These scenarios are selected to evaluate what the optimization acquisition policy is for time varying rates of change.

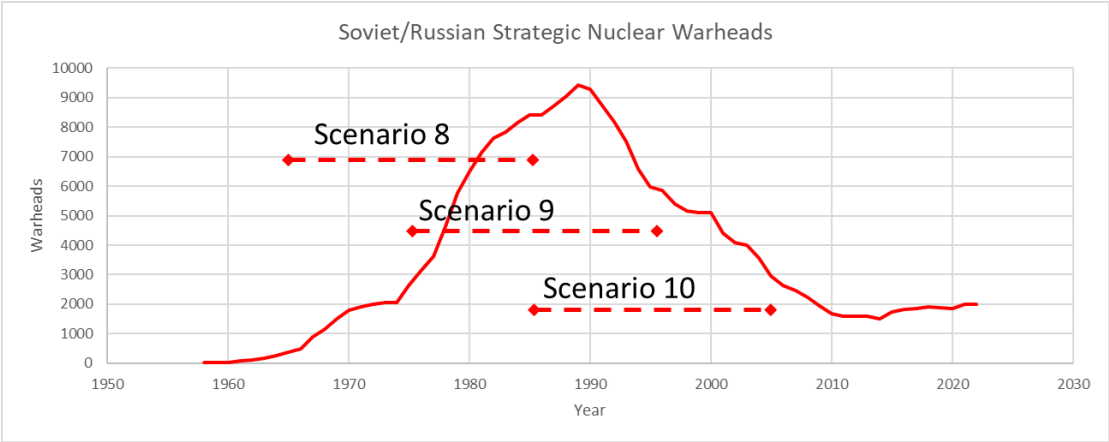

Figure 3-16. Historically Informed Scenarios

The final three scenarios are historical. They are selected to illustrate different geopolitical epochs. Scenario 8 is a period of aggressive growth where the distribution the future distribution is modeled from the period starting in 1965. The second scenario represents a transition from tension to cooperation. The final scenario a period of relaxation of tensions reflecting the onset of arms control and the collapse of the Soviet Union with the with the distribution beginning in 1985. Note that rates of change are not

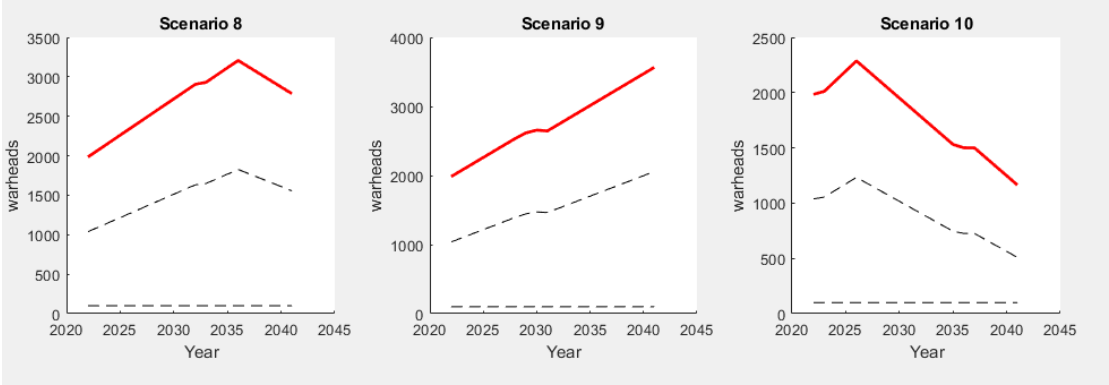

Figure 3-17. Scenarios 8-10

equivalent to those shown in Figure 3-18. They are scaled to the initial value of the Red order of battle to keep the trajectory from going negative.

3.4.2 Development of Data

The final step in the process of generating data involves the final integration of all the analytic elements outlined in this chapter. With a given set of design parameters (ICBM warhead load and SSBN missile tubes) and other parameters and constraints including the attributes of the Russian strategic nuclear forces, we generate an optimal acquisition policy. This policy is applied to various future scenarios, each recommending an optimal (or least worst) acquisition plan. The remaining step is to compare the actual recommended plan to the two parts of the objective functions, the Blue_ceiling and Blue_floor and evaluate how closely they are met.

To show how this is done, we consider the case of an ICBM with 3 warheads and SSBN with 16 missile tubes evaluated against a scenario where the Russian order of battle changes by the mean, i.e, Scenario 1, with all other parameters set to their nominal values. As indicated in this scenario, there's a monotonic increase in the Russian order of battle, which yields in turn, the Blue ceiling and floor indicated in Figure 3-18. The next step is to evaluate how the Blue order of battle changes over the same time period.

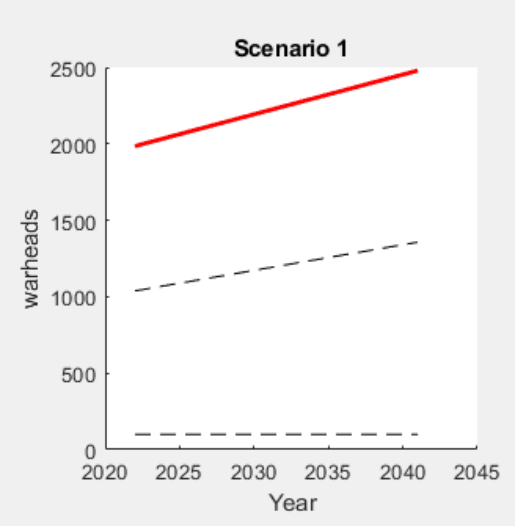

Figure 3-18. Example Scenario

3.4.2.1 Legacy Blue weapon systems

Figure **3-2** shows the Legacy Blue weapon systems. The existing submarines and missiles will eventually leave the fleet order of battle through retirement. This is shown in the green lines as indicated in Figure 3-19. Note that there is no change through 2027 because that is the period through which the New START treaty is valid. For realism, the assumption is made that the US would not begin to change the maximum loading of missiles and submarines until treaty expiration. This is reflected in the rise, and then precipitous fall of their contribution to the warhead maximum as the missiles and submarines are eliminated from active use. It is essentially a fixed element of the optimization. It represents the changing context for optimization of Blue warhead inventory on new systems.

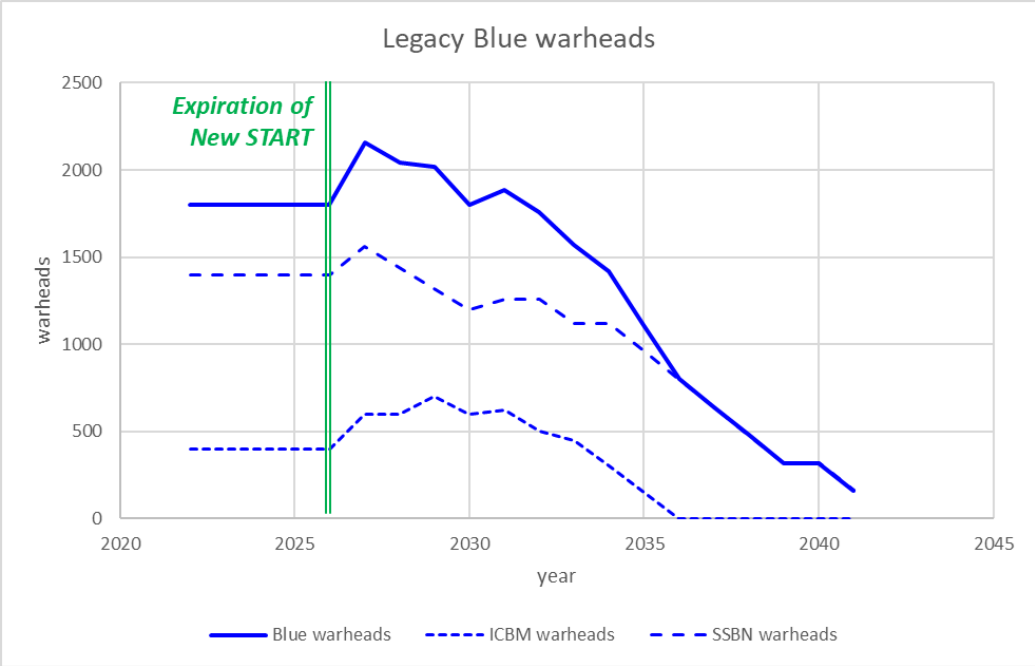

Figure 3-19. Legacy Blue Warheads

3.4.2.2 New Weapon Systems

Against that backdrop, the optimization policy recommended acquisition of new submarines and missiles and the subsequent build up in new warhead capacity is shown in Figure 3-20. This is the acquisition recommend through the sequential reveal of the Russian order of battle. The final figure shows the aggregation of legacy and newly acquired system into the total Blue maximum warhead capacity (and similarly derived minimum survivable capacity as well).

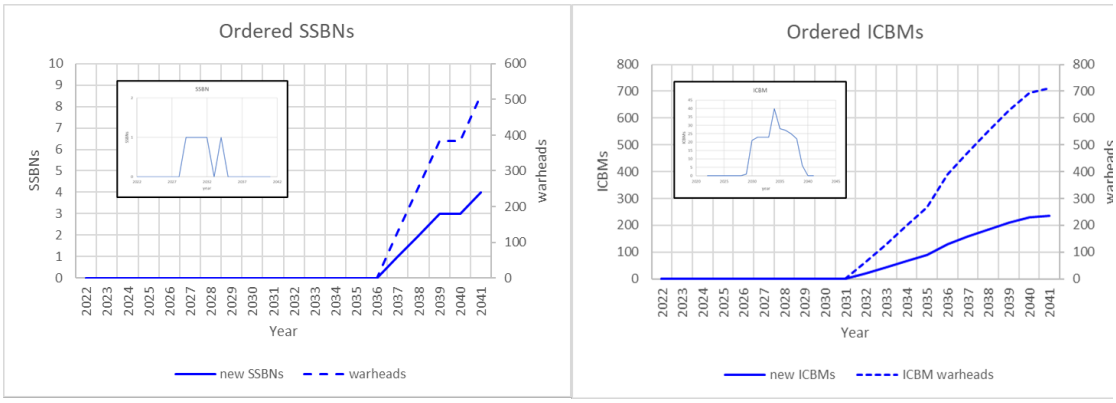

Figure 3-21. Robustly Optimized Ordering Quantities

These elements are brought together in Figure 3-21. Blue_minimum is the minimal warhead count of the entire Blue order of battle. Blue_survivable_minimum

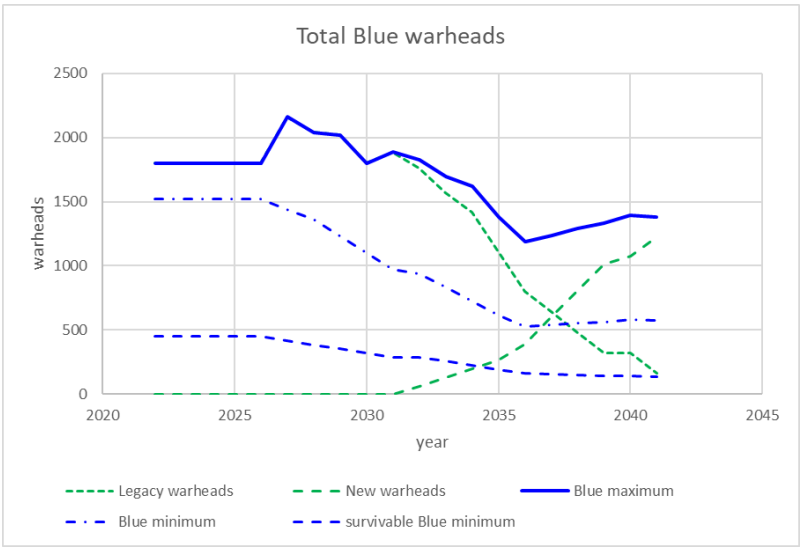

Figure 3-20. Blue warheads: Legacy and Optimized

represents only those Blue warheads that are invulnerable to a Russian first strike and is the measure compared to the overall minimum Blue warhead requirement.

The Blue max and Blue survivable min are then compared to the ceiling and floor as shown in Figure 3-22. In this example, as the new force is acquired the maxima and minima are well aligned to the ceiling and floor respectively.

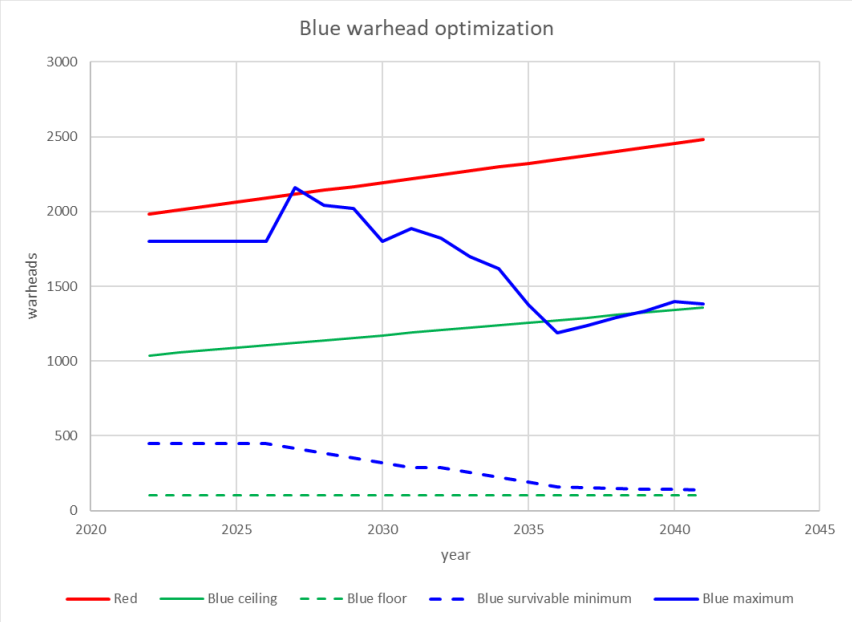

Figure 3-22. Comparison of Optimized Acquisition and Objective Function

From this, we count the cumulative excess warheads above the ceiling, and that is accounted. as the Blue_maximum_regret. Similarly, we tally the cumulative warhead count above the floor as the Blue_minimum_regret. This data captures the optimality of this weapon system design set and the constraints. This process is then applied to all designs for all constraints and all scenarios as necessary for hypothesis evaluation.

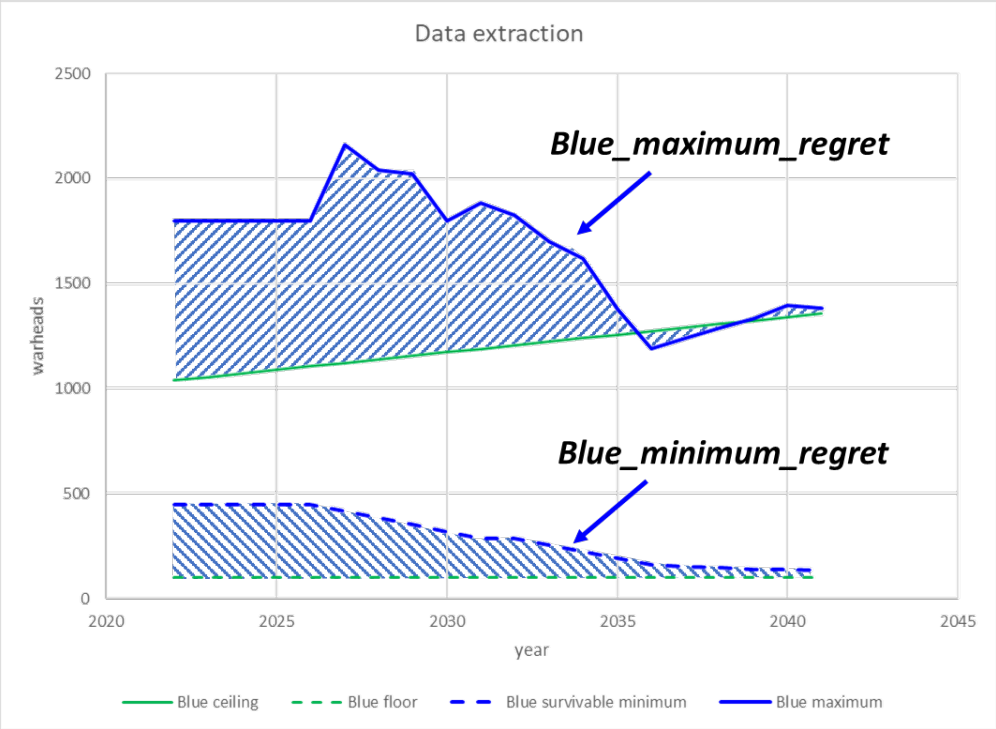

Figure 3-23. Determination of Regret

From this plot, we extract the following information as the datum.

Table 3-16. Data Example

Design		Years	Regret (warheads)	
IC	BN		minimum	maximum
3	16	20	3836	9297

3.4.3 Hypothesis Testing

They hypotheses developed in support of the three research questions of Chapter 1 require appropriate tests with which to evaluate their validity.

3.4.3.1 Hypothesis 1

Hypothesis 1:

> **The optimal weapon design combination for nuclear-only and nuclear/conventional constraints is the heavier ICBM (10 warheads) and largest SSBN (24 missile tubes)**

Evaluation

This hypothesis is focused on the core premise of the praxis, that robust optimization can identify the best choice of available design options. It will be evaluated through a comparison of the span of objective function regret across the design parameters for the both the nuclear-only objective function and the nuclear-conventional mix. The hypothesis will be confirmed if the 10 warhead ICBM and 24 tube SSBN represent the Pareto optimal design option over the range of potential scenarios and the nuclear and nuclear/conventional objective functions.

For each hypotheses' data set, the Pareto dominance of each point will be assessed and tallied for an overall measure of the frequency with which the data fall on the pareto frontier and are dominant. Ordinal Pareto dominance is the primary measure for hypothesis evaluation, where efficiencies are rank ordered (Iancu & Trichakis, 2014). A secondary measure of mean dominance is also calculated. Across a sample, it is the mean number of points that dominate the design. It helps to identify designs that may not fall on the Pareto frontier but are not far from it.

3.4.3.2 Hypothesis 2

Hypothesis 2:

> **More rapid missile and submarine production rates will improve design optimality for all scenarios**

Evaluation:

This hypothesis compares the impact of the industrial base's manufacturing capability to the design parameters. The production timeline variants are show in Table 3-19. The impact on Blue design will be measured against the ICBM and SSBNs in a nuclear-only optimization. The hypothesis will be confirmed if the rapid acquisition ICBM/SSBN designs are the Pareto dominant solutions.

Table 3-17. Hypothesis 2

Acquisition variant	ICBM	SSBN
Rapid	1 year	5 years
Nominal	2 years	7 years
Slow	3 years	9 years

3.4.3.3 Hypothesis 3

Hypothesis 3:

The optimum designs are insensitive to Russian force design philosophy

Evaluation:

This hypothesis relates to two principal properties of the Red warhead distribution, the numbers of specific weapon systems and their respective warhead carrying capacity and stresses the sensitivity to Russian designs for the force and individual weapon systems. This will be evaluated by varying the distribution ratios and their nominal per system warhead capacity and the Red design philosophy it represents. The impact on Blue design will be measured against the ICBM/SSBN design combinations in a nuclear-only optimization. The parametric variation is illustrated in Table 3-20. The hypothesis will be confirmed if the Pareto dominance of Blue ICBM/SSBN design combinations are invariant over the range of Russian force designs.

Table 3-18. Hypothesis 3

	Russian force ratios		
Design Emphasis	SSBN	ICBM, silo	ICBM, mobile
Survivability	0.4	0.2	0.4
Nominal	0.4	0.3	0.3
Heavy Lift	0.3	0.4	0.3

Chapter 4 —Results

4.1 Introduction

This chapter presents the data developed to evaluate this praxis' research questions and associated hypotheses. The optimization is coded into MATLAB, using the Robust Optimization Made Easy 9.0 toolbox, running CPLEX Optimization Studio 12.0 as the solver engine. As discussed in Chapter 3, the basis of hypothesis tests is the identification of the Pareto frontier within the select data and evaluating the degree of design dominance relative to the hypothesis. Hypothesis 1 tests the core premise of the praxis, that is, that robust optimization techniques can determine the optimal design choice under future ambiguity. Hypothesis 2 explores the sensitivity of the optimization to parameters of the design space within the control of the designer, specifically the impact of accelerated production timelines. Hypothesis 3 looks at the optimization sensitivity to parameters of the design space that are beyond the control of the designer, in this case the broad attributes of the Russian nuclear force.

For notional clarity, the six design combinations will be 'abbreviated' in this chapter according to the Table below.

Table 4-1. Design 'Abbreviations'

'Abbreviation'	3/16	3/20	3/24	10/16	10/20	10/24
ICBM warhead capacity	3	3	3	10	10	10
SSBN missile tubes	16	20	24	16	20	24

4.2 Hypothesis 1

Hypothesis 1 states:

> **The optimal weapon design combination for nuclear-only and nuclear/conventional constraints is the heavier ICBM (10 warheads) and largest SSBN (24 missile tubes)**

This was tested by developing optimal design acquisition policies for both nuclear-only and nuclear-conventional contexts and evaluating them against the ten scenarios. The minimum and maximum regret for each design in each scenario (measured in total warheads) for the nuclear-only constraint is shown in Figure 4-1 and the nuclear/conventional constraints are in Figure 4-2. The larger plot shows the aggregate of all designs and scenarios with the individual scenarios shown in the lower part of the figure.

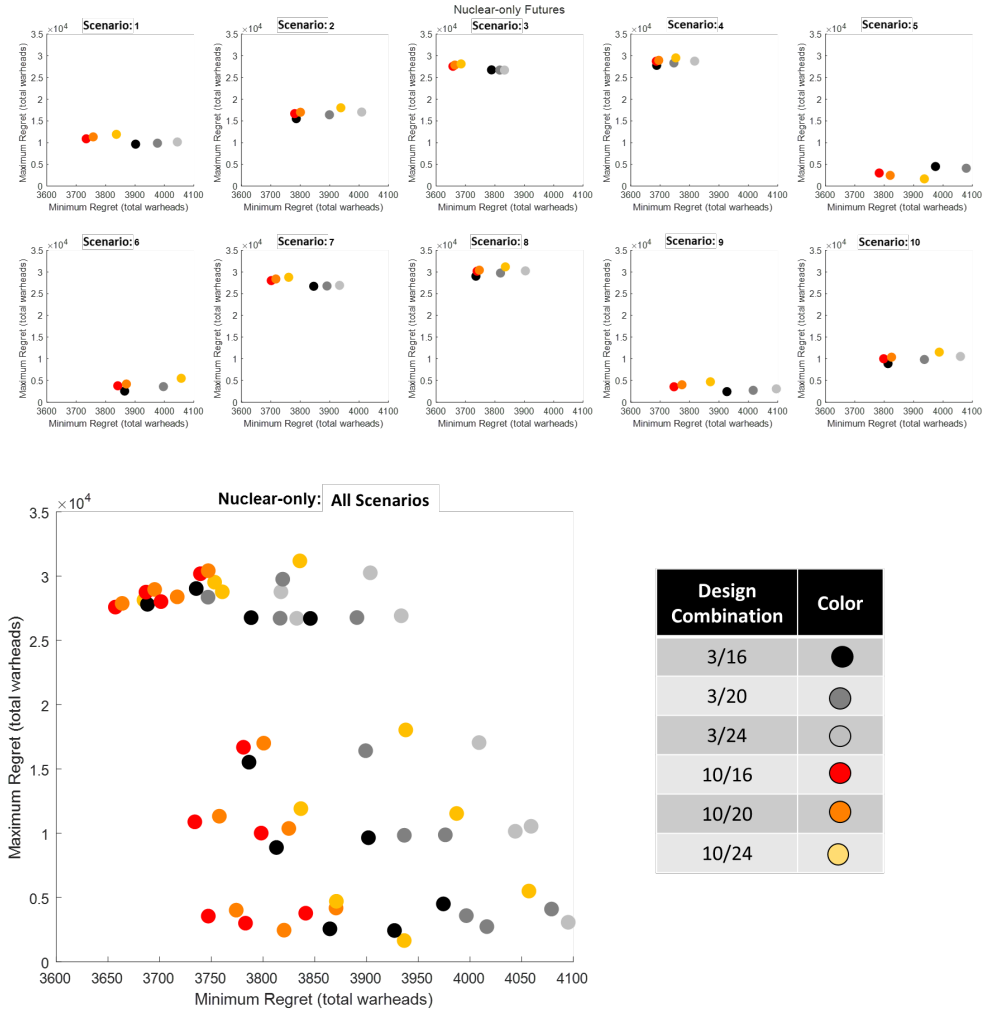

Figure 4-1. Hypothesis 1 Results: Nuclear-only Scenarios

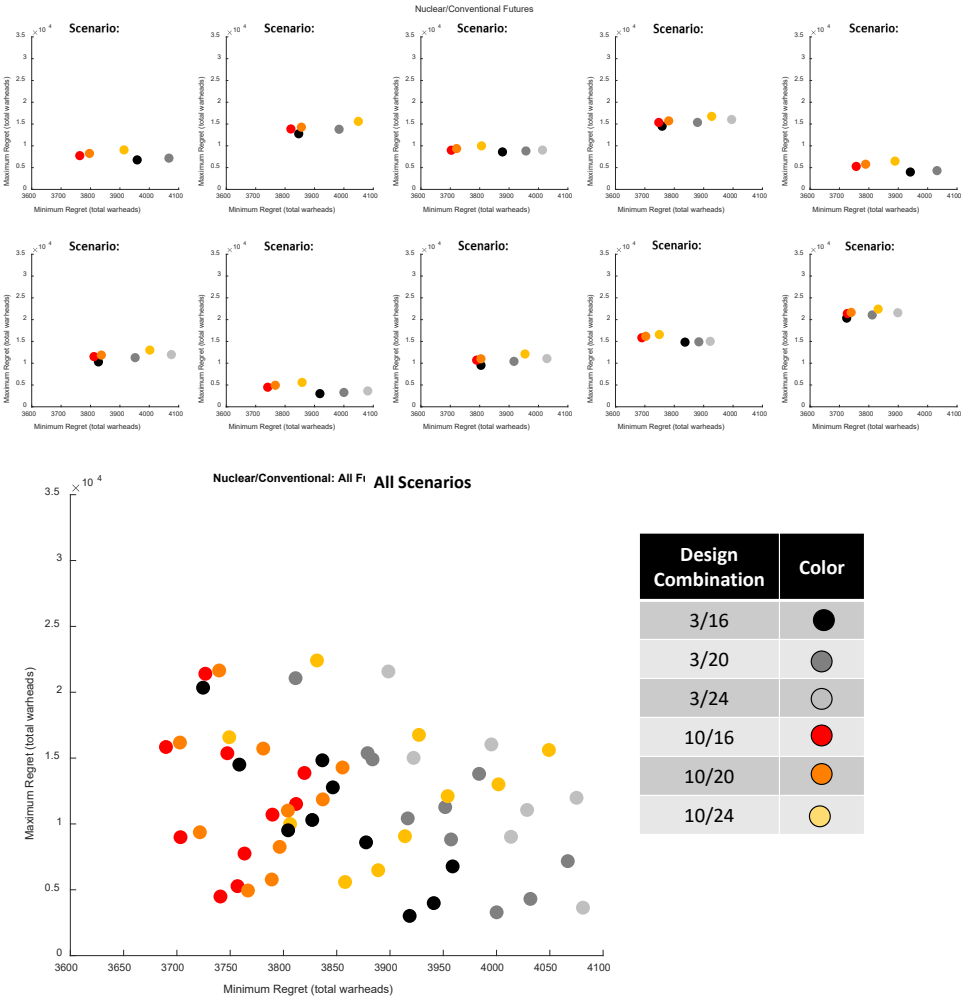

Figure 4-2. Hypothesis 1 Results: Nuclear/Conventional Scenarios

The stochastic dominance of the data is presented in Table 4-2 and Table 4-3. For each combination of design options and scenario (the source of randomness), the value shown indicates the number of other points that dominate it. A design dominated by all others, i.e., the least possible optimal, would show a value of "5". An undominated design, that is, one that lies on the Pareto frontier, shows a value of "0".

Table 4-3. Design Alternative Dominance in Nuclear-Only Scenarios

Design		Future									
IC	**BN**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
3	16	0	0	2	0	0	0	0	0	0	0
3	20	1	1	1	1	1	1	1	1	1	1
3	24	2	2	0	2	3	3	4	2	2	2
10	16	0	0	0	0	0	0	0	0	0	0
10	20	1	0	1	1	1	1	1	1	1	0
10	24	3	2	4	3	4	4	3	4	3	3

Table 4-2. Design Alternative Dominance in Nuclear and Conventional Scenarios

Design		Future									
IC	**BN**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
3	16	0	0	0	0	0	0	0	0	0	0
3	20	1	1	1	1	1	1	1	1	1	1
3	24	3	2	3	3	3	3	3	3	3	3
10	16	1	1	1	0	1	1	1	1	0	1
10	20	3	3	3	3	3	3	3	3	3	3
10	24	4	4	4	4	4	5	4	4	4	4

The summary statistics are shown in Table 4-4. The count indicates the number of scenarios where the design was not dominated by any others and therefore was on the Pareto frontier. The mean column shows for a given design, the average number of other designs that dominate it. A smaller number is indicative of overall proximity to the frontier and is a secondary measure of dominance.

Table 4-4. Hypothesis 1 - Summary Statistics

Design		Nuclear only				Nuclear and Conventional			
		Dominance				Dominance			
IC	BN	Count	%	mean	Rank	Count	%	mean	Rank
3	16	9	90%	0.2	2	10	100%	0.0	1
3	20	0	0%	1.0	4	0	0%	1.0	3
3	24	1	10%	2.2	5	0	0%	2.9	4
10	16	10	100%	0.0	1	2	20%	0.8	2
10	20	2	20%	0.8	3	0	0%	3.0	5
10	24	0	0%	3.3	6	0	0%	4.1	6

This data shows the best performing designs across the two constraint sets are the lightest combination, the 3 warhead ICBM/16 missile tube SSBN and the 10 warhead ICBM/16 missile tube SSBN. It also shows that not only is the combination of the heavier ICBM and the largest SSBN not Pareto dominant, is the most dominated of all designs for all scenarios and both strategic contexts.

Hypothesis 1 is unsupported by the data.

4.3 Hypothesis 2

Hypothesis 2 states:

> **More rapid missile and submarine production rates will improve design optimality for all scenarios**

This hypothesis is tested by evaluated by developing optimal design acquisition policies for all scenarios using three different sets of values for missile and production rate (Table 4-5). These policies are evaluated only within the nuclear-only constraint set.

Table 4-5. Production Rate Variations

Acquisition variant	ICBM production time	SSBN production time
Rapid	1 year	5 years
Nominal	2 years	7 years
Slow	3 years	9 years

The minimum and maximum regret for each design in each scenario (measured in total warheads) for the Nuclear-only constraint is shown in Figure 4-3. The larger plot shows the aggregate of all designs and scenarios with the individual scenarios shown in the lower part of the figure.

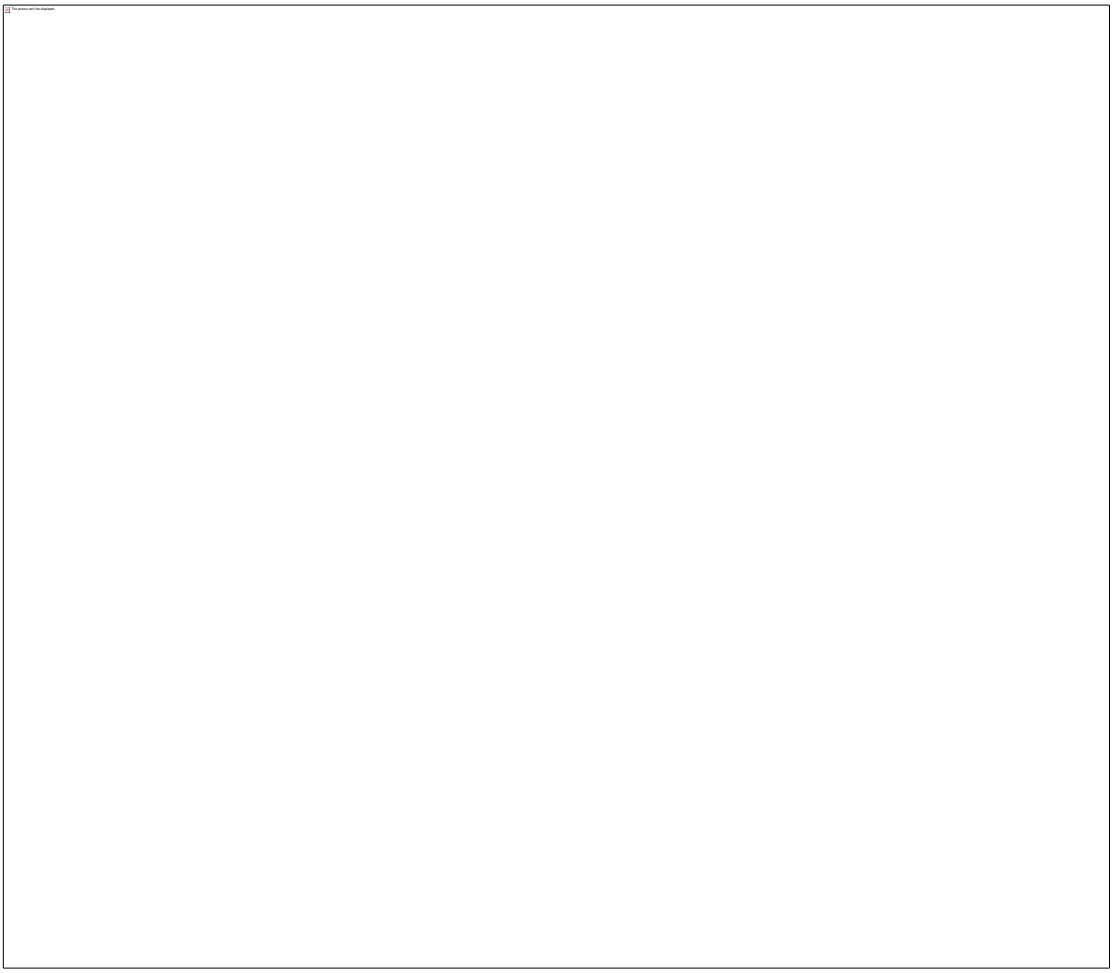

Figure 4-3. Hypothesis 2 Results

The stochastic dominance data are show in Table 4-6. For each combination of design options and scenario, the value shown indicates the number of other points that dominate it.

Table 4-6. Design Alternative Dominance over Acquisition Rates

	Design		Scenario									
	IC	**BN**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Rapid	3	16	11	11	7	11	11	11	11	11	11	11
Rapid	3	20	13	13	13	13	13	13	13	13	13	13
Rapid	3	24	14	14	14	14	14	14	14	14	14	14
Rapid	10	16	11	11	11	12	11	11	11	11	11	11
Rapid	10	20	13	13	13	13	13	13	13	13	13	13
Rapid	10	24	16	14	17	15	17	17	16	17	16	15
Nominal	3	16	3	3	5	3	3	3	3	3	3	3
Nominal	3	20	5	4	2	4	6	6	6	6	5	5
Nominal	3	24	8	7	1	8	9	9	10	8	8	8
Nominal	10	16	3	3	1	3	3	3	3	3	3	3
Nominal	10	20	4	3	3	4	4	5	4	5	4	3
Nominal	10	24	9	8	10	9	10	10	9	10	9	9
Slow	3	16	0	0	3	0	0	0	0	0	0	0
Slow	3	20	1	2	4	1	1	1	1	1	1	1
Slow	3	24	2	1	5	2	2	2	2	2	2	2
Slow	10	16	0	0	0	0	0	0	0	0	0	0
Slow	10	20	1	1	0	1	1	1	1	1	1	1
Slow	10	24	2	2	0	2	2	2	2	2	2	2

The summary statistics are shown in Table 4-7. The count indicates the number of futures where the design was not dominated by any others and therefore was on the Pareto frontier. The mean column shows for a given design, the average number of other designs that dominate it

Table 4-7. Hypothesis 2 - Summary Statistics

Design			Dominance			
Production Rate	IC	BN	Count	Percentage	mean	Overall Rank
Rapid	3	16	0	0%	10.6	13
Rapid	3	20	0	0%	13.0	15
Rapid	3	24	0	0%	14.0	17
Rapid	10	16	0	0%	11.1	14
Rapid	10	20	0	0%	13.0	15
Rapid	10	24	0	0%	16.0	18
Nominal	3	16	0	0%	3.2	8
Nominal	3	20	0	0%	4.9	10
Nominal	3	24	0	0%	7.6	11
Nominal	10	16	0	0%	2.8	7
Nominal	10	20	0	0%	3.9	9
Nominal	10	24	0	0%	9.3	12
Slow	3	16	9	90%	0.3	2
Slow	3	20	0	0%	1.4	4
Slow	3	24	0	0%	2.2	6
Slow	10	16	10	100%	0.0	1
Slow	10	20	1	10%	0.9	3
Slow	10	24	1	10%	1.8	5

The data show that for all future scenarios, the Pareto frontier is defined exclusively by designs having the slowest production rate. Contrary to the expectations of the hypothesis, the rapid production designs were dominated by both nominal and slow production. The least optimal option for all designs and all futures was for the rapid production rate.

Hypothesis 2 is unsupported by the data.

4.4 Hypothesis 3

Hypothesis 3 states:

The optimum designs are insensitive to Russian force design philosophy

This hypothesis tests the sensitivity of the design optimization to the choices the Russians make about the composition of their force as shown in Table 4-8. These policies are evaluated only within the nuclear-only constraint set.

Table 4-8. Russian Force Design Variants

	Russian force ratios		
Design Emphasis	SSBN	ICBM, silo	ICBM, mobile
Survivability	0.4	0.2	0.4
Nominal	0.4	0.3	0.3
Heavy Lift	0.3	0.4	0.3

The minimum and maximum regret for each design in each future (measured in total warheads) for the Nuclear-only constraint is shown in Figure 4-4. The larger plot shows the aggregate of all designs and futures with the individual futures are shown in the lower part of the figure.

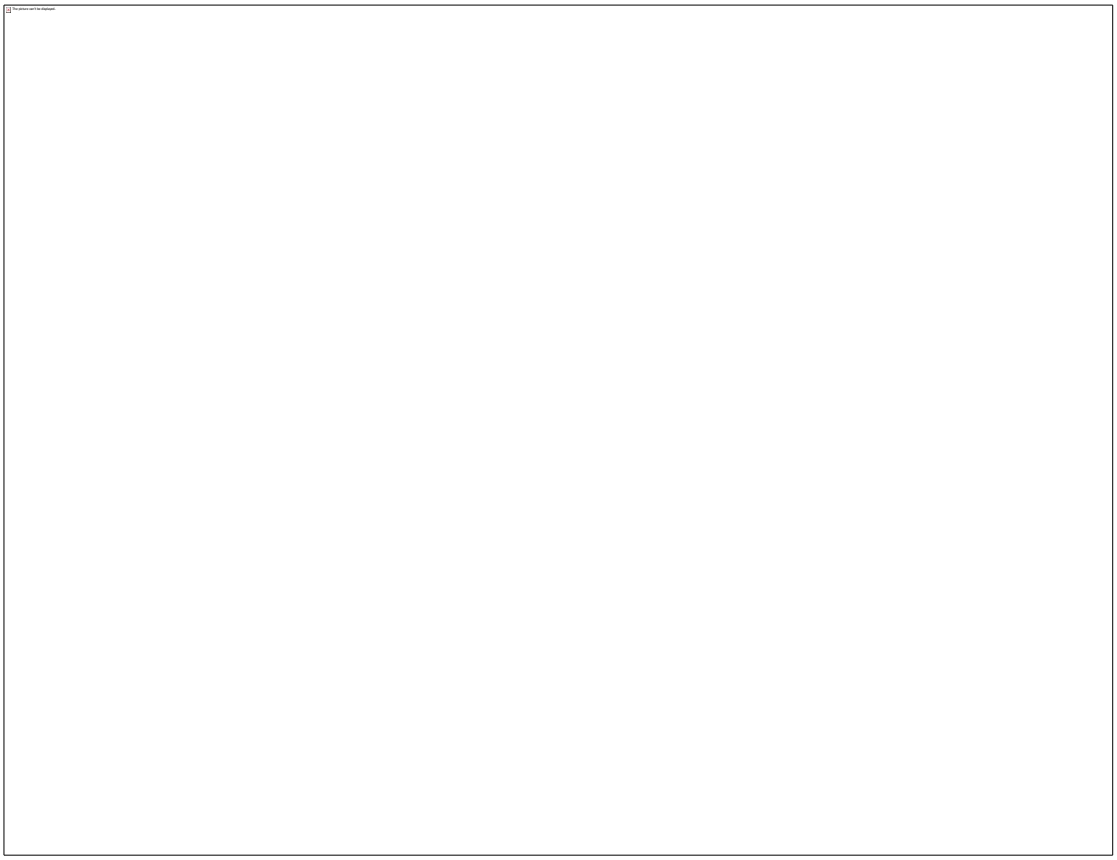

Figure 4-4. Hypothesis 3 Results

The stochastic dominance data are show in Table 4-9. For each combination of design options and scenario, the value shown indicates the number of other points that dominate it.

Table 4-9. Design Alternative Dominance over Russian Force Design Variations

Russian Variants	Design		Scenario									
	IC	**BN**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Survivable	3	16	0	0	0	0	0	0	0	0	0	0
	3	20	1	1	0	1	1	1	2	1	1	1
	3	24	2	2	1	2	2	3	3	2	2	2
	10	16	0	0	0	0	0	0	0	0	0	0
	10	20	1	1	1	1	1	1	1	1	1	1
	10	24	2	2	3	2	3	3	2	3	2	2
Nominal	3	16	0	0	2	0	0	0	0	0	0	0
	3	20	1	1	1	1	1	1	1	1	1	1
	3	24	2	2	0	2	3	3	4	2	2	2
	10	16	0	0	0	0	0	0	0	0	0	0
	10	20	1	0	1	1	1	1	1	1	1	0
	10	24	3	2	4	3	4	4	3	4	3	3
Heavy	3	16	0	0	2	0	0	0	0	0	0	0
	3	20	1	1	2	1	1	1	1	1	1	1
	3	24	2	2	1	2	2	2	3	2	2	2
	10	16	0	0	0	0	0	0	0	0	0	0
	10	20	1	1	1	0	1	1	1	1	1	1
	10	24	3	3	3	3	3	4	3	3	3	3

The summary statistics are shown in Table 4-10. The hypothesis optimization design is highlighted in yellow. The count indicates the number of scenarios where the design was not dominated by any others and therefore was on the Pareto frontier. The mean column shows for a given design, the average number of other designs that dominate it.

Table 4-10 Hypothesis 3 Statistics

Russian Design	Design		Dominance			
	IC	BN	Count	%	mean	Overall Rank
Survivable	3	16	10	100%	0	1
	3	20	1	10%	1	3
	3	24	0	0%	2.1	5
	10	16	10	100%	0	1
	10	20	0	0%	1	3
	10	24	0	0%	2.4	6
Nominal	3	16	9	90%	0.2	2
	3	20	0	0%	1	4
	3	24	1	10%	2.2	5
	10	16	10	100%	0	1
	10	20	2	20%	0.8	3
	10	24	0	0%	3.3	6
Heavy	3	16	9	90%	0.2	2
	3	20	0	0%	1.1	4
	3	24	0	0%	2	5
	10	16	10	100%	0	1
	10	20	1	10%	0.9	3
	10	24	0	0%	3.1	6

Table 4-11. Sensitivity to Russian Force Design

Design Rank	Survivable	Nominal	Heavy
1	3/16, 10/16	10/16	10/16
2	-	3/16	3/16
3	3/20, 10/20	10/20	10/20
4		3/20	3/20
5	3/24	3/24	3/24
6	10/24	10/24	10/24

The rank of each design combination is show in Table 4-11. For every scenario and every Russian system design, system design combination 10/16 is on the Pareto frontier. Design 3/16 is either on the frontier or closely adjacent for all circumstances as well. Similarly, design 10/20 is consistently the 3rd ranked design followed closely by 3/20. Designs 3/24 and 10/24 are always ranked 5th and 6th respectively. This table shows that there is very little meaningful variation in the design ranks across the three Russian design philosophies evaluated.

Hypothesis 3 is supported by the data.

Chapter 5 - Discussion and Conclusions

5.1 Discussion

It is a feature of optimization that results can be readily interpretable while simultaneously defying casual explanations. In the case of the hypotheses investigated for this praxis, particular designs may be optimal or non-optimal, but the reason for outcomes cannot be definitively determined without additional analysis. Nevertheless, the data can provide contextual indicators to inform subsequent investigations. The disconfirmation results for Hypotheses 1 and 2 were unanticipated because they were designed to evaluate conventional wisdom about military systems, i.e., that more capability sooner is always preferable to less capability later. Hypothesis 3, while confirmed, was exploratory.

As a test of convention wisdom, it was surprising that Hypothesis 1 showed that the largest intercontinental ballistic missile (ICBM) and the largest missile carrying submarine (SSBN) were the least optimal design pair. This is unexpected because a common thread in military thought is one cannot have too much capability. However, the objective function for this optimization is atypical in that it is imposes penalties for having 'too much', i.e., the Blue_ceiling. This speaks to the fundamentally political nature of nuclear weapons. So, in the doubly bounded optimization as implemented by this praxis, maximal designs impose a reduction in flexibility when having to deal with future uncertainty. It is difficult for them to remain unpenalized when Blue_ceiling drops, especially late in a time sequence. While this hypothesis was unsupported by the data, upon further consideration, it is less unexpected given the nature of the objective

function. It further suggests that a measure of flexibility should be developed for this type of design problem. As a final note on Hypothesis 1, the optimal design for the combination of nuclear-only and nuclear-conventional context is the 3-warhead ICBM and the 16-tube SSBN, which coincidentally are the potential attributes of the Sentinel missile and those of the actual Columbia-class submarine being built today.

The results for Hypothesis 2 are more difficult to explain than Hypothesis 1. It also bears further investigation, but one can still speculate as to reasons why faster acquisition is not optimal. It may be that the optimality of acquisition rate may be determined by the relationship between system production rate and the rate of change of the objective function, which in turn depends upon the temporal properties of the central distribution of Russian force structure (Figure 3-6). Were that the case, if Blue's acquisition rate outpaced that of Red, it could allow the optimization policy to recommend a larger force structure, an excess of which cannot be remedied by the removal of missile or submarines from the inventory. Similar, optimality could depend upon the relationship between the acquisition rates of the two American weapon systems. Either way, the existing data does not support identification of a specific causal mechanism for this hypothesis.

Hypothesis 3 explored the sensitivity of the optimization to factors that are entirely outside of the designers' control. History has shown that design philosophies for forces do change over time (Figure 3-9). Given both the magnitude of the expense of nuclear weapon systems and the non-existent appetite for diversified force, it is especially important that the basic design choices are insensitive to exogenous factors. The test of Hypothesis 3 shows that design choice is robust to a variety of different Russian choices

about their own force. The question of why a particular design choice was always optimal is the subject of Hypothesis 1 but the fact that the optimal choice is optimal under a variety of circumstance affirms the robustness of the design.

5.2 Conclusions

The limitations of this praxis preclude drawing hard conclusions from this body of work. Those limitations derive from several factors including the lack of authoritative public information about adversary capabilities, the simplistic nature of the acquisition model compared to actual practice (OSCAM, 2011), and the equal value of the future scenarios. Nevertheless, it offers insight into a novel approach for engineering design.

We can conclude from this praxis that design of the basic attributes of an engineering system can be effectively evaluated against complex but ambiguously uncertain requirements using robust optimization. From that, we can further conclude that requirements ambiguity need not preclude their inclusion into the design space. The robust approach allows the use of the best available information, regardless of its quality, by quantifying its distributional properties, thereby allowing more freedom of analysis than one would get through a purely stochastic approach.

For the missile and submarine systems under consideration, we can conclude that the major attribute of nuclear warhead loading cannot be treated as independent, relative to the implicit counterforce requirement of the American nuclear force (Chapter 1). In the adversary's frame of reference, they are alternatives of the same threat. The evaluation of Hypothesis 1 that contrasted an American view (nuclear only) to a Russian view (nuclear and conventional) allows us to further conclude that it is possible to

evaluate the consequences of competing frames of reference on design even under uncertainty.

5.3 Contributions to Body of Knowledge

This research was conducting in a domain with few priors, which allowed the praxis to make several contributions to the field. They include:

1. A demonstration that system design requirements that depend upon uncertain exogenous requirements, production rate, and the quantity in inventory can be evaluated quantitatively
2. A demonstration that an inventory management model can be used to account for explicit and implicit requirements in long-term planning, in essence, a new process for system evaluation in the early stages of weapon system design.
3. A demonstration that design criteria for different engineered systems can be evaluated simultaneously when they symbiotically serve the same requirement.
4. A method with which future uncertainty can be modeled beyond the time horizon of traditional Intelligence Community tradecraft
5. A kernel upon which a higher fidelity model of design and acquisition can be built within a computing environment appropriate for the information that populates it
6. A means to generate insights about different views of optimal American force structure

5.4 Recommendations for Future Research

This praxis serves as a proof of concept for dealing with design requirements that are not addressed by existing Defense acquisition guidelines. For that reason, there are several worthwhile directions for future research. They include:

- Model refinement
    - Incorporate government models of ships and missile design into the optimization
    - Work with the government to generate a prioritized weighting schema for a utility function to overlay on the different scenarios (Kang et al, 2012)
    - Develop an estimate of the error of Red(z,t) (de Ruiter et al., 2016) by evaluating how the accuracy of Intelligence Community assessments of adversary force levels change over time. This should include a characterization of errors associated with different information sources, e.g., spies vs satellites.
    - Determine how to treat parameters such as those in Tables 3-12 and 3-15 as probability distributions rather than scalar-valued
- Expand the optimization to include the emerging third player, the Chinese and their rapidly expanding nuclear force
- Adapt the model to include the emerging area of robust game-theoretic optimization (Bertsekas, 2021). That could allow incorporation of decades of prior work on two-party nuclear game theory. New work on three-party nuclear games will permit inclusion of China into this construct. Robust optimization may make it easier to find equilibria and plausibly be a means to capture

emerging rules and dynamics, while also incorporating robust formulations for future force structure.

- Apply robust optimization techniques for new systems such as the new attack submarine or SSN(X) (Burgess, 2022).
- Add the B-21 Raider heavy bomber into the objective function and re-evaluate the hypotheses with the addition of new American capability

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