

**A Dynamic Stochastic Optimization
for Recharging of Plug-in Electric Vehicles**

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Abstract of Thesis

A Dynamic Stochastic Optimization for Recharging of Plug-in Electric Vehicles

Plug-in electric vehicles (PEVs) grow quickly in recent years. The Electric Power Research Institute (EPRI) expects that the PEV market penetration may be 80% by the year 2050. The governments also make policies to stimulate the PEV sales.

However, there are some drawbacks in an unregulated PEV recharging control algorithm. An unwanted load peak that is caused by the PEV recharging could lead to a disruption, such as a brownout or a blackout in the grid.

The thesis proposes a two-stage dynamic stochastic optimization method in order to minimize recharging cost without raising the peak load. The method is an online scheduling algorithm that bases on an offline optimization. Our dynamic algorithm depends on the knowledge of driving behaviors and marketing information to take stochastic factors from different angles into account. The method is robust to several kinds of stochastic parameters, has a low communication requirement, and benefits both users and the power utility. In the paper, the system structure, data models, and mathematical formulation of the proposed method are introduced.

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Chapter 1 - Introduction

1.1 Background

Plug-in Electric Vehicle (PEV) or plug-in hybrid electric vehicle (PHEV) is a type of vehicles that has a battery to store power and has a motor rather than an internal combustion engine for propulsion. In recent years, with political factors, technical improvement, climate change issues and rising energy prices, PEV has gained extensive attention from researchers, users, power utilities and automobile manufacturers.

There is no doubt that there will be a considerable growth of the PEV penetration into the market. Some forerunner automobile manufacturers, e.g. Tesla Motors and Nissan, are in the process of making and selling PEVs. According to Electric Drive Transportation Association, there are nearly 400,000 PEVs and PHEVs in the United States. Figure 1.1.1 shows cumulative PEV and PHEV sales in recent years.

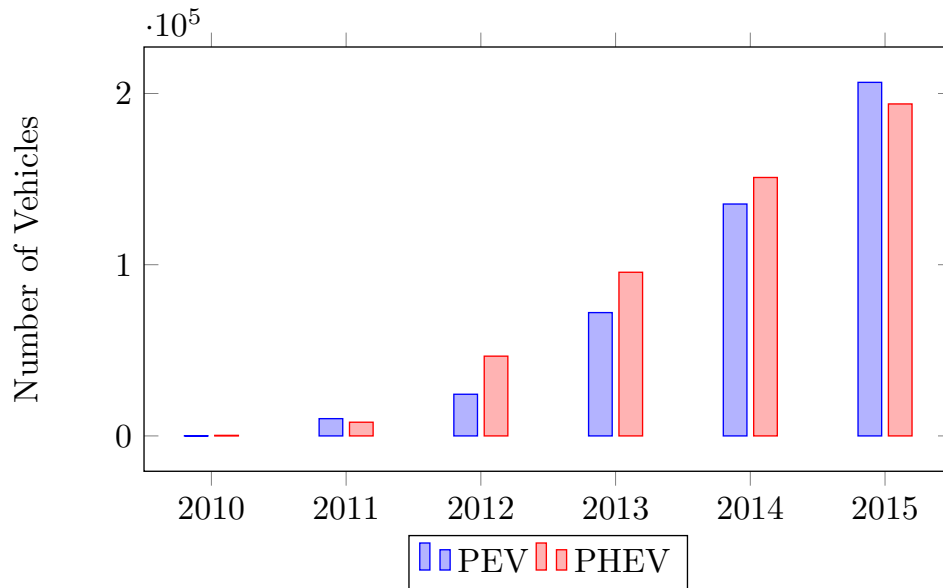


Figure 1.1.1: Total Number of PEVs and PHEVs in the United States

On the other hand, there are some favorable policies in most states that benefit PEV users[1]. The governments put a lot of effort into promoting the construction

of charging stations or charging piles[1]. Drivers of PEVs are allowed to use high occupancy vehicle lanes in most states, such as Maryland and Florida[1]. The Chinese government launched the Electric Vehicle Subsidy Scheme (EVSS) in Jan 2009 so as to stimulate sales of PEVs[2]. Over 35% of the total vehicles in the United States will be PHEV by 2020 according to Electric Power Research Institute (EPRI) in the moderate expectations, and by 2050, up to 80% of vehicles will be PHEV[3]. Table 1.1.1 shows the expectation scenarios of the PHEV market penetration from EPRI's report.

Table 1.1.1: PHEV Market Penetration Based on Scenarios in 2050

Vehicle Market in 2050		Vehicle Type		
		Conventional	Hybrid	Plug-in Hybrid
PHEV Scenarios	Low	56%	24%	20%
	Medium	14%	24%	62%
	High	5%	15%	80%

A high PEV rate brings multiple benefits to society, the environment and users. It makes the contribution of reducing carbon-dioxide emissions and decreasing fuel consumption when consumers drive PEVs instead of internal combustion engine vehicles[3, 4]. A maximum reduction of 612 million metric tons of greenhouse gas will be reached in 2050 according to EPRI's report, shown in Table 1.1.2[3]. Additionally, some policies come as an enormous boon for users, for instance, public law 112-240 offers tax credits to those who buy PEVs[5].

Table 1.1.2: Annual Green House Gas Emissions Reductions Scenarios in 2050

Annual Green House Gas Reduction in 2050 (million tons)		CO_2 Intensity		
		High	Medium	Low
PHEV Scenarios	Low	163	177	193
	Medium	394	468	478
	High	474	517	612

1.2 Motivation

PEV brings benefits as well as troubles. Nowadays the recharging schedules are assigned by third-party aggregators, such as parking lots and power retailers. These aggregators generally use a default recharging algorithm. At the time that a PEV is plugged into the grid, the vehicle charges in a maximum possible rate.

On the one hand, this default recharging algorithm introduces enormous problems to the grid in its current status. Since PEV owners who share the same community are very likely to keep the same schemes of their lifestyles, a huge amount of energy demand from PHEV batteries will rush into the grid when people get home from their workplaces. Even with a low vehicle market penetration, the PEVs in this maximum rate recharging scheme lead to an unwanted baseload demand peak or disruptions in the grid (e.g. brownouts and blackouts)[6].

On the other hand, it is not convenient for PEV owners to recharge their vehicles in default plans. Most parking lots charge customers in time instead of in energy recharged. Most PEV owners have to calculate how much time they would like to recharge their vehicles, and then they have to drive their vehicles away so as to avoid unnecessary charges.

Moreover, PEV introduces several random factors to the grid, such as the time a PEV connects to the grid and the amount of power the PEV needs. These stochastic factors may affect the reliability of the grid and may cause troubles in the power dispatch or the power flow calculation.

Not only devices such as charging piles, but also recharging scheduling methods are needed to reduce the impacts on the power system and society. Today, there is still no standard way of recharging control. Hence, the utilities and PHEV owners have an urgent need to develop a reliable charging strategy deciding "when" and "how much" to recharge.

1.3 Literature Review

In this section, the existing research on PEV fleet recharging problem is introduced and classified. Moreover, the stochastic factors considered in these studies are further discussed.

1.3.1 Classification of Existing Studies

A considerable amount of studies on the PEV fleet recharging problem have been done in the last decade. Different methodologies, the point of views, system structures and time distinctions (online or offline) are adopted in the literature[7, 8]. Therefore, it is worth mentioning various classification criteria.

Firstly the methodologies used differ from study to study. For example, Galus et al. and Ma et al. use a game theory in their study[9, 10] while Huang et al. use a scheduling algorithm[11]. However, most researchers still apply various types of optimization approaches[12–20]. Optimization aims to maximize or minimize a particular objective function by choosing a set of variables from the feasible area. For instance, in [21] the authors control charging and discharging rate to minimize the fluctuation of demand. The optimization can be solved in a programming way[22], an intelligent algorithm way[23], or an optimal control way[12]. Compared with other approaches, optimization strategies are explicit and self-consistent, which means an optimization always gives one or a group of best answers regarding the objective function. On the contrary, it is hard to prove systematically that other approaches, such as scheduling, are working well.

Secondly, objectives of algorithms also vary a lot among literature. PEV fleet recharging problem can be settled from either a user’s point of view, a third party aggregator’s point of view, or even a power utility’s point of view. Most of the user-perspective studies target the minimum economic costs of recharging[24, 25] while Yudovina et al. and Huang et al. pursue the minimum waiting time[26]. Wen et al.

consider user convenience which involves the final state of charge and total recharging time[27]. In the aggregator’s perspective, a maximum profit and some kind of load obligation need to be reached, such as [16] and [28]. The others choose to benefit the grid and society from a power utility’s point of view, for example minimizing energy loss on the distribution system, minimizing the peak load, minimizing the risk of mismatch, filling load valleys, or providing the frequency regulation service[29–33].

Some researchers are confident about a power-utility-perspective algorithm because the batteries in PEV could store power which can support the grid and improve power quality[30]. However, this kind of approaches is impractical for now, since most PEV owners charge their vehicles in the third party parking lots near their work place. The paramount factor users and the profit-making parking lots are concerned with is still economic cost in a low PEV penetration scenario. The power utilities have to pay PEV owners a lot of money to take good use of the energy stored in the PEV batteries. Moreover, bidirectional vehicle-to-grid (V2G) devices are expensive[34], which can become an obstacle of deploying recharging schedules in a power utility’s point of view.

A user’s point of view and an aggregator’s perspective also have drawbacks without any grid-support service. If a recharging algorithm only stands in the user’s perspective, an unwanted load peak will occur at the time that has the minimum electricity price[35]. The aggregator’s point of view seems to be the most practical one. However, a real-life aggregator aims to make maximum profits and sell power as much as possible. Therefore, those studies that have objectives to reach maximum profits with minimum recharging power are not realistic either[36].

Thirdly, when considering the time distinctions of literature, the existing studies can be divided into two categories: offline methods and online methods[37]. Offline algorithms are also known as day-ahead algorithms, which means the problem has to be solved before users recharge their PEVs[38]. The main advantage of an offline algorithm is that it provides a globally optimal solution within a defined horizon. This

feature will ensure that the PEV recharging scheme performs well on the condition that the objective function is proper. Moreover, time complexity is not so relevant in an offline model, since the problem is settled one day ahead. In other words, the PEV recharging problem can be modeled in a larger scale in an offline way than in an online way. While offline approaches solve the PEV fleet recharging problem one day ahead, online ones provide real-time schemes as the vehicles are recharging[8, 24, 39]. A PEV recharging scheme can be deployed by an online algorithm regarding some real-time information, such as the real-time electricity price, actual load, and actual capacity, which provide a precise recharging scheme and raise overall performance. However, these online algorithms have to deploy schedules at the time PEVs are recharging, which means an online method has trouble processing a large-scale centralized problem with many variables. Besides, the communication requirement can be a bottleneck of online approaches.

1.3.2 Stochastic factors

Aside from distinct classification criteria, stochastic factors are taken into account in the majority of literature. Most studies consider the uncertainty of user behaviors in the PEV recharging deployment problem.[40–42]. These studies manage to extract user’s driving patterns, such as time spent on driving PEV per day, and the initial state of charge (SOC) when PEV is plugged in the grid[43]. Mohamad et al., Ma et al. and Tan et al. use a set of characteristic probability density functions (PDF) of departure time and driving distance to describe a PEV fleet[42], while stochastic PEV energy demand is considered as diverse scenarios by [41]. Besides, Grahn et al. put forward transition states of driving[44]. Stochastic renewable energy generation is also considered in some research[18].

These studies utilize statistical analysis, fuzzy control, stochastic programming and model predictive control to deal with the uncertainty depending on the mathematical models[40–45].

However, researchers do not pay much attention to uncertainties in the power market. Most algorithms are developed under an assumed day-ahead and regulated market, where users and the aggregator could get exact electricity prices and baseload demand information[28]. On the contrary, real-time power markets and power auctions are pretty universal now. Electricity prices, power capacity, and even baseload demand can be stochastic. Therefore, the algorithms based on deterministic parameters will not be robust to the power market anymore, which means the solution of a deterministic algorithm is sometimes not the optimal solution. Even worse, the aggregator has no access to know how bad the recharging plan is and how the recharging plan affects economic cost or any other objective. Although consideration is given to stochastic electricity prices by Halvgaard et al. in a model predictive control approach[46], the method needs a large communication bandwidth. A heavy communication duty makes the method could not serve a wide area. Moreover, a detailed recharging schedule is not deployed to each PEV in the algorithm. Consequently, it is necessary to take these stochastic factors into account in a new way which could cover a wider area and offer a more detailed solution.

1.4 Contribution of the Thesis

In this thesis, a novel online stochastic linear programming (SLP) based on research [35] is proposed. Online elements and offline elements, a user's point of view and a utility's point of view are combined in the thesis with a consideration of stochastic factors.

There are two stages in the method. In the first stage, an offline linear programming (LP) is used to pursue minimum costs. It takes uncertain user behaviors as well as stochastic real-time marketing parameters into account. A load obligation is also satisfied with a consideration of stochastic load demand and the effect of renewable energy generation. In the second stage, a group of constraint-adjusted prices, which are related to the optimal solution in stage one, will be broadcasted to each

PEV. Dynamic model-based recharging schedules will be created by charging piles or personal smart meters.

Our work has five contributions that have not been mentioned in previous studies. Firstly, the scheme is an online and near-optimal algorithm based on offline results. Secondly, stochastic marketing parameters are considered in the method. Thirdly, the rolling initial and last SOC are applied to avoid end effects in the finite horizon problem. Fourthly, a load obligation is met so that the algorithm could reduce impacts on the grid. Finally, the method only needs a low bandwidth of communication and can be used in a wide area, because the aggregator only communicates to each PEV once.

The thesis is divided into three main parts. Chapter 2 proposes the structure of the model. Chapter 3 introduces data modeling and features of stochastic parameters. Chapter 4 discusses the static SLP formulation and the dynamic recharging scheduling algorithm. Results and conclusion are in Chapter 5.

Chapter 2 - Structure and Procedures

2.1 Introduction

In this chapter, the composition of the model and the procedures of the method will be introduced. The structure of the system is discussed in Section 2.2 and the procedures of the method are introduced in Section 2.3.

2.2 Structure of Recharging System

In a PEV recharging problem, there will be a conflict of interest between the power utility and PEV owners. A PEV owner places paramount importance on recharging costs while the power utility concerns about the reliability of the grid and electricity storage function of PEV batteries. Thus, there must be a third party standing between PEV owners and the power utility. An aggregator company could play a major role in benefiting both sides.

Figure 2.2.1 displays the basic structure of the PEV recharging system in the future. The aggregator stands between the power utilities and users. Not only PEV owners can be users in this structure. Some parking lots or whoever owns charging piles can also be users of the aggregator. The arrows in the diagram represent directions of communication. Only a relatively small communication bandwidth is needed among the aggregator, the power utility, and users.

In the model, the aggregator needs both day-ahead information and real-time information. Day-ahead information includes historical electricity prices, historical baseload, historical user behaviors and prediction of renewable energy generation in the next day. Except for day-ahead prediction of renewable energy generation, all data is used to create mathematical models that profile the power market. The aggregator generates a PDF of electricity prices, scenarios of baseload demand and typical driving profiles. The market models only need to be updated every month or

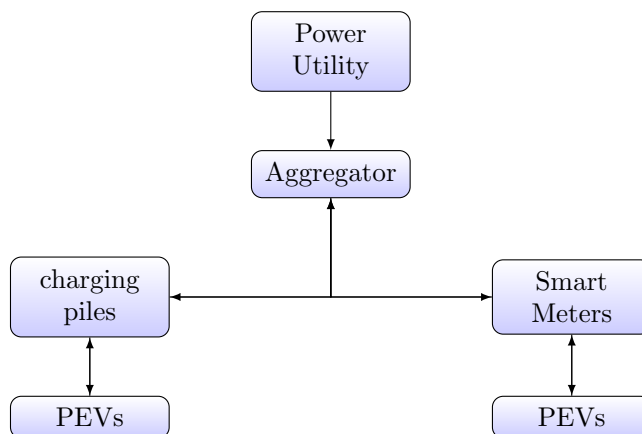


Figure 2.2.1: Physical Structure of the PEV Recharging System

week. The mathematical models of the data are constructed in Chapter 3.

Unlike day-ahead information, real-time information interchange is bidirectional. The aggregator broadcasts a group of constraint-adjusted prices[35] to all charging piles. Each charging pile assigns a recharging schedule to PEV separately. Then the recharging schedules are sent back to the aggregator.

2.3 Procedure of the Method

The procedures of the method are shown in Figure 2.3.1. There are two stages in the method. One is the day-ahead stage, which is shown in light blue boxes in the figure, and the other is the real-time stage, which is shown in dark blue boxes in the figure. In the day-ahead stage, the aggregator collects diverse kinds of data to create mathematical models. Then a stochastic linear programming is solved.

In the real-time stage, constraint-adjusted prices are updated according to the total recharging energy flow. Then they are broadcasted to the parking lots and users. A charging pile determines the recharging schedule at the time the PEV is plugged into the grid. After that, charging piles send the schedules to the aggregator. Once the recharging schedule of a PEV has been decided, the PEV would not communicate

with the aggregator anymore.

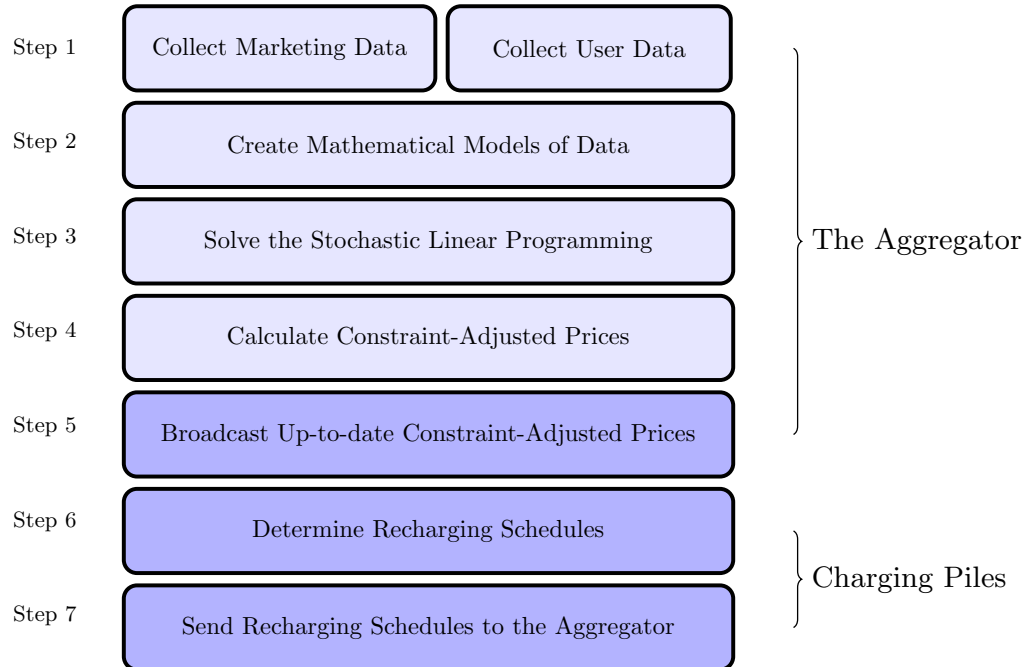


Figure 2.3.1: Procedure of the Method

The brackets on the right side indicate that the procedures are processed by either the aggregator or charging piles. The aggregator processes the majority of the method in the day-ahead part. Besides, the exact recharging schedules are assigned by users themselves. Thus, the communication requirement is not strict, though there are some real-time elements in the method.

Chapter 3 - Historical Data Modeling

3.1 Introduction

In this chapter, the data used in the thesis including vehicle transport data and marketing data is introduced. Typical features are extracted from historical data to characterize the uncertainty in the recharging problem. These features play vital roles in the constraints and the objective function in static stochastic linear programming. Two assumptions are made in Section 3.2. Vehicle transport data is discussed in Section 3.3. Marketing data is discussed in Section 3.4.

3.2 Assumptions

To model marketing data and vehicle transport data as parameters which can be used in the SLP and to simplify the problem, two assumptions are made in the thesis.

The first assumption is that all data is decoupled from each other or any other factors. Under the real-life circumstance, PEV transport demand may affect marketing. For instance, the electricity price will stand in a high position if baseload demand is heavy and power capacity is small. When an aggregator deploys recharging plans for PEVs, the power scarcity is increasing, which means the electricity price is rising depending on PEV recharging. Even worse, the electricity price is not a linear function of power demand and capacity, because pricing is auction-based. In that case, it is too complicated either to dig out the relationship among data or to solve the recharging problem. Other factors may be coupled with vehicle transport data and marketing data too. For example, marketing data and PEV transport data can be influenced by weather. During a snowstorm, there is no PEV energy demand generally while baseload demand is high. Then the electricity price is probably high. Weather is coupled with marketing data and vehicle transport data at the same time.

Therefore, it is easier to build mathematical models under the decoupling assumption.

The second assumption is that the electricity price and stochastic part of renewable energy generation are independent of their status in the past. For example, when the electricity price is high, it is very likely that the electricity price will stay high in next several hours. It is possible to model the property of marketing data in the Markov chain form. However, it limits the methodologies that can be used in the study as well as the scale of the problem. In the thesis based on the SLP, the property is ignored and marketing data at different time points is independent of each other.

3.3 Vehicle Transport Data

Considering a workplace parking lot equipped with charging piles, diverse types of vehicles will park in the garage during consumer's working hours. These vehicles have various battery capacities, different energy consumption rates. The worst thing is that these vehicles come from different places so they have entirely different arrival time and battery energy demands. Thus, it is important to profile and classify the vehicle transport data.

3.3.1 Driving Profiles

An enormous number of driving profiles are taken from the National Household Travel Survey (NHTS) in the thesis. This survey that held in 2009 updates data of 2001 NHTS. There are several surveys in prior, including National Personal Transportation Surveys (NPTS) conducted in 1969, 1977, 1983, 1990 and 1995. Data is collected from over 150,000 households with diverse features of vehicles and trips. Personal information about family members and their real driving data, such as trip lengths, driving purposes and even driving areas, are recorded in 2009 NHTS. In the thesis, we only use trip lengths, arrival time and departure time to construct driving profiles.

Besides, data of various types of vehicles is collected in 2009 NHTS, including

internal combustion engine vehicles, PHEVs, and PEVs. However, a PEV has a weak capability of driving comparing to a combustion engine vehicle because of the low battery capacity and the relatively high energy consumption rate. Additionally, PEV owners buy their vehicles only for daily commute rather than long distance traveling. Travelling ranges of several PEV models are listed in Table 3.3.1. Most PEVs only have the ability to run under 80 miles. Only vehicles that travel under 75 miles in a day in 2009 NHTS are considered as PEVs in our study.

Table 3.3.1: Ranges of Different PEV Models

PEV Model	Range (miles)
BMW i3	81
Ford Focus Electric	76
Mitsubishi i-MiEV	62
Nissan LEAF	107

Each driving profile consists of driving distances for every 15 minutes in 24 hours, which means there are 96 time intervals. The profile of an individual vehicle j is shown in a vector format:

$$t_j = [t_{j,1}, t_{j,2}, \dots, t_{j,96}] \in \mathbb{R}^{96} \quad (3.1)$$

for $j = 1, \dots, m$, where m represents the number of PEVs considered in the problem.

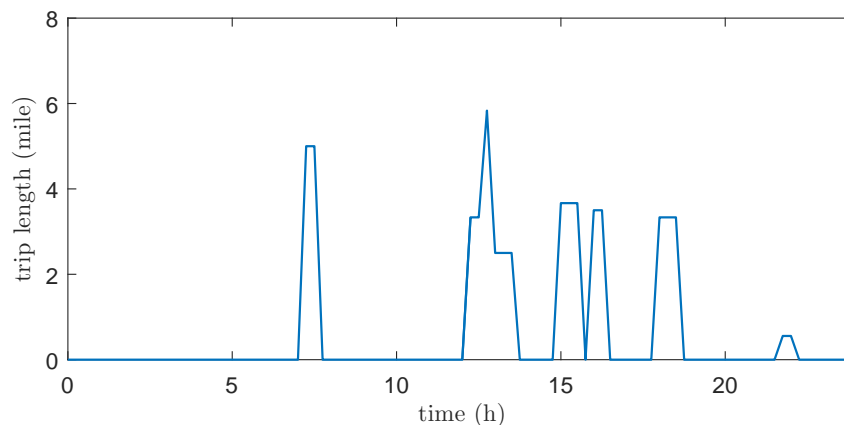


Figure 3.3.1: An Example of Driving Profile

For example, Figure 3.3.1 shows the driving profile of a PEV. The consumer drives 59 miles in total, mainly in the early morning and in the afternoon.

Aside from driving profiles, there are subtle differences in battery capacities and energy consumption rates among PEVs. These variations are not taken into account since they will not effect recharging energy demand of a PEV a lot.

3.3.2 User Behavior Clustering

If an aggregator could not foresee each vehicle's future transport load and schedule, the solution of LP is not valid anymore because of the bad model construction. However, it is impossible for the aggregator to get precise data on total energy demand and recharging time schedules of PEVs directly. Therefore, it is significant for the aggregator to create a mathematical model which shows PEV short-term future performance.

Fortunately, the driving profiles are not distributed arbitrarily. Similar PEV owner's information, similar driving periods and same region are likely to shape the driving patterns similarly. For example, if two PEV owners have the same community and the same workplace, their PEVs will perform the same driving profiles. Most costumers commute by their PEVs, while some users, such as Uber drivers, travel during the whole daytime.

The k-means clustering method is used to evaluate the expected future energy demand of PEVs. Given a time horizon $n = 96$ (the number of 15-minute intervals in 24 hours), we could cluster some driving patterns from all driving profiles. A classification of driving profiles and a bunch of centroids concerning Euclidean distance are then obtained. Finally, each driving profile will be set to the centroid of its cluster in the SLP. The physical meaning of a centroid is the typical driving profile which could represent all other profiles in its cluster. It is worth noticing that the proportion of each PEV cluster is also estimated by the number of point in the cluster and the total number of driving profiles.

These typical driving profiles are only used in the SLP, and the precise driving profiles are considered in the dynamic scheduling part. Thus, the diversity of driving

profiles is taken into account in the thesis.

In k-means clustering method, k random vectors will be set in the feasible area initially as the centroids of k clusters. Each profile will examine and allocate to the centroid to which it is closest. Then k centroids are recalculated according to points that are allocated to the clusters. After several iterations, a set of converged centroids will be obtained.

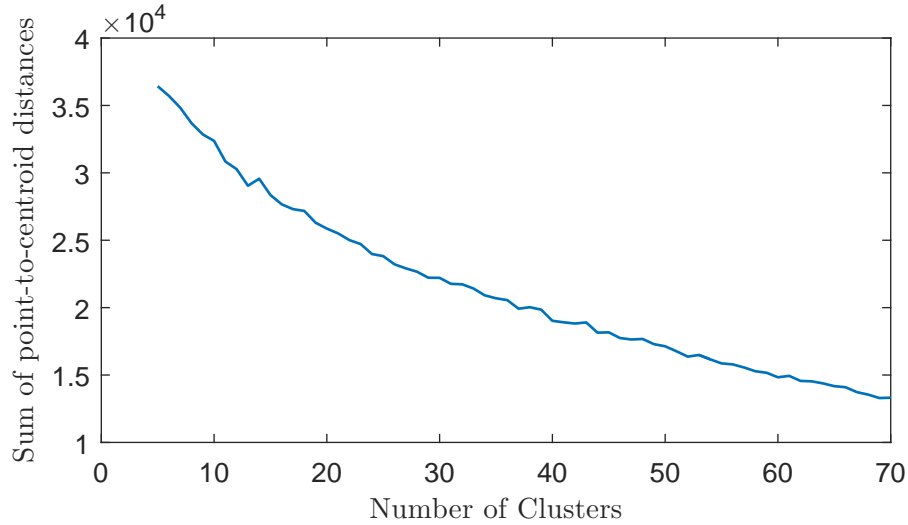


Figure 3.3.2: Sum of Points-to-Centroid Distances for Given Numbers of Clusters

There are two subtle problems of k-means clustering method that need to be mentioned. The one is that the number of clusters k must be determined in advance. A range of k has been tested in our study. Figure 3.3.2 shows that the trend of summation of point-to-centroid distances with increasing the number of clusters in the 500-profile clustering. k is chosen to be 40 in consideration of both the performance of clustering and the scale of the linear programming in the study.

Another problem is that the results of k-means clustering are not unique due to the random initial centroid selection. Some of the centroids even cannot be converged. Thus, it is significant to identify the group of the centroids that have the best performance in terms of point-to-centroid distances. 500 driving profiles are clustered for 500 times to ensure that the group of centroids is the near-optimal one.

For example, It is displayed in Figure 3.3.3 that three of the forty centroids in

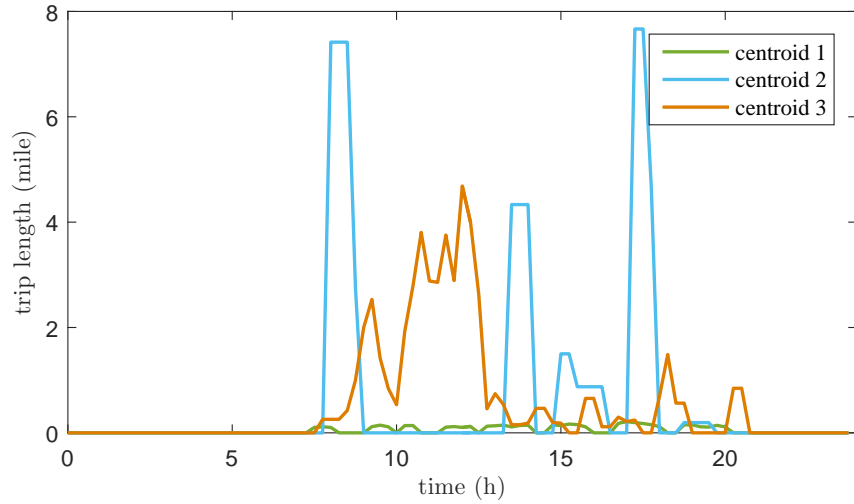


Figure 3.3.3: Three Centroids in the Best Result of Clustering

the best results of clustering. The driving profiles which are in the corresponding clusters are set to the centroids approximately.

3.4 Marketing Data

Marketing data, like historical electricity prices, is used as critical parameters in most existing studies, because it is indispensable for calculating economic costs or other power system indices. However, the marketing data is always assumed to be pre-known and deterministic in the existing studies.

In a real-time power market, marketing data tends to be uncertain. A real-time power marketing is a marketing plan that is depending on up to date events. Rather than setting a marketing plan in advance and executing it based on a fixed schedule, real-time power marketing is building a dynamic strategy centering on current trends and feedback from customers. The data in a real-time market is all dynamic and auction-based. Therefore, marketing data is hard to forecast.

Generally, there is no access for an aggregator to know the precise short-term future marketing data. In other words, the aggregator will not find out when the electricity price reach its minimum or when baseload demand has a peak in a real-

time power market.

Historical data used in the thesis is the real data from Electric Reliability Council of Texas (ERCOT). ERCOT manages power flow for 24 million Texas customers, which are 90 percent of the electric load in Texas. As a nonprofit organization governed by a board of directors, ERCOT consists of consumers, cooperatives, generators, electric utilities, power marketers, and retail electric providers. Diverse types of marketing data, including electricity prices and baseload demand, are posted to the Internet by ERCOT.

In this section, electricity prices, baseload, and renewable energy generation will be modeled as parameters in the SLP. Electricity prices are discussed in 3.4.1 and baseload demand is discussed in 3.4.2. Renewable power generation is briefly introduced in 3.4.3

3.4.1 Real-Time Electricity Price

ERCOT provides both local marginal prices (LMPs) and settlement point prices (SPPs) in the real-time market in Texas. We take historical SPPs in Houston area to create the electricity pricing profiles.

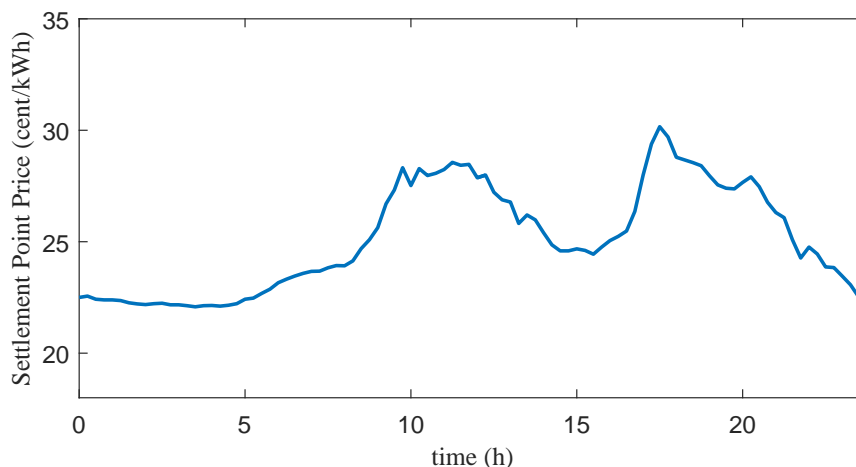


Figure 3.4.1: Settlement Point Prices of Houston in January 1, 2015

Each pricing profile consists of electricity prices for every 15 minutes in 24 hours.

The profile of electricity prices in a certain day r is shown in a vector format:

$$p_r = [p_{r,1}, p_{r,2}, \dots, p_{r,96}] \in \mathbb{R}^{96} \quad (3.2)$$

for $r = 1, \dots, 365$. Figure 3.4.1 exhibits the pricing profile in January 1, 2015.

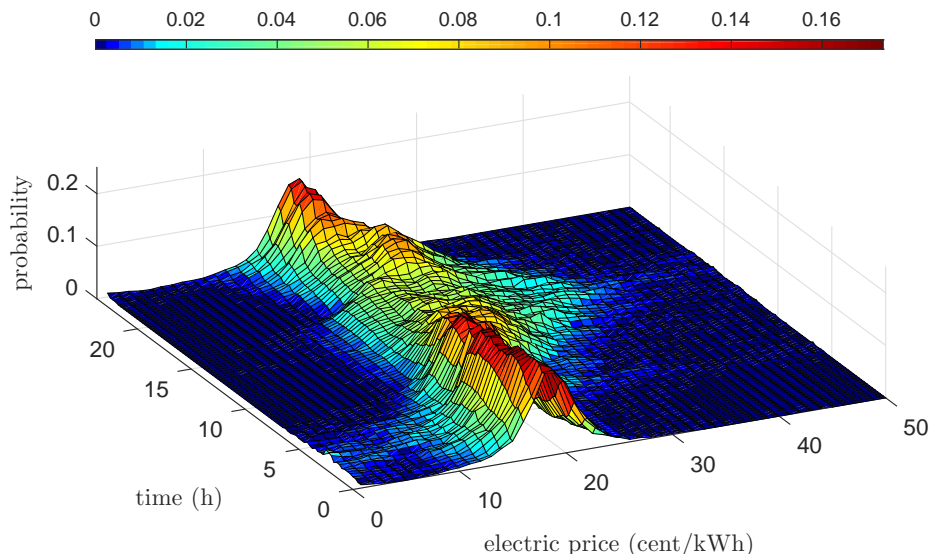


Figure 3.4.2: Probability Density Function of Electricity Prices during 24 Hours

We take pricing profiles in the year 2015 as samples to estimate the PDF of electricity prices in the real-time power market. The PDF is discrete in time, which has a 15-minute minimum unit, as well as in price, which has a 1 *cent/kWh* minimum unit. Extreme cases, in which the electricity prices are over 100 *cent/kWh* or under 0 *cent/kWh*, are ignored. It causes a drop in performance when considering the worst case whose probability is very low.

The main part of the time-variant PDF is displayed in Figure 3.4.2. Probabilities are shown in the figure corresponding to different time intervals in a day and different electricity prices. It is very unlikely for electricity price to raise up over 50 *cent/kWh*, though the possibility of the electricity prices from 50 *cent/kWh* to 100 *cent/kWh* are considered in the thesis. From 21:00 to 10:00 in the next day, which are off peak hours, the PDF tends to be spiky. In other words, electricity prices are stable at

off-peak hours. There is a flatter PDF in the peak hours, which means the pricing is harder to predict at that point.

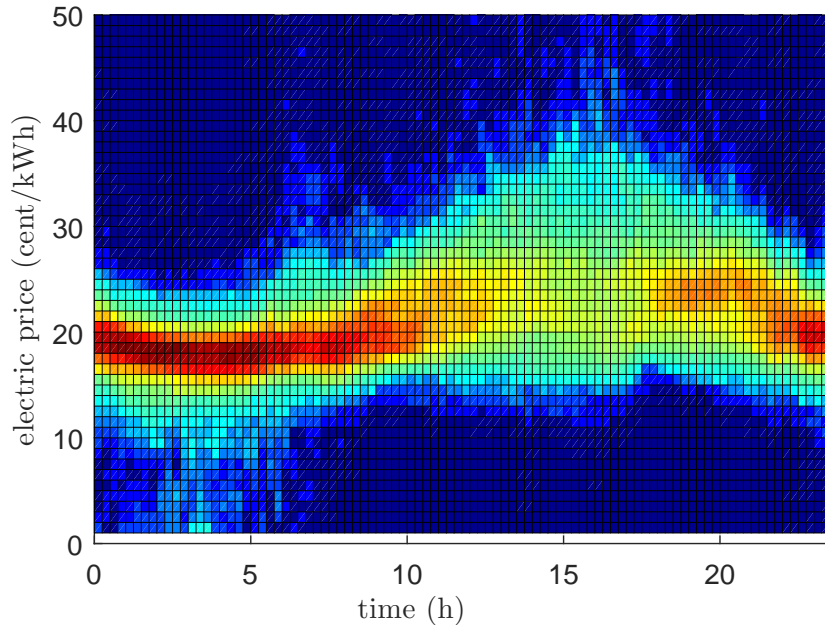


Figure 3.4.3: Heat Map of Electricity Prices PDF during 24 Hours

Figure 3.4.3 shows the PDF as a heat map from a different angle. It offers a clearer vision of the time-variant pricing PDF during 24 hours. The real-time electricity prices in peak hours tend to not only be more uncertain but also be higher than the prices in off-peak hours. Electricity prices reach a minimum at below 20 *cent/kWh* in the early morning, around 4 am.

3.4.2 Baseload Demand

The aggregator would construct a mathematical model of baseload to meet load obligation. In the thesis, baseload demand data is also obtained from ERCOT's website. We take load profiles in the Coast area, where Houston is located, from April 2015 to September 2015. The profiles of weekends and holidays are filtered out. Therefore, all weekday load profiles in 2015 summer in the Texas Coast area are collected.

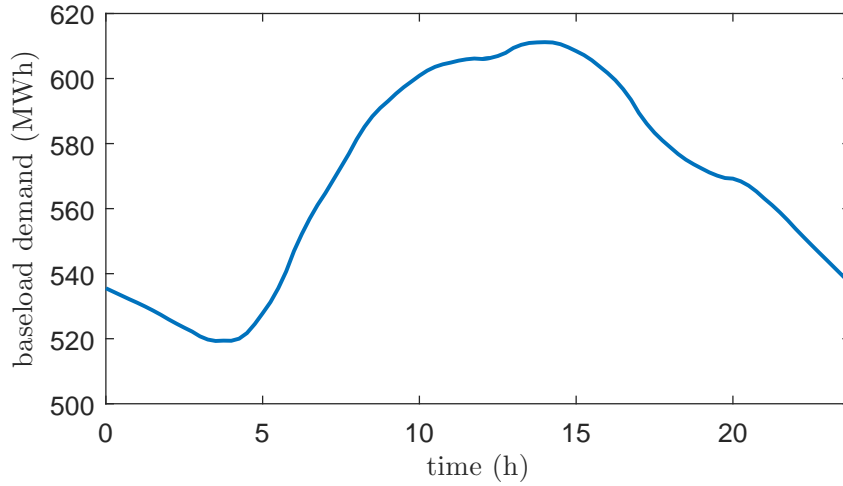


Figure 3.4.4: Baseload Profile of April 1, 2015

A general vector-form of a baseload profile is also represented here:

$$L_r = [L_{r,1}, L_{r,2}, \dots, L_{r,96}] \in \mathbb{R}^{96} \quad (3.3)$$

for $r = 1, \dots, 128$, where r represents a weekday number and 128 is the number of weekdays in summer 2015. Figure 3.4.4 shows the baseload profile of April 1, 2015. It is worth noting that the baseload demand curve is in a 15-minute interval, which means the peak load in the day is more than 2000 *MWh*.

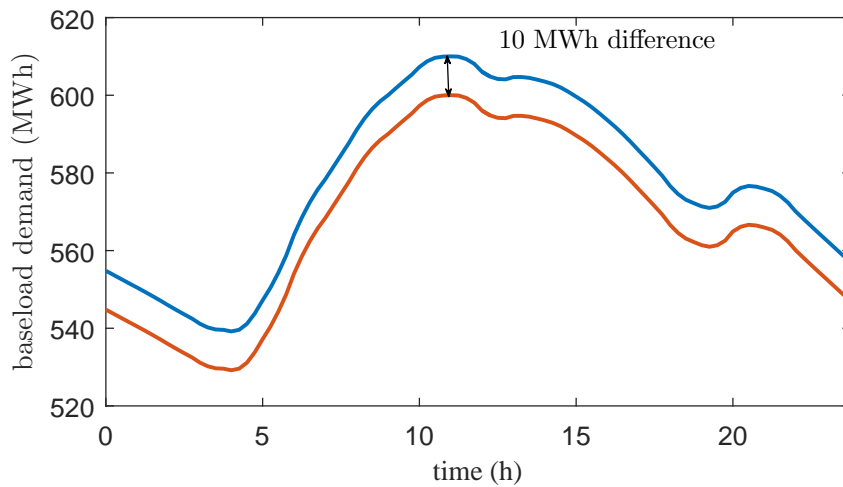


Figure 3.4.5: Two Baseload Profiles with Different Heights and the Same Shape

Because the aggregator could not reshape the total load curve arbitrarily, the exact height of baseload demand curve is not relevant at all. For example, considering two curves in Figure 3.4.5, the two profiles have 10 *MWh* differences in all time intervals while they have the same shape. V2G service is not considered in the thesis, so the only thing an aggregator can do is providing valley-fill service to the grid. Consequently, a particular valley-fill recharging schedule is valid for two different baseload profiles, on condition that they have the same valley shape.

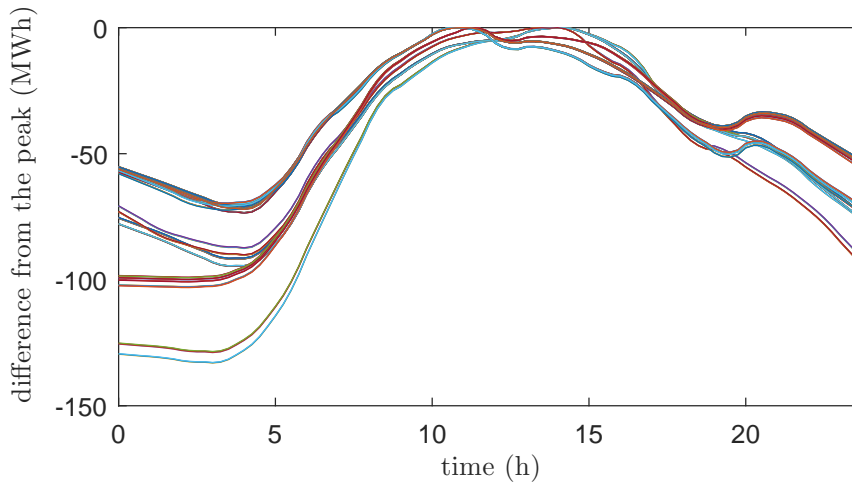


Figure 3.4.6: Shifted Baseload Profiles

Thus before extracting the features from all baseload profiles and constructing a reliable model, a preconditioning has to be taken. The baseload demand profiles are shifted to the same peak level, as it is shown in Figure 3.4.6.

Apparently, these baseload demand profiles are not distributed evenly in range. There are several bands in the morning and evening. Additionally, demand peak occurs from 10:15 to 14:45. Hence, scenarios of baseload demand are generated based on the baseload demand trends.

Four examples of baseload scenarios are listed in the Figure 3.4.7. The curves follow the possible directions in the morning and the evening while they are at peak at noon. It is worth noticing that we generate sixteen possible scenarios in the thesis.

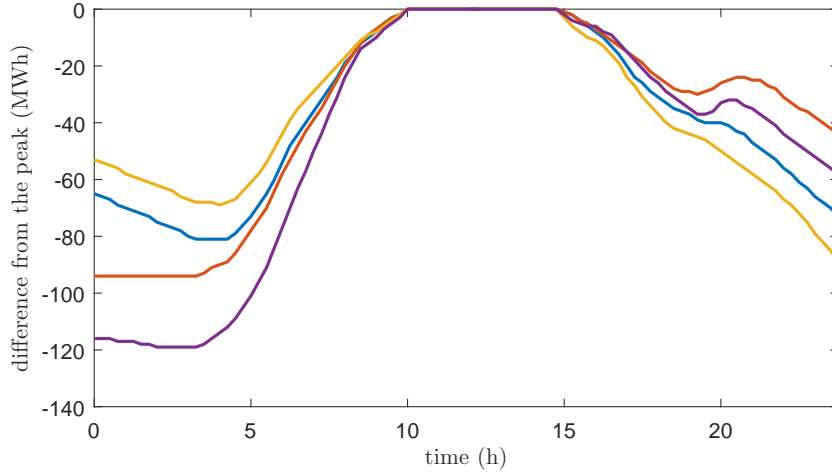


Figure 3.4.7: Scenarios of Baseload

3.4.3 Renewable Energy Generation

Renewable energy generation capacity is growing in the decade. Also, it brings uncertainty into the system. However, the features of renewable power generation are totally different from the uncertainty in the baseload demand or the electricity prices. Renewable power generation, such as the wind and solar power, is mainly determined by weather, which can be predicted. Thus, we assume that the renewable energy production consists of a deterministic part and a stochastic part. In the thesis, a day-ahead prediction of renewable energy generation is chosen as the deterministic part. To fit the baseload data, its power is scaled down to about 200MW.

The vector form of renewable energy generation prediction is represented here:

$$\bar{R} = [\bar{R}_1, \bar{R}_2, \dots, \bar{R}_{96}] \in \mathbb{R}^{96} \quad (3.4)$$

We also assume that the stochastic part of renewable generation is a white noise. The vector form of the white noise on the renewable energy generation is that:

$$\tilde{R} = [\tilde{R}_1, \tilde{R}_2, \dots, \tilde{R}_{96}] \in \mathbb{R}^{96} \quad (3.5)$$

In order to apply the stochastic part directly in next chapter, 81 scenarios have been constructed from the PDF of the white noise.

Chapter 4 - Mathematical Model Formulation

4.1 Introduction

In the chapter, the detailed mathematical model formulation of the two-stage method is discussed. The first stage of the method, the SLP, is introduced in Section 4.2. The second stage, dynamic algorithm, is introduced in Section 4.3.

4.2 Linear Programming Formulation

Linear programming is the core of day-ahead part in the method. Stochastic parameters modeled in Chapter 3 are used in the LP so as to get an overall optimal solution, which is robust to the fluctuance in the real-time power market.

In the Subsection 4.2.1, a CLP that mentioned in research [35] is briefly introduced. Electricity prices and baseload are assumed to be deterministic in the CLP. In the Subsection 4.2.2, the stochastic parameters, including stochastic electricity prices, stochastic baseload demand and stochastic renewable energy generation, are considered. The CLP is developed to be the SLP. Constraint-adjusted price is defined and obtained in the Subsection 4.2.3.

4.2.1 Clustered Linear Programming Formulation

In this subsection, the primal LP and the dual LP of PEV recharging problem are formulated.

Assume that $k = 40$ is the number of clusters and $n = 96$ is the time horizon in a 15-minute unit. For each cluster ℓ , $t_\ell \in \mathbb{R}^{96}$ represents the driving profile of the centroid of the cluster.

Table 4.2.1 lists the decision variables of the CLP, and Table 4.2.2 contains the parameters of the CLP, which are gathered from either PEV owners or the utility.

Table 4.2.1: List of Variables in LP

Variable	Description	Dimension
s_ℓ	state of charge	n
c_ℓ	charging schedule	n
c_{cap}	charge cap	1

A static clustered linear programming is formed with the parameters and variables listed above. The solution to the linear programming will provide the recharging schedules $c_\ell \in \mathbb{R}^n$, that minimize the total economic costs while meeting the load obligation. The solution also includes the optimal charge cap, which is the peak value of the total load in the horizon. A low charge cap ensures that there would not be a new load demand.

Table 4.2.2: List of Parameters in LP

Parameter	Description	Dimension
\bar{s}_ℓ	storage capacity	n
\bar{c}_ℓ	maximum charging rate	n
t_ℓ	cluster typical driving profile	n
L	baseload demand	n
p	electricity price	n
ρ	weight coefficient of charging cap	1
α	consumption rate ($kWh/mile$)	1

In the thesis, we use a Matlab-like vector notation, where $x_{a:b}$ means the a th to the b th elements of vector x . The formulation of CLP is shown in (CLP).

The first five constraints in (CLP) show the dynamics of PEV battery energy flow, and the last constraint ensures that the PEV fleet recharging meets the charge cap. The objective function consists of the total recharging cost of PEV and a penalty regarding charge cap.

The first constraint updates the changing of SOC based on charging schedules and driving profiles. The SOC of a vehicle is related to the SOC in the last time interval, recharging schedule and driving profile. If a vehicle is traveling, the SOC will go down. On the contrary, if the vehicle is recharging, the SOC will go up.

$$\begin{aligned}
& \min_{c_\ell, s_\ell, \forall \ell, c_{cap}} \sum_{\ell=1}^k p^T \cdot c_\ell + \rho \cdot c_{cap} & (\text{CLP}) \\
& \text{s.t.} \quad s_{\ell,2:n} = s_{\ell,1:n-1} + c_{\ell,2:n} - \alpha \cdot t_{\ell,2:n} & : \mu_{\ell,2:n} \\
& \quad s_{\ell,1} = s_{\ell,n} + c_{\ell,1} - \alpha \cdot t_{\ell,1} & : \mu_{\ell,1} \\
& \quad \text{if } t_{\ell,h} > 0 \text{ then } c_{\ell,h} = 0 & : \eta_\ell \\
& \quad 0 < s_\ell < \bar{s}_\ell & : \nu_\ell^s, \gamma_\ell^s \\
& \quad 0 < c_\ell < \bar{c}_\ell & : \nu_\ell^c, \gamma_\ell^c \\
& \quad \sum_{\ell=1}^k c_\ell + L \leq c_{cap} & : \theta
\end{aligned}$$

The second constraint can be seen as a special case of the first one so as to eliminate end effects in the linear programming. End effect is a kind of problem which occurs in a finite horizon optimization. The optimal solution of last several time intervals would deflect to other unwanted directions due to end effects. In PEV recharging problem, end effects perform as that all PEVs tend to use up all their power in the batteries in the last several hours. A solution with empty batteries at midnight is not realistic for PEV recharging problem, and it leads the LP in next day to be unfeasible. To handle the end effects, we assume that all PEV owners drive their vehicles periodically since they drive through the same path to the workplace in weekdays. Thus, the initial SOC of a certain PEV is likely to keep the same every day on condition that the PEV has an certain everyday driving profile. Generally, the initial SOC should be the same as the last SOC, which is the initial SOC in the next day so that we have two equation below, where $s_{\ell,0}$ is the initial SOC:

$$s_{\ell,0} = s_{\ell,n} \quad (4.1)$$

$$s_{\ell,1} = s_{\ell,0} + c_{\ell,1} - \alpha \cdot t_{\ell,1} \quad (4.2)$$

Substitute (4.1) into (4.2) and then the second constraint is derived.

The if-then constraint limits the recharging occasion. In other words, a vehicle cannot charge and travel at the same time. It is a linear constraint, although it is written in a more comprehensive but nonlinear form. Finally, the last constraint shows how the recharging schedules meet load obligation. The maximum of the total load, c_{cap} , is minimized in the objective function.

In order to explain the method more clearly, the dual problem of the LP is shown in (Dual CLP), where $I_d^\ell \in \mathbb{R}^{96}$ is derived from the if-then constraint. It is the identity matrix whose columns correspond to the hours that the typical driving profile of cluster ℓ is not zero. The dual problem has the equivalent optimal solution to the primal problem, as a result that we can solve either the dual problem or the primal problem to get the solution.

$$\begin{aligned}
& \max_{\substack{\mu_\ell, \eta_\ell, \nu_\ell^c, \gamma_\ell^c \\ \nu_\ell^s, \gamma_\ell^s \forall \ell, \theta}} - \sum_{\ell=1}^k \alpha \cdot t_\ell^T \mu_\ell + \bar{s}_\ell \cdot \gamma_\ell^s + \bar{c}_\ell \cdot \gamma_\ell^c - \theta \cdot L & \text{(Dual CLP)} \\
& \text{s.t. } \mu_{\ell,1:n} - \begin{bmatrix} \mu_{\ell,2:n} \\ \mu_{\ell,1} \end{bmatrix} + \gamma_\ell^s - \nu_\ell^s = 0 & : s_\ell \\
& p - \mu_\ell + \theta + I_d^\ell \cdot \eta_\ell + \gamma_\ell^c - \nu_\ell^c = 0 & : c_\ell \\
& \rho - \theta^T \cdot \mathbf{1} = 0 & : c_{cap} \\
& \theta, \nu_\ell, \gamma_\ell \leq 0
\end{aligned}$$

4.2.2 Stochastic Linear Programming Formulation

An SLP with only stochastic electricity prices is discussed first detailly. It is remarkable to notice that the electricity prices only appear in the dual constraint of c_ℓ as parameters instead of coefficients. Therefore we can move p to the right-hand side and rewrite the constraint in (4.3), where \tilde{p} represents the stochastic electricity

prices vector:

$$-\mu_\ell + \theta + I_d^\ell \cdot \eta + \gamma_\ell^c - \nu_\ell^c = -\tilde{p} \quad (4.3)$$

Then the dual problem is the SLP where only the right-hand side values are random variables. The general form of this kind of problem is listed in (4.4), where ξ represents the right-hand side random vector.

$$\begin{aligned} \min_x \quad & v^T x \\ \text{s.t.} \quad & Ax = b \\ & Tx = \xi \\ & x \geq 0 \end{aligned} \quad (4.4)$$

The SLP in (4.4) cannot be solved directly with the constraint $Tx = \xi$, since ξ is not a deterministic value. Whatever the variables are, there is always at least a certain set of possible values of ξ that would break the equality of the constraint.

However the stochastic electricity prices, \tilde{p} , are described as a discrete finite PDF, so that the problem can be solved through a series of simple linearization techniques mentioned in research [47, 48].

Prekopa et al. transform the constraint with random variable into a penalty function in the objective, and give a near-equivalent form of the SLP, which is listed in (4.5). In the representation, n means the dimension of the right-hand side stochastic vector ξ . Additionally q^+ and q^- are weight factors of the penalty on condition of $q^+ + q^- \geq 0$.

$$\begin{aligned} \min_x \quad & \{v^T x + \sum_{i=1}^n q_i^+ E[\xi_i - T_i x]^+ + \sum_{i=1}^n q_i^- E[T_i x - \xi_i]^+\} \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned} \quad (4.5)$$

The problem can be linearized further. It transforms into the λ -representation problem in (4.6). For a finite discrete stochastic parameter ξ_i , $z_{i,j}$ is the j th smallest possible value among all v_i possible values. It is easy to derive that $z_{i1} < \dots < z_{i,v}$.

$$\begin{aligned}
\min_{x,\lambda} \quad & \{v^T x + \sum_{i=1}^n \sum_{j=1}^{v_i} w_{i,j} \lambda_{i,j}\} & (4.6) \\
\text{s.t.} \quad & Ax = b \\
& T_i x - \sum_{j=1}^{v_i} z_{i,j} \lambda_{i,j} = 0 \\
& \sum_{j=1}^{v_i} \lambda_{i,j} = 1 \\
& x \geq 0, \quad \lambda_{i,j} \geq 0 \\
& j = 1, \dots, v_i, \quad i = 1, \dots, n
\end{aligned}$$

In the programming, $w_{i,j}$ is a penalty element defined in (4.7). Besides $\lambda_{i,j}$ is a variable that needs to be decided in the optimization.

$$w_{i,j} = q_i^+ E [\xi_i - z_{i,j}]^+ + q_i^- E [z_{i,j} - \xi_i]^+ \quad (4.7)$$

It is worth noticing that a quadratic form of the electricity price penalty factors w^p is used in the PEV recharging problem in this thesis, which is shown in (4.8), where q_i^p is the weight factor of the stochastic electricity price penalty.

$$w_{i,j}^p = q_i^p E [(\tilde{p}_i - z_{i,j}^p)^2]^+ \quad (4.8)$$

Consequently, it is easy to develop the λ -representation of the dual problem in consideration of stochastic electricity prices in (Dual SLP'), where p^{eq} is the equivalent electricity prices and $z_{i,j}^p$ is the j th smallest possible value of \tilde{p}_i .

$$\begin{aligned}
& \max_{\substack{\mu_\ell, \eta_\ell, \nu_\ell^s, \gamma_\ell^c \\ \nu_\ell^s, \gamma_\ell^s \forall \ell, \theta, \lambda^p, p^{eq}}} & - \sum_{\ell=1}^k \alpha \cdot t_\ell^T \mu_\ell + \bar{s}_\ell \cdot \gamma_\ell^s + \bar{c}_\ell \cdot \gamma_\ell^c - \theta \cdot L & & \text{(Dual SLP')} \\
& & - \sum_{i=1}^n \sum_{j=1}^{v_i^p} w_{i,j}^p \cdot \lambda_{i,j}^p & & \\
\text{s.t.} & \mu_{\ell,1:n} - \begin{bmatrix} \mu_{\ell,2:n} \\ \mu_{\ell,1} \end{bmatrix} + \gamma_\ell^s - \nu_\ell^s = 0 & & : s_\ell \\
& p^{eq} - \mu_\ell + \theta + I_d^\ell \cdot \eta_\ell + \gamma_\ell^c - \nu_\ell^c = 0 & & : c_\ell \\
& p_i^{eq} = \sum_{j=1}^{v_i^p} z_{i,j}^p \cdot \lambda_{i,j}^p & & : \kappa_i \\
& \sum_{j=1}^{v_i^p} \lambda_{i,j}^p = 1 & & : \tau_i \\
& \rho - \theta^T \cdot \mathbf{1} = 0 & & : c_{cap} \\
& \theta, \nu_\ell, \gamma_\ell \leq 0 \\
& \lambda^p \geq 0 \\
& i = 1, \dots, n, \quad j = 1, \dots, v_i^p
\end{aligned}$$

The typical value of $\lambda_{i,j}^p$ is 0 or 1. Essentially, the λ -representation of the SLP is a mixed 0-1 linear programming, where a certain possible value $z_{i,j}^p$ is chosen as the equivalent electricity price p_i^{eq} of i th time interval in the third constraint. The corresponding penalty $w_{i,j}^p$ in i th time interval is determined at the same time. The quadratic form of the possible penalty $w_{i,j}^p$ ensures that the minimum occurs when $z_{i,j}$ is around the expectation value of \tilde{p}_i .

The duality of linear programming is applied again, and the primal problem in consideration of stochastic electricity prices lists in (SLP'):

$$\begin{aligned}
& \min_{c_\ell, s_\ell, \forall \ell, c_{cap}, \tau, \kappa} \sum_{i=1}^n \tau_i + \rho \cdot c_{cap} & (\text{SLP}') \\
& \text{s.t.} \quad s_{\ell,2:n} = s_{\ell,1:n-1} + c_{\ell,2:n} - \alpha \cdot t_{\ell,2:n} & : \mu_{\ell,2:n} \\
& \quad s_{\ell,1} = s_{\ell,n} + c_{\ell,1} - \alpha \cdot t_{\ell,1} & : \mu_{\ell,1} \\
& \quad \kappa = - \sum_{\ell=1}^k c_\ell & : p^{eq} \\
& \quad \tau_i \geq -z_{i,j}^p \cdot \kappa_i - w_{i,j}^p & : \lambda_{i,j}^p \\
& \quad \text{if } t_{\ell,h} > 0 \text{ then } c_{\ell,h} = 0 & : \eta_\ell \\
& \quad 0 < s_\ell < \bar{s}_\ell & : \nu_\ell^s, \gamma_\ell^s \\
& \quad 0 < c_\ell < \bar{c}_\ell & : \nu_\ell^c, \gamma_\ell^c \\
& \quad \sum_{\ell=1}^k c_\ell + L \leq c_{cap} & : \theta
\end{aligned}$$

The third and the fourth constraint about κ and τ can be rewritten as below:

$$\tau_i \geq \max \left\{ z_{i,j}^p \cdot \sum_{\ell=1}^k c_{\ell,i} - w_{i,j}^p \right\} \quad (4.9)$$

The constraint (4.9) shows a risk management process. In the SLP, one of the goals is to figure out the optimal equivalent electricity prices, p^{eq} . However, there is an opportunity cost (OC) of choosing the equivalent electricity price vector. Considering there is a certain scenario the real-life electricity price has a larger value than p_i^{eq} in the i th time interval, there will be an additional cost caused by the surplus part of the actual price. The additional cost is the OC of p_i^{eq} in the scenario. Thus in the objective function, the expectation of OC should be added in the objective function.

The OC is a value which means the risk that recharging cost goes beyond the budget. When the OC is larger than the threshold $w_{i,j}^p$, it would be better if the policy maker raise the budget instead of taking a chance. The essence of budget raising is

that the policy maker believes there will be a higher equivalent price p_i^{eq} than the expectation value of electricity price $E[\tilde{p}_i]$ in the i th time interval. The more energy is recharged at the same time, the higher p_i^{eq} is. Thus, the policy maker tends to recharge PEV evenly among time intervals instead of putting all eggs in one basket to keep all p_i^{eq} low. This kind of recharging strategy benefits users, the aggregator, and the power utility.

The SLP with all three kinds of stochastic marketing parameters, including the electricity prices, the baseload and the renewable energy generation is derived then.

The intention of the baseload constraint is to avoid creating a new load demand and to fill load valley, as a result that the reliability of power system increases and an optimal power dispatch can be reached easily. However in the power system involving PV arrays or wind generators, the renewable energy ought to be used as much as possible. The free renewable energy reshapes the power needed from conventional generators. Thus, it can be considered as a negative load in the constraint (4.10).

$$\sum_{\ell=1}^k c_{\ell} + \tilde{L} - \bar{R} - \tilde{R} \leq c_{cap} \quad (4.10)$$

Before transforming the SLP with stochastic baseload and renewable energy generation into λ -representation, the penalty w^R and w^L have to be decided. There is no explicit relationship between L , R and the objective function, and L , R will not affect the economic cost. Therefore, it is necessary to determine that which kind of scenario is the most unwanted one.

It is a fact that V2G service is not provided in the method and the PEVs tend to recharge in off-peak hours. Under this circumstance, the baseload with the smallest valley is the worst scenario in the SLP, since it is likely for the aggregator to create a new load demand with a small valley baseload. Similarly, the renewable energy generation scenario which has the highest generation in off-peak hours and lowest generation in peak hours is considered to be the worst one.

Define $w_{S_g}^{\tilde{R}}$ and $w_{S_f}^L$ as (4.11) and (4.12), where S_f is the scenario of baseload demand for $f = 1, \dots, 16$, and S_g is the scenario of renewable energy generation for $f = 1, \dots, 81$. It is worth noticing that we construct baseload scenarios whose peak always occurs in $i = 42, \dots, 59$ while the scenarios of white noise on renewable energy are distributed randomly. Thus different signs are used for peak and off peak hours in (4.12).

$$w_{S_f}^L = \sum_{i=1}^n q^L (L_{peak, S_f, i} - L_{S_f, i}) \quad for \quad f = 1, \dots, 16 \quad (4.11)$$

$$w_{S_g}^R = - \sum_{i=1}^{41} q^R \tilde{R}_{S_g, i} + \sum_{i=42}^{59} q^R \tilde{R}_{S_g, i} - \sum_{i=60}^{96} q^R \tilde{R}_{S_g, i} \quad for \quad g = 1, \dots, 81 \quad (4.12)$$

Then a λ -representation of SLP can be written in (SLP). Notice that the additional costs to consider a worse case of baseload is very low, which means that if the PEV penetration is not extremely high, the valley is always assumed to be the smallest among all scenarios.

$$\begin{aligned}
& \min_{\substack{c_\ell, s_\ell, \forall \ell, c_{cap}, \tau, \kappa \\ L^{eq}, \tilde{R}^{eq}, \lambda^L, \lambda^R}} \sum_{i=1}^n \tau_i + \rho \cdot c_{cap} + \sum_{f=1}^{16} w_{S_f}^L \cdot \lambda_f^L + \sum_{g=1}^{81} w_{S_g}^R \cdot \lambda_g^R & \text{(SLP)} \\
& \text{s.t.} \quad s_{\ell,2:n} = s_{\ell,1:n-1} + c_{\ell,2:n} - \alpha \cdot t_{\ell,2:n} \\
& \quad s_{\ell,1} = s_{\ell,n} + c_{\ell,1} - \alpha \cdot t_{\ell,1} \\
& \quad \tau_i \geq \max \left\{ z_{i,j}^p \cdot \sum_{\ell=1}^k c_{\ell,i} - w_{i,j}^p \right\} \\
& \quad \text{if } t_{\ell,h} > 0 \text{ then } c_{\ell,h} = 0 \\
& \quad 0 < s_\ell < \bar{s}_\ell \\
& \quad 0 < c_\ell < \bar{c}_\ell \\
& \quad L^{eq} = \sum_{f=1}^{16} L_{S_f} \cdot \lambda_f^L \\
& \quad \tilde{R}^{eq} = \sum_{g=1}^{81} \tilde{R}_{S_g} \cdot \lambda_g^R \\
& \quad \sum_{\ell=1}^k c_\ell + L^{eq} - \bar{R} - \tilde{R}^{eq} \leq c_{cap}
\end{aligned}$$

The programming shows in (SLP) is the stochastic programming used in the first stage of the method.

4.2.3 Constraint-Adjusted Price

A constraint-adjusted price is defined as the marginal price which is related to the dual constraint of variable c_ℓ . In the research [35], Taheri et al. use the constraint-adjusted prices in order to get a near-optimal online solution. Thus the constraint-adjusted prices are the key to bond the first stage and the second stage of the method together.

$$p^{eq} - \mu_\ell + \theta + I_d^\ell \cdot \eta_\ell + \gamma_\ell^c - \nu_\ell^c = 0 \quad (4.13)$$

In the constraint (4.13) , γ_ℓ^c and ν_ℓ^c are slack variables, so that if $c_\ell = 0$, then $\nu_\ell^c > 0$ and if $c_\ell = \bar{c}_\ell$, then $\gamma_\ell^c > 0$ on condition of $\nu_\ell^c \cdot \gamma_\ell^c = 0$, where \bar{c}_ℓ is the maximum charge rate of cluster ℓ . The value

$$d_\ell = p^{eq} - \mu_\ell + \theta + I_d^\ell \cdot \eta_\ell \quad (4.14)$$

can be defined as the constraint-adjusted prices over next n time intervals for cluster ℓ . The charge cap, the equivalent prices, the SOC and the vehicle's driving profiles are taken into account.

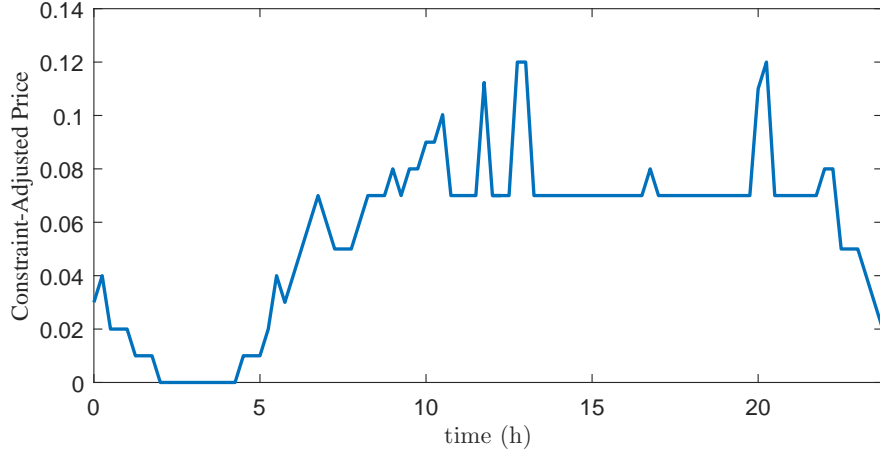


Figure 4.2.1: A Example of Normalized Constraint-Adjusted Price

A normalized constraint-adjusted price is displayed in Figure 4.2.1. The constraint-adjusted prices show the marginal energy cost when considering the stochastic electric prices and load obligation. When the vehicle is driving, or the baseload demand is high, the constraint-adjusted price tends to be high.

It is worth mentioning that the equivalent price in constraint-adjusted price, p^{eq} is related to both thresholds $w_{i,j}^p$ and the charging schedule c_ℓ . In the constraint-adjusted prices which are broadcasted to all charging piles, the up-to-date p^{eq} rather than the optimal p^{eq} in the solution is used. It means constraint-adjusted prices broadcasted to the charging piles may change according to up-to-date charging scheduling. The more the aggregator recharges PEV, the high constraint-adjusted prices are.

4.3 Dynamic Scheduling

A naive dynamic algorithm is used in the second stage. The procedures of the dynamic scheduling are listed below in Table 4.3.1.

Table 4.3.1: Constraint-Adjusted Price Scheduling Algorithm

Algorithm 1 Constraint-Adjusted Price Dynamic Scheduling

—Obtain the centroids of clusters from the aggregator
for each vehicle that plugs in
 —Decide the cluster ℓ that the vehicle belongs to
 —Obtain the constraint-adjusted price d_ℓ
 while the vehicle needs energy and it is not travelling
 —Charging the vehicle at the time which has
 the smallest constraint-adjusted price d_ℓ
 end while
 —Send recharging scheduling to the aggregator
 —The aggregator updates constraint-adjusted prices
 when there is a new equivalent price p^{eq}
end for

Firstly, the parking lot gets the results of clustering from the aggregator every season or every half year, since the driving profiles in a certain region would not change in a long time. Secondly, the aggregator broadcasts constraint-adjusted prices to the parking lots when the equivalent prices p^{eq} are changing according to the total recharging energy flow in the day. When a vehicle plugs in through a charging pile, it reports its driving profile to the charging pile and then gets a recharging schedule at once regarding the minimum constraint-adjusted price. Then the parking lot sends the recharging schedule to the aggregator.

The algorithm processes PEVs one by one. Once a charging pile decides the recharging schedule for a vehicle, the schedule would not change later and the aggregator would not communicate with the occupied charging pile anymore until the PEV leaves.

Chapter 5 - Results and Conclusion

5.1 Introduction

In the chapter, two algorithms are compared, which are the minimum cost algorithm and the default recharging algorithm, to the constraint-adjusted price dynamic scheduling algorithm. The comparison algorithms are introduced in Section 5.2. The simulation results of the three algorithms in Matlab are listed and analyzed in Section 5.3. Finally, Section 5.4 gives the conclusion of the thesis research.

5.2 Comparison Algorithms

In this section, two universal PEV recharging algorithms are introduced. One is the default recharging algorithm, and the other is the minimum cost algorithm. Using these two algorithms leads to severe consequences. Even a small amount of PEVs which were recharged in the two algorithms can create a new unwanted load peak.

Table 5.2.1 lists the procedures of the default recharging algorithm, which is recharging PEVs at a maximum rate. It is worth noticing that a PEV only stops recharging until the battery is full.

Table 5.2.1: Default Recharging Algorithm

Algorithm 2 Default Recharging Algorithm

```
for each vehicle that plugs in
    while the vehicle needs energy and it is not travelling
        —Charging the vehicle at a maximum recharging rate
    end while
end for
```

Table 5.2.2 shows the procedures of the minimum cost algorithm. In this algorithm, all PEVs recharge at the time the expectation of electricity price is minimum.

Table 5.2.2: Minimum Cost Algorithm

Algorithm 3 Minimum Cost Algorithm

```

for each vehicle that plugs in
    while the vehicle needs energy and it is not travelling
        —Charging the vehicle at the time which has
        the smallest expectation of electricity price
    end while
end for
  
```

5.3 Results

In order to compare three different algorithms in the Matlab, certain scenarios have been built. Here a random baseload curve and a random renewable energy generation prediction in Texas Coast area are chosen as the baseload profile and renewable energy profile.

We compare three algorithms mentioned above in three PEV market penetration scenarios, which are a 40,000-PEV scenario, an 80,000-PEV scenario, and a 160,000-PEV scenario. According to Department of Energy (DOE), there are about 40,000 PHEVs and PEVs sold cumulatively in the United States until the end of 2015[49]. Thus, the 40,000 PEV scenario in Texas Coast area is the most realistic scenario. It demonstrates the power system with PEVs in next five years. The 80,000-PEV scenario can be an estimation of the power system in the ten-year future. The 160,000-PEV scenario represents a very extreme situation. It may happen in 2030. However, the baseload data and the electricity price PDF will be entirely different from the mathematical model in this thesis. Thus, the high PEV penetration scenario is a stress testing more than a realistic scenario.

In these three scenarios, the driving profiles are randomly sampled from 2009 NHTS. We use the same data set for all three algorithms. The initial SOC of PEV battery is generated randomly in the range of 40% to 60% total capacity. At the end of the day, both the constraint-adjusted algorithm and the minimum cost algorithm

manage to recharge the batteries to their initial SOC. The default algorithm recharges the vehicles as much as possible.

Figure 5.3.1 exhibits the results in the 40,000 PEV scenario. The baseload-renewable energy curve shows the difference between baseload and renewable energy generation, which is the demand of conventional power generation.

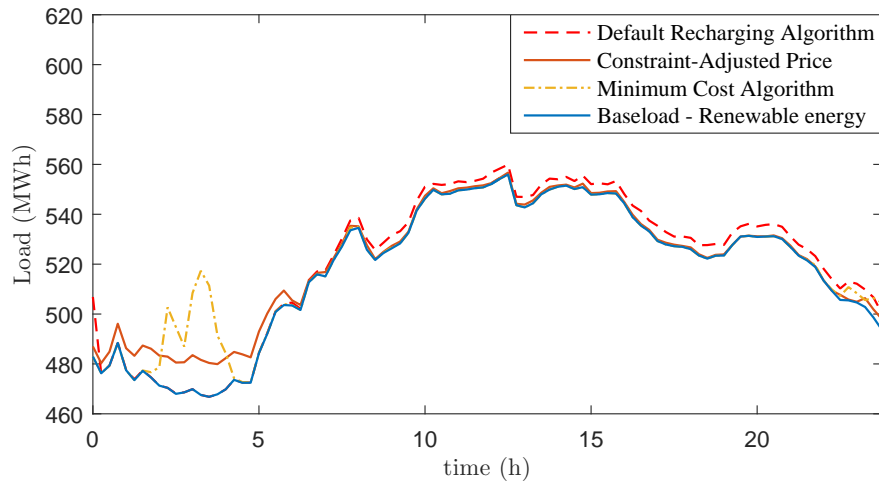


Figure 5.3.1: Results in the 40,000 PEV Scenario

The default recharging algorithm curve represents the total load with the default recharging schedules. The peak load is increasing. The minimum cost algorithm curve shows the total load with minimum cost algorithm. There is no load rising during the daytime. However, a new spike occurs in the early morning and the reliability of the power system may be severely damaged. The constraint-adjusted price curve is the total load after recharging the 40,000 PEV by using the method proposed in this thesis. The load valley from 0 am to 5 am is filled well and there is no new load peak at all.

Figure 5.3.2 indicates the optimal recharging schedules of the static SLP in the first stage and the recharging schedules deployed in the second stage by using dynamic scheduling. On the one hand, the centroids of the clusters are used in the SLP and individual driving profiles in the simulation, but the two curves are the same. Therefore, it proves the subtle differences among vehicle driving profiles can be ig-

nored and the vehicle clusters are good enough to model the overall driving profiles. On the other hand, the diagram demonstrates that the dynamic scheduling algorithm executes the plan solved in the first stage strictly. Thus, the method can be called a near-optimal online optimization.

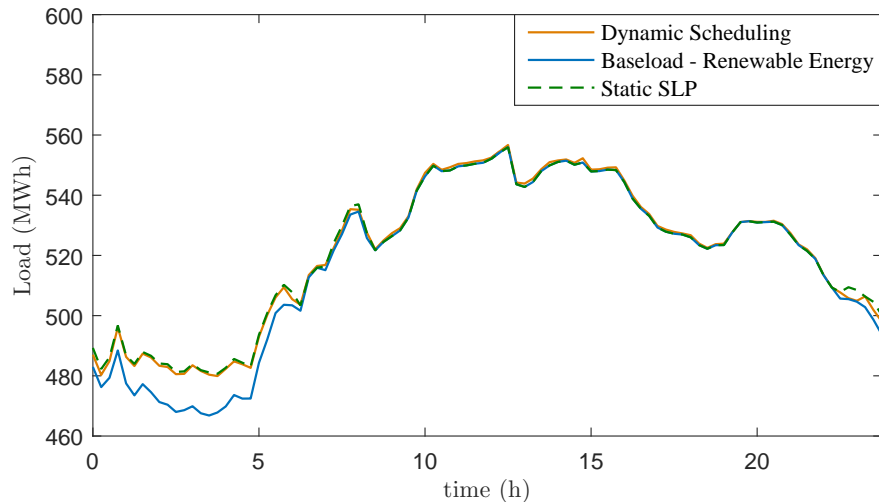


Figure 5.3.2: Results of SLP and Dynamic scheduling in the 40,000 PEV Scenario

Table 5.3.1 lists the total energy costs of three algorithms. The cost of constraint-adjusted price algorithm is not the least but acceptable. Besides, the default recharging algorithm costs far more than the other two algorithms.

Table 5.3.1: Total Energy Costs in the 40,000 PEV scenario

Algorithm	Total Cost Expectation (\$)
Constraint-Adjusted Price Algorithm	52,884
Default Recharging Algorithm	75,456
Minimum Cost Algorithm	48,974

Figure 5.3.3 shows the situation in the medium PEV market penetration scenario. In the plot, the default recharging algorithm and the constraint-adjusted price algorithm keep the total load in its original shape, although the default recharging plan raises the peak load in the daytime. However, the minimum cost algorithm creates a huge peak, which is unacceptable.

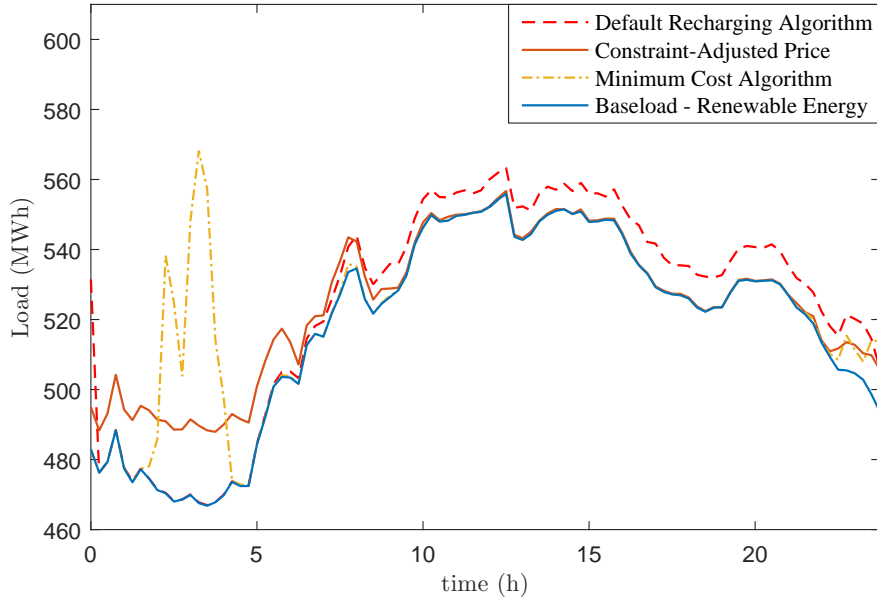


Figure 5.3.3: Results in the 80,000 PEV Scenario

Table 5.3.2 shows exactly the same story as the situation in the 40,000 PEV scenario. The total energy costs of the constraint-adjusted price algorithm and the minimum cost algorithm are very close. The default recharging algorithm costs a lot more.

Table 5.3.2: Total Energy Costs in the 80,000 PEV scenario

Algorithm	Total Cost Expectation (\$)
Constraint-Adjusted Price Algorithm	105,286
Default Recharging Algorithm	150,405
Minimum Cost Algorithm	97,771

Figure 5.3.4 displays the load curves in the 160,000 PEV scenario, where the minimum cost algorithm is totally intolerable, and the default recharging algorithm is not good either. In this extreme situation, the constraint-adjusted price algorithm not only provides practical recharging schedules but also flattens the total load without creating new demand peak. Because the performance of two comparison algorithms are not acceptable, the energy costs are not compared here.

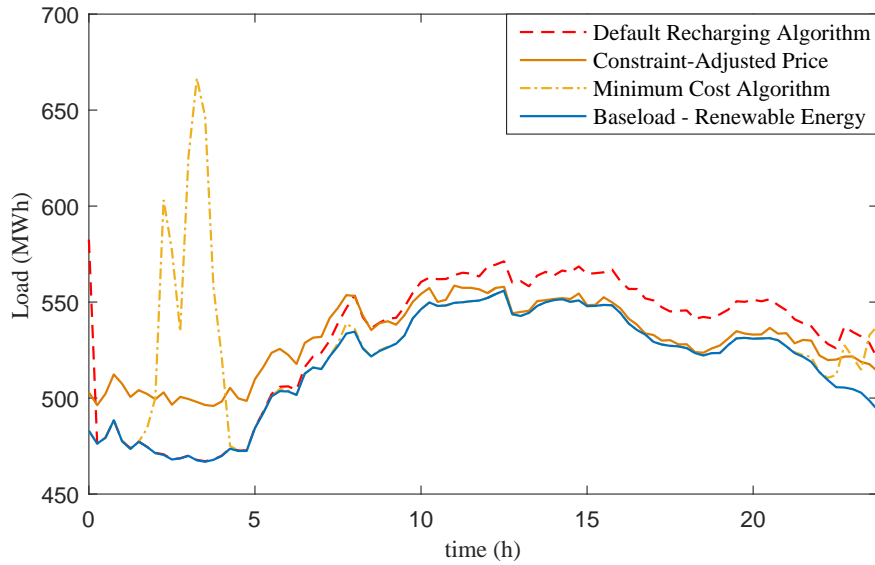


Figure 5.3.4: Results in the 160,000 PEV Scenario

5.4 Conclusion

The market penetration of PEV will grow quickly in the future. In the thesis, we propose a novel online optimization algorithm to recharge a fleet of PEVs in minimum the economic cost without creating a new peak load. A two-stage method is used in the thesis, which has a static linear programming part and a dynamic scheduling part.

There are five contributions of the method. The first is the algorithm deploys a near-optimal solution online. The second is that the stochastic marketing data is taken into account. The third is that the rolling initial and last SOC are used to reduce end effects. The fourth is that the algorithm would not create a new peak load. The last contribution is that the aggregator could deploy recharging schedules in a fast speed with a low communication bandwidth.

In the future work, we will focus on how to reduce the data needed in the method. Besides the uncertainty of PEV numbers and the detailed driving profiles can be discussed in the future.

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