Front Axle Car Suspension Control using Fuzzy Logic

By

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Dedication

The author wishes to dedicate this work to all the friends, family members, and colleagues who have provided moral support and guidance through the years it has taken to complete my masters. I would also like to dedicate this work to a fine individual Dr. Lotfi Zadeh, whose invention of fuzzy sets and logic made this work possible. I thank all of you for your good wishes, your friendship, and the faith you have had in me.

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I would like to thank my parents Mr. Amrish Shah and Ms. Kirti Shah, my sister Ms. Neha Joshi and my brother Mr. Prarthan Shah and his family for their continued support, encouragement throughout the process that surely seemed interminable to them. I sincerely hope that the result will have been worth the effort and wait.

Abstract

Active Suspensions for automobiles is considered to improve ride quality and handling of the vehicle**.** The study reported in this thesis deals with developing an active suspension taking into consideration different road irregularities on both side of the vehicle. For the purpose a fuzzy logic controller for each wheels of vehicle is considered as a control methodology of active suspension.

A front axle car suspension model is considered in the study to test the performance during different road irregularities on both side of the car. The Fuzzy logic controller suggested in the study is compared with Passive Suspensions, Active Suspensions using Linear Quadratic Regulator. The requirement of the proposed controller configuration is also proved by simulating the results for only one side fuzzy controlled. The thesis includes simulation results generated using Matlab-Simulink for each comparisons.

Table of Contents

Dedication……....i
Acknowledgements……....ii
Abstract……iii
Table of Contents……..iv
List of Figures……viii
List of Tables……....xii
Chapter 1: Introduction……1
1.1 Vehicle Suspension System……...1
1.1.1 Passive Suspension System……1
1.1.2 Semi-Active Suspension System……2
1.1.3 Active Suspension System……...2
1.2 Suspension Types……...3
1.2.1 Dependent Suspension Systems……...3
1.2.2 Independent Suspension Systems……....4
1.3 Fuzzy Logic……...5
1.4 Problem Synthesis……6
1.4.1 Ride versus Handling.……7
1.4.1.1 Model Description……...7
1.4.1.2 Spring Conflict……9
1.4.1.3 Damper Conflict……12
1.5 Objectives……14
Chapter 2: Vehicle Suspension System.……...……..15
2.1 Vehicle Coordinate Systems……15
2.2 Vehicle Vibrations……...16
2.2.1 Road Model……..17
2.2.2 Dynamic Loads……18
2.2.3 Acceleration & Braking……21
2.2.4 Cornering……..23

2.3 Passive Suspension System…….26

2.3.1 Springs……..26

2.3.2 Shock Absorbers……..……..29

2.3.3 Development of Passive Suspension……31

2.4 Active Suspension……...35

2.4.1 Active Suspension versus Passive Suspension…….35

2.4.2 Control Methodologies…….38

2.5 Literature Review……38

2.5.1 Introduction……..38

2.5.2 Brief Review of Significant Papers……..39

Chapter 3: Control Methodologies.……48

3.1 Linear Quadratic Regulator (LQR)…….48

3.2 Fuzzy Logic…….49

3.2.1 Choosing Fuzzy Controller Inputs and Outputs……...51

3.2.2 Linguistic Variables, Values and Rules……51

3.2.2.1 Universe of Discourse……51

3.2.2.2 Linguistic Variables……...52

3.2.2.3 Linguistic Values……...52

3.2.2.4 Linguistic Rules…….52

3.2.3 Fuzzy Sets, Fuzzy Logic and Fuzzy Rule Base……53

3.2.3.1 Membership Functions...……53

3.2.3.2 Fuzzy Sets……..54

3.2.3.3 Fuzzy Logic……...54

3.2.3.3.1 Fuzzy Subset……...54

3.2.3.3.2 Fuzzy Complement……54

3.2.3.3.3 Fuzzy Intersection……..54

3.2.3.3.4 Fuzzy Union (OR)……..55

3.2.3.3.5 Fuzzy Cartesian Product……55

3.2.3.4 Fuzzy Quantification of Rules……..56

3.2.4 Fuzzification……56

3.2.5 The Inference Mechanism……57

3.2.5.1 Matching……58

3.2.5.2 Inference Step……...……59

3.2.6 Defuzzification……61

3.2.6.1 Implied Fuzzy Sets……61

3.2.6.1.1 Center of Gravity……61

3.2.6.1.2 Center of Average……...61

3.2.6.2 The Overall Implied Fuzzy Set……..62

3.2.6.2.1 Max Criterion……..63

3.2.6.2.2 Mean of Maximum……63

3.2.6.2.3 Center of Area……64

3.2.7 Mathematical Representations of Fuzzy Systems……64

3.2.8 Mamdani's Model……65

3.2.9 Takagi-Sugeno Fuzzy Model……67

3.2.10 Comparison between Sugeno and Mamdani Method……69

3.2.11 Advantages of Mamdani's Method……69

3.2.12 Advantages of Sugeno Method……..70

Chapter 4: Design of Front Axle Car Suspension Model...……..71

4.1 Model Description……...74

4.2 State Space Representation…….74

Chapter 5: Fuzzy Logic Controller Design……....78

5.1 Step I: Fuzzify the Non Fuzzy Variables into the Fuzzy Variables…….78

5.2 Step II: Establish the Membership Function of the Fuzzy Variables……..80

5.3 Step III: Establish the Fuzzy Reasoning Rules……...82

5.4 Step IV: Apply Fuzzy Reasoning Process……...84

5.5 Step V: Defuzzify the Fuzzy Output into the real Variable……....84

Chapter 6: Simulation & Results……...85

6.1 Proposed Front Axle Car Suspension using Fuzzy Logic Controllers……....85

6.2 Simulation I: Active Car Suspension System vs Passive Car Suspension System.....87

6.3 Simulation II: Fuzzy Logic Controller vs Linear Quadratic Regulator……..95

6.4 Simulation III: System with Two Fuzzy Logic Controller vs System with only One Fuzzy Logic Controller with only Left Wheel Input…….106

6.5 Simulation IV: System with Two Fuzzy Logic Controller vs System with only One Fuzzy Logic Controller with only Right Wheel Input……...115

Chapter 7: Conclusions & Future Expansions……126

7.1 Conclusions……126

7.2 Future Expansions……..126

References……..127

List of Figures

Figure 1.0: Quarter Car Model……..8

Figure 1.1: Sprung Mass Acceleration Response versus road input frequency for various strut spring rates…….10

Figure 1.2: A typical relation between the tire lateral force and vertical force……...11

Figure 1.3: The force response versus the input frequency for various strut spring rates……...12

Figure 1.4: Sprung mass acceleration response versus various input frequency for various damping rates……..13

Figure 1.5: Tire force response versus road input frequency for various damping rates……13

Figure 2.1: Earth-fixed axis system(X,Y,Z) and vehicle axis system (x,y,z)……...16

Figure 2.2: Power Spectral density of various terrains…….18

Figure 2.3: Significant forces acting on the vehicle body during driving(side view)…..19

Figure 2.4: Axle Wind up and Wind Down…….22

Figure 2.5: Response of a vehicle during cornering…….23

Figure 2.6: Significant forces involved in cornering at tire……..24

Figure 2.7(a): Single-beam leaf spring…….27

Figure 2.7(b): Semi-elliptical leaf spring…….27

Figure 2.8: Usage of a Torsion Bar……..28

Figure 2.9: Typical Dampers used in a vehicle suspension

(a): Emulsified Damper……....30

(b): Gas Pressurized Damper……....30

(c): Twin-tube Damper……....30

Figure 2.10: One Example of Solid-Axle Suspension: The Hotchkiss rear suspension...32

Figure 2.11: Left Side of a Four link Rear Suspension……33

Figure 2.12: Basic Structure of a Short-Long arm Independent Suspension……...34

Figure 2.13: Basic structure of Macpherson Strut Independent Suspension……35

Figure 2.14: A typical passive suspension model and the effects of various damping rates……..36

Figure 2.15: Suspension Deflection, Body Velocity and Suspension velocity for the paper by Yahaha[29]……..41

Figure 2.16(a): The Deflection of front wheel for a paper by Yahaha[31]……..43

Figure 2.16(b): The Deflection of rear wheel for a paper by Yahaha[31]……..…..43

Figure 2.16(c): The acceleration of front body for a paper by Yahaha[31]...…..……..43

Figure 2.16(d): The acceleration of rear body for a paper by Yahaha[31]...…..……..43

Figure 2.17: Front & Rear Body Displacement for a paper by Chiou[32]……...45

Figure 2.18: Tire deflection with Fuzzy Logic Control & Optimal Control for a paper by Medhi[37]…….47

Figure 3.1: Fuzzy Controller…….50

Figure 3.2: A two input, two rule Mamdani FIS with crisp inputs……...66

Figure 3.3: Sugeno Rule……...68

Figure 4.1: Front Axle Car Suspension Model……71

Figure 5.1: The procedure of Fuzzy Control Process……...79

Figure 5.2: Suspension Deflection Membership Function...……80

Figure 5.3: Suspension Deflection membership Function……81

Figure 5.4: Ouput Control Force Membership Function……..81

Figure 5.5: Output Surface……83

Figure 6.0 Front Axle Car Suspension System using
proposed Fuzzy Logic Controllers……...86

Figure 6.1 Passive Front Axle Car Suspension System……... 88

Figure 6.2 Sprung Mass Displacement & Velocity for Active vs Passive Suspensions...90

Figure 6.3 Sprung Mass Displacement & Velocity for Active vs
Passive Suspensions with only Right wheel displacement.....92

Figure 6.4 Sprung Mass Displacement & Velocity for Active vs
Passive Suspensions with only Left wheel displacement……94

Figure 6.5 Front Axle Car Suspension Control using Linear Quadratic Control…….96

Figure 6.6 Sprung Mass Displacement & Velocity for FLC vs LQR and Control
Outputs for both the system……99

Figure 6.7 Sprung Mass Displacement & Velocity for FLC vs LQR and Control
Outputs for both the system, when displacement
is only on the right wheel…….102

Figure 6.8 Sprung Mass Displacement & Velocity for FLC vs LQR and Control
Outputs for both the system, when displacement
is only on the left wheel……...105

Figure 6.9 Front Axle Car Suspension system using Fuzzy Logic Controller
only for the Left Wheel……...107

Figure 6.10 Sprung Mass Displacement & Velocity for two FLC's vs one FLC with
only left wheel input and Control Outputs for both the system……109

Figure 6.11 Sprung Mass Displacement & Velocity for two FLC's vs one FLC with only left wheel input and Control Outputs for both the system, with displacement only on the right wheel……..112

Figure 6.12 Sprung Mass Displacement & Velocity for two FLC's vs one FLC with only left wheel input and Control Outputs for both the system, with displacement only on the left wheel……114

Figure 6.13 Front Axle Car Suspension system using Fuzzy Logic Controller only for the Right Wheel……116

Figure 6.14 Sprung Mass Displacement & Velocity for two FLC's vs one FLC with only right wheel input and Control Outputs for both the system……..119

Figure 6.15 Sprung Mass Displacement & Velocity for two FLC's vs one FLC with only right wheel input and Control Outputs for both the system, with displacement only on the right wheel……121

Figure 6.16 Sprung Mass Displacement & Velocity for two FLC's vs one FLC with only right wheel input and Control Outputs for both the system, with displacement only on the left wheel..……124

List of Tables

Table 2.1: Fuzzy Rules for a Paper by Medhi and team [37]……..46

Table 3.1: Mathematical Characteristic of Gaussian Membership Function……..53

Table 5.1: Fuzzy Reasoning Rule Base……...83

Chapter1: Introduction

1.1 Vehicle Suspension System

The Suspension System is among the most important system on any road vehicle or all terrain vehicles (ATV). Its primary function is to maintain proper tire contact with the road surface and support the chassis while the vehicle is maneuvering [2,3]. Its secondary function is to minimize the effect of road disturbances on the vehicle ride, passengers and cargo. Unfortunately, these functions require a balance between conflicting design tradeoffs. In the first case, a firm suspension is required to minimize chassis roll and pitch motions and to keep the wheels firmly on the ground. In the second case, a soft suspension is required to minimize the transmission of high frequency road surface displacements to the passenger compartment. Therefore, the design process leads to a compromise between the two functions. There have been numerous suspension designs created since the invention of the automobile. Still, these designs can be categorized into three different types: Passive, Semi-active and Active.

1.1.1 Passive Suspension Systems

The desired response and handling characteristics of modern passenger vehicle suspension have been determined by decades of experimentation and refinement [4]. The suspensions are composed of springs, dampers and linkages that have fixed properties. A passive suspension system has the ability to store energy via a spring and to dissipate it via a damper. Its parameters are generally fixed, being chosen to achieve a certain level of compromise between road holding, load carrying and comfort. The springs and dampers serve as energy managers while the linkages are designed to ensure that the

wheels and chassis move in a specific manner during road maneuvers. This type of system is called 'Passive' because its characteristics cannot be adjusted based on a measurable change in the operating conditions for the system.

1.1.2 Semi-Active Suspension Systems

Semi-active suspension systems are different from passive suspensions because they provide a mechanism for changing some characteristics of the system based on a change in the operating conditions. Semi-active suspensions include devices such as air springs and switchable shock absorbers, various self-leveling solutions, as well as systems like Hydro pneumatic, Hydrolastic, and Hydro gas suspensions A great deal of theoretical work is focused on adjustment of damping forces include optimal, fuzzy, sliding mode, model reference and self tuning adaptive control [1,7,8,9,10,11]. A defining characteristic of these types of systems is that they are only capable of producing forces that oppose relative motion between the chassis and wheel. In a plot of forces versus relative velocity, this means that these suspensions would only produce force in two quadrants [12], hence the term 'Semi-Active'.

1.1.3 Active Suspension Systems

Due to the conflicting demands, suspension design has had to be something of a compromise, largely determined by the type of use for which the vehicle was designed. Active suspensions are considered to be a way of increasing the freedom one has to specify independently the characteristics of load carrying, handling and ride quality. The term 'active' specifies the class of suspension systems in which the typical spring and damper design is augmented or replaced with an actuator that provides forces regardless of the direction of relative motion between the chassis and wheel. In theory, this means

that the system dynamics could be altered in any fashion within the performance limits of the controller/actuator system. An active suspension system has the ability to store, dissipate and to introduce energy to the system. It may vary its parameters depending upon operating conditions and can have knowledge other than the strut deflection the passive system is limited to. Therefore, active suspensions can provide more control capability than is realizable with a semi-active suspension.

The first Active suspensions, which were hydraulically or pneumatically actuated, appeared during the late 1950's and early 1960's [13]. The majority of active suspension systems today are now controlled by sophisticated electronics. The hydraulic/pneumatic systems can be used in conjunction with standard springs to help reduce the amount of power consumed by the actuator [14]. A large number of automobile manufacturers have conducted research and experimentation with active suspension systems.

1.2 Suspension Types

Suspension systems can also be broadly classified into two subgroups - dependent and independent. These terms refer to the ability of opposite wheels to move independently of each other.

1.2.1 Dependent Suspension Systems

A dependent suspension normally has a beam (a simple 'cart' axle) or (driven) live axle that holds wheels parallel to each other and perpendicular to the axle. When the camber of one wheel changes, the camber of the opposite wheel changes in the same way (by convention on one side this is a positive change in camber and on the other side this a negative change). Dependent systems may be differentiated by the system of

linkages used to locate them, both longitudinally and transversely. Often both functions are combined in a set of linkages.

Examples of location linkage includes, Trailing Arms, Mumford linkage, Watt's linkage, leaf springs used for location (transverse or longitudinal).Fully elliptical springs usually need supplementary location links and are no longer in common use. Longitudinal semi-elliptical springs used to be common and still are used in heavy-duty trucks. They have the advantage that the spring rate can easily be made progressive (non-linear). A single transverse leaf spring for both front wheels and/or both back wheels, supporting solid axles was used by Ford Motor Company, before and soon after World War II, even on expensive models. It had the advantages of simplicity and low unsprung weight (compared to other solid axle designs).

1.2.2 Independent Suspension Systems

An independent suspension allows wheels to rise and fall on their own without affecting the opposite wheel. Suspensions with other devices, such as anti-roll bars that link the wheels in some way are still classed as independent. Independent suspension is a broad term for any automobile suspension system that allows each wheel on the same axle to move vertically (i.e. reacting to a bump in the road) independently of each other. A fully independent suspension has an independent suspension on all wheels. Some early independent systems used swing axles, but modern systems use Chapman or Macpherson struts, trailing arms, multiple links, or wishbones.

Independent suspension typically offers better ride quality and handling characteristic, due to lower sprung weight and the ability of each wheel to address the road undisturbed by activities of the other wheel on the vehicle. Independent suspension

requires additional engineering effort and expense in development versus a live axle or beam axle arrangement. The key reason for lower unsprung weight relative to a live axle design is that, for driven wheels, the differential unit does not form part of the unsprung elements of the suspension system. Instead it is either bolted directly to the vehicle's chassis, or more commonly to a sub frame.

1.3 Fuzzy Logic

Fuzzy logic is a form of multi-valued logic derived from fuzzy set theory to deal with reasoning that is approximate rather than precise. Just as in fuzzy set theory the set membership values can range (inclusively) between 0 and 1, in fuzzy logic the degree of truth of a statement can range between 0 and 1 and is not constrained to the two truth values {true, false} as in classic predicate logic. And when linguistic variables are used, these degrees may be managed by specific functions, as discussed below.

The concept of Fuzzy Logic (FL) was conceived by Lotfi Zadeh, a professor at the University of California at Berkley, and presented not as a control methodology, but as a way of processing data by allowing partial set membership rather than crisp set membership or non-membership. This approach to set theory was not applied to control systems until the 70's due to insufficient small-computer capability prior to that time. Professor Zadeh reasoned that people do not require precise, numerical information input, and yet they are capable of highly adaptive control. If feedback controllers could be programmed to accept noisy, imprecise input, they would be much more effective and perhaps easier to implement.

Fuzzy Logic is a problem-solving control system methodology that lends itself to implementation in systems ranging from simple, small, embedded micro-controllers to

large, networked, multi-channel PC or workstation-based data acquisition and control systems. It can be implemented in hardware, software, or a combination of both. Fuzzy logic provides a simple way to arrive at a definite conclusion based upon vague, ambiguous, imprecise, noisy, or missing input information. Fuzzy Logic approach to control problems mimics how a person would make decisions, only much faster.

1.4 Problem Synthesis

A vehicle suspension is a compromise between ride and handling and such an aim of the vehicle dynamicist to find settings that give the best of both worlds. The following section is a rephrase from "Active Suspension simulation through software interfacing" by Joseph [39]. Since passive suspension has excited throughout the age of the automobile, much is known of the system of the system and the engineers today are very good at extracting the most performance from it. In fact it is commonly thought that the performance of passive suspension has little more improvement left in it. This, combined with a highly competitive automotive market, with hard to please consumers, has sent companies searching for alternative solutions to the conventional system.

As defined by Wong [19] ride is concerned with the sensation or feel of the passenger in the environment of a moving vehicle. Problems arise mainly from vibrations of the vehicle body, induced by sources such as aerodynamic forces and vibration from the power train, drive train and road. According to Hrovat [20], vertical ground input disturbances caused by road roughness are the most relevant for ride studies. Handling, on the other hand, deals with the vehicle's response to inertial body input forces. The main criteria for optimizing the dynamics of a vehicle are:

- Body motion caused from road surface irregularities.

- Body motion caused from road inputs, aerodynamic loading and inertial forces.
- Road holding related to the contact force between the tire and road surface (through controlling wheel motion).
- Maintain directional stability during maneuvers.

An active suspension is better able to find a compromise between these conflicting requirements and as such can offer greater performance than passive suspension. However even with the most sophisticated system, no suspension can simultaneously optimize each criterion since they are all coupled.

As opposed to optimal control theory, fuzzy logic control has been considered by many authors as an alternative control methodology to Active Suspension System. An important objective in utilizing fuzzy logic control (FLC) for Active Suspension System is to determine the control rules such that satisfactory performance under different road profiles can be achieved.

1.4.1 Ride versus Handling

1.4.1.1 Model Description

A vehicle is a complex multi-degree of freedom system that requires sophisticated multibody dynamic algorithms to describe its behavior. However, to reveal its fundamental response and limitations, a simple lumped mass model of a quarter of the vehicle may be considered; in this way the following discussion will be similar to outline.

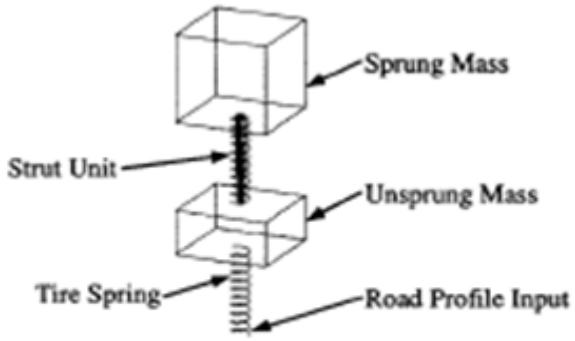


Figure 1.0 Quater Car Model
"Active Suspension simulation through software interfacing" by Joseph[39]

From figure 1.0, the sprung mass represent the vehicle body; in this model it is roughly one quarter the weight of the entire body, while the unsprung mass represents the mass of one wheel unit. Here the strut unit contains both the suspension spring and damper; it provides the link between the two masses. At the same time the tire spring represents the radial stiffness of the tire with negligible damping. The road input is specified at the end of the tire spring in the form of a vertical displacement and since both masses are constrained to only move vertically, the system has two degrees of freedom system.

During operation, it is the suspension spring that absorbs most of the disturbance movement, while the damper dissipates this energy input from the road. Of the variables that affect the vehicle response, the damper and spring rate of the strut unit are two that the dynamicist has direct control over; their chosen parameters contribute to the conflict between ride and handling.

1.4.1.2 Spring Conflict

The equations of motion for the Quarter car model are as follows:

$m_{sprung}\ \ddot{z}_s = -k_{spring}\ (z_s - z_u)$

$m_{unsprung}\ \ddot{z}_u = -k_{spring}\ (z_s - z_u)\ -k_{tire}\ z_u$

where m_{sprung} is the Sprung Mass of the quarter car model in figure 1.0, $m_{unsprung}$ is the Unsprung Mass of the quarter car model in figure 1.0, z_s & z_u are the sprung mass displacement and the unsprung mass displacement respectively, k_{spring} is the spring constant for the strut unit in figure 1.0 & k_{tire} is the sprig constant for the tire in figure 1.0.

When solving the equations of motion for the quarter car model, the damped natural frequencies may be approximated by the undamped natural frequencies of:

$w_{sprung} = [(RR)/m_{sprung}]^{1/2}$

$w_{unsprung} = [(k_{spring} + k_{tire})/m_{unsprung}]^{1/2}$

$RR = [\ (k_{spring} \cdot k_{tire})/\ (k_{spring} + k_{tire})]$

where w_{sprung} is undamped sprung mass natural frequencies and $w_{unsprung}$ is undamped unsprung mass natural frequency.

From these equations, the suspension spring rate is related to both natural frequencies and is the only value in the equations that the dynamicist has control over. Due to the difference in mass, the natural frequency of the unsprung mass is an order higher than the sprung mass becomes excited by the unsprung mass according to Joseph [39]. As a result, for high frequency inputs when the unsprung mass is much lower. However for low frequency inputs, disturbances are passed by the wheel to the body, even amplified at times.

In examining the characteristics of a typical random road profile the vertical acceleration intensity increases with road frequency. As a result, minimizing the natural frequency of the sprung mass decreases the intensity of its response when excited and results in a better ride. This implies that the strut spring should be as soft as possible.

Another way to see the effects of the suspension spring rate on the sprung mass is by considering the frequency response plot of the sprung mass acceleration [39] of figure 1.1. Frequency plots generate the sinusoidal response amplitude to a vertical sine wave road profile of variable frequency. As shown, the softer suspension the lower the acceleration levels of the sprung mass, especially in the area of its natural frequency. However the softness of the strut spring is limited by the amount of space in which the suspension can move within-both while the car statically sits and when it encounters road disturbances.

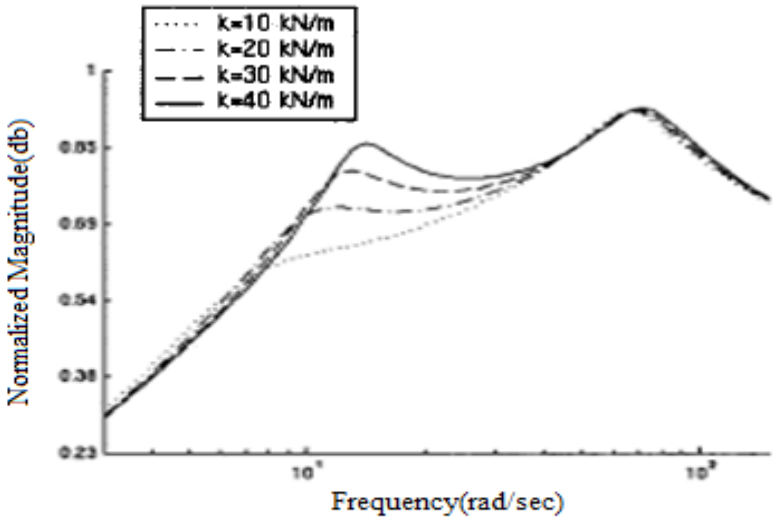


Figure 1.1 Sprung Mass accleration response versus road input frequency for various strut spring rates
"Active Suspension Simulation through Software interfacing "by Joseph [39]

Although a softer suspension produces less sprung mass motion, it also leads to deterioration in handling due to a loss of tire grip. When the vehicle performs a cornering

maneuver the sprung mass shifts to the outside tires increasing the normal tire loading on the outside tires and decreasing it on the inside. The generated lateral tire force is related to the normal tire load, yet at a decreasing it on the inside. The generated lateral tire force is related to the normal tire load, yet at a decreasing rate[39] as illustrated in figure 1.2. As such, the more the vehicle rolls the less lateral tire grip is produced from the combined outside and inside lateral tire force. Hence for handling purposes stiff springs should be used.

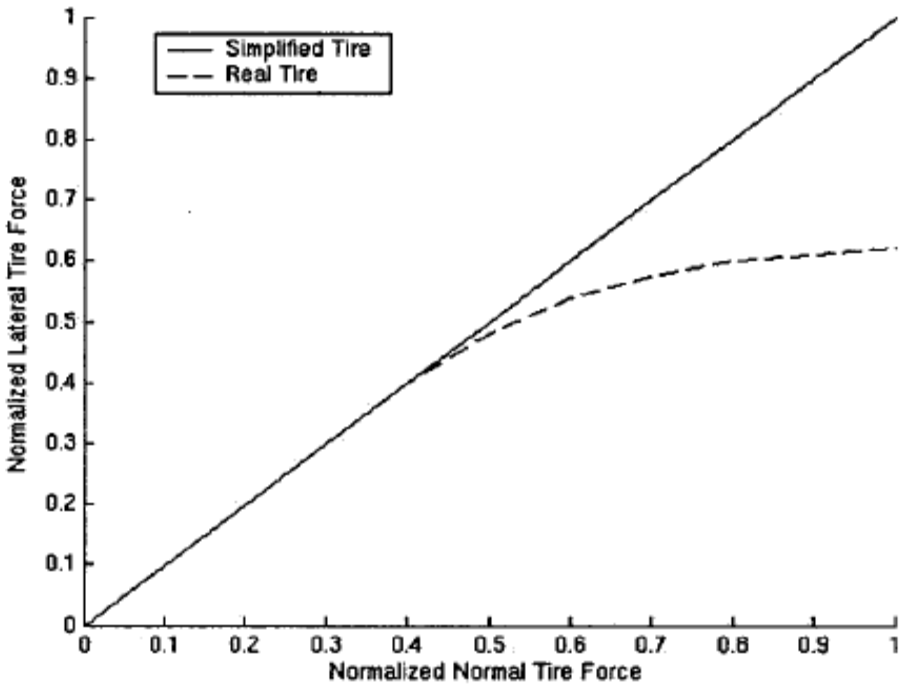


Figure 1.2 A typical relation between the tire lateral force and vertical force "Active Suspension Simulation through software interfacing" by Joseph[39]

For ride maneuvers [39], softer springs lead to greater wheel movement which will increase the fluctuation in tire force. This is confirmed in figure1.3, which shows the frequency response of the tire force. For road input frequencies near the unsprung mass resonant frequency, a softer suspension leads to greater tire force fluctuation, which decreases the longitudinal grip of the vehicle.

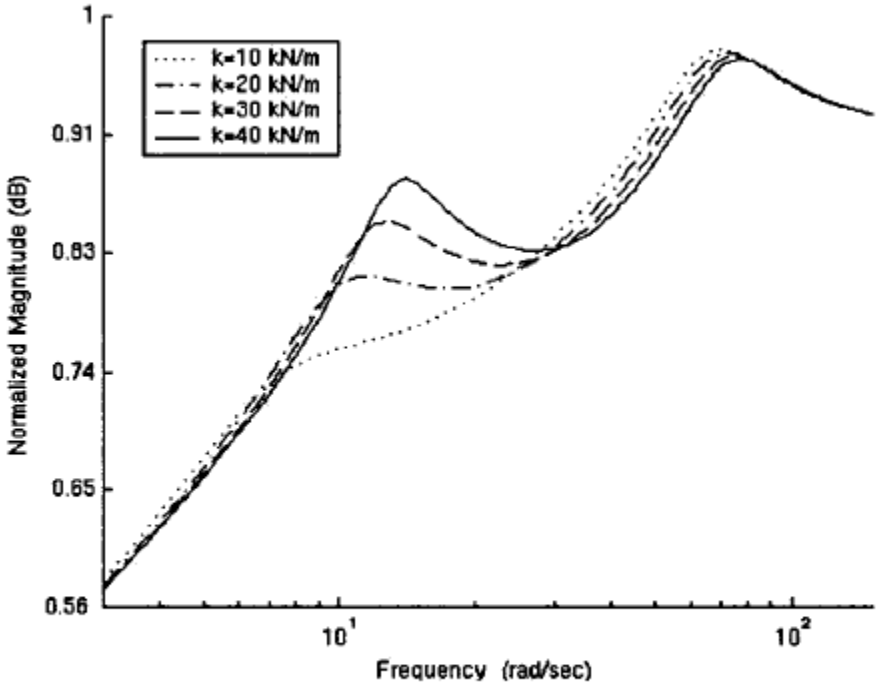


Figure 1.3 Tire force response versus road input frequency for various strut spring rates
"Active suspension similation through software" from Joseph [39]

1.4.1.3 Damper Conflict

The ideal damping coefficient depends on the natural frequencies of the system and the road input frequency and so it continuously changes as the vehicle operates. In looking at the frequency response of the body acceleration [39] in figure1.4 for different damping rates, for input frequencies around the natural frequencies high damping is best, yet at all other frequencies low damping is preferred. A similar pattern emerges for the tire force frequency response [39] in figure1.5, high damping is ideal near the body and wheel natural frequencies with low damping ideal everywhere else in minimizing the deflection.

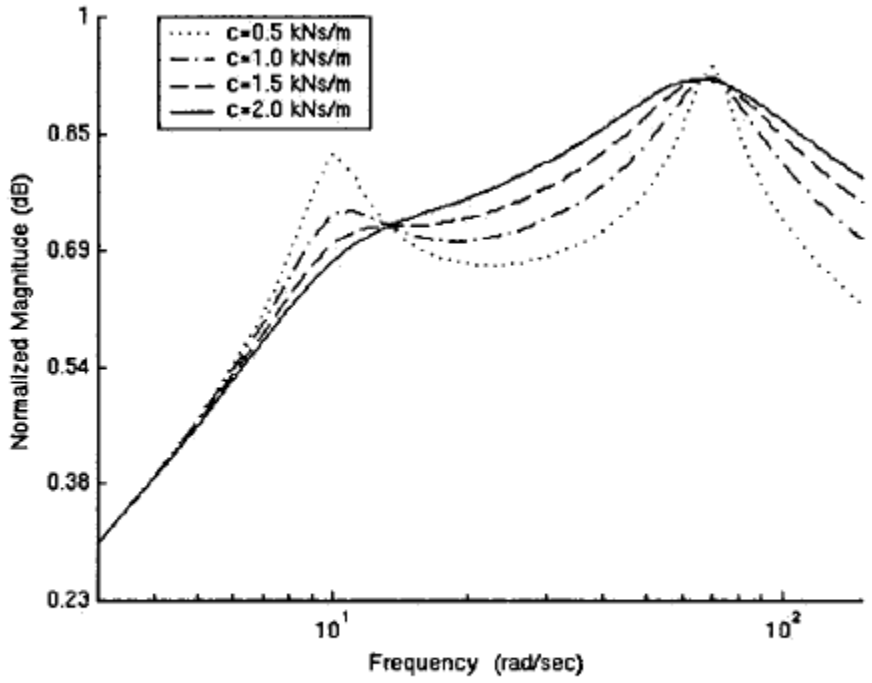


Figure 1.4 Sprung mass accelaration response road input frequency for various damping rates
"Active Suspension Simulation through software interface" by Joseph[39]

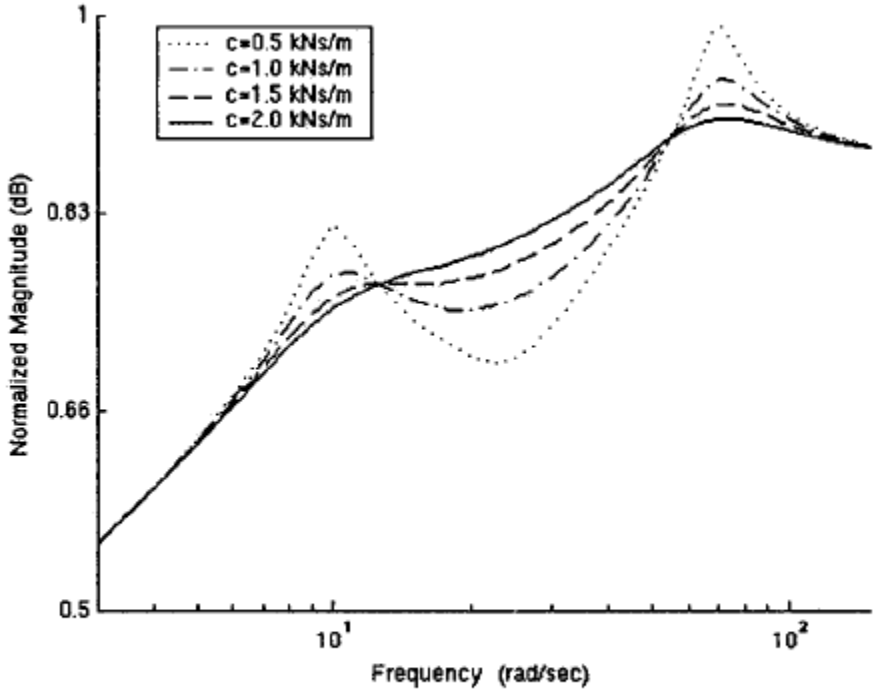


Figure 1.5 tire Force response versus road input frequency for various damping rates
"Active Suspension Simulation through Software interfacing" by Joseph[39]

1.5 Objectives

The objective of the research is to show a method to control the vehicle using fuzzy logic controller for active car suspension system, to evaluate the vehicle stability when left and right tires of the vehicle come across different road disturbances and compare it with different methods. Different road disturbances on left and the right tires of a vehicle are quite common and very few research works emphasize on such disturbances. An independent front axle vehicle suspension with fuzzy logic controller for active suspension system is proposed in the research.

More specifically to:

- Implement a front axle car suspension model of a mass passenger vehicle.
- Implement a fuzzy logic control for the active car suspension system.
- Model the vehicle in Matlab-Simulink and generate simulation results with different road disturbances for left and right tires for the front axle car suspension systems.

And to compare:

- The differences of the various procedures developed if any.
- Explore the differences between the active suspension method for front axle and the method proposed in the research.
- Explore the differences between LQR and Fuzzy Logic Control for car suspension.
- Explore the differences between Passive and fully-active suspension.

Chapter 2: Vehicle Suspension System

Chapter 1 described the purpose of the automotive suspension, different vehicle suspension systems and the problems in designing the vehicle suspension system. This Chapter begins with the introduction of vehicle coordinate systems, the major excitation sources and the responses of a vehicle to these excitations. Then, the chapter will briefly discuss the development of passive and active suspension systems in their attempts to reduce the effects of road disturbances. With the progress of control and electricity, the concept of active-suspension receives wide interests for its potential in improving some inherent drawbacks of a passive suspension system. The following discussion is a rephrase of "Design of Active suspension with a controllable damper" by Chang

2.1 Vehicle Coordinate Systems

According to the convention of the Society of Automotive Engineers (SAE), the motion of a vehicle is referred to an earth- fixed axis system (X, Y , Z). As shown in fig. 2.1, the earth- fixed axis system is a right hand orthogonal axis system fixed on the earth. The X-axis and the Y-axis lie on the horizontal plane and the Z-axis is directed downwards. Relative to the earth-fixed axis system, the orientation of the vehicle is described by a vehicle axis system (x, y, z) located at the center of the mass of vehicle. The x-axis of the vehicle axis systems is defined as the forward direction of the vehicle while the y-axis extends towards the right side of the vehicle. Following the right-handed set of vectors, the z-axis is specified by the vector cross product and downward with respect to the vehicle. Fig 2.1 also defines the component of the velocity vector of a vehicle.

In addition to the vehicle coordinate systems, the term sprung and unsprung mass are usually used to represent a vehicle body in the coordinates. Sprung mass denotes the mass of the vehicle body supported by the suspension, including the mass of the suspension components. In contrast to the sprung mass, unsprung mass is the mass that is not carried by the suspension system, but is supported by the wheel.

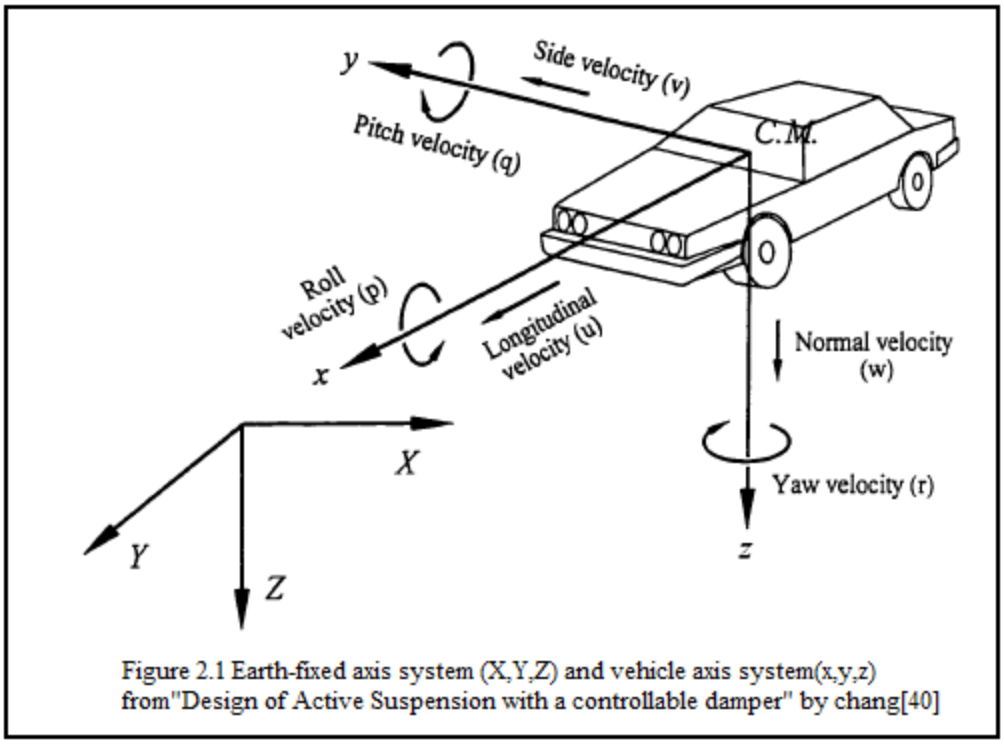


Figure 2.1 Earth-fixed axis system (X,Y,Z) and vehicle axis system(x,y,z) from"Design of Active Suspension with a controllable damper" by chang[40]

2.2 Vehicle Vibrations

To most car buyers, the degree of vibration perceived in riding is the top criteria in evaluating the quality of a car. According to the terminology defined by the Society of Automotive Engineers [21], vibration can be categorized according to its frequency. Ride is a low frequency vibration under 5 Hz. Shake is an intermediate frequency vibration between 5 to 25 Hz. The vibrations above 25 Hz include harshness that can be perceived tactually and/or audibly and boom that is a high intensity vibration perceivable only

audibly and characterized as a sensation of pressure by the ear. The design of such a suspension system deals with the vibration less than 25 Hz.

2.2.1 Road Model

The road roughness possibly accounts for the majority of the excitation sources for vibrations. By defining roughness as the elevation profile of a road surface, the measurement of the road roughness can be described by a Fourier series [22-24]. For each spatial frequency, there is corresponding amplitude. From the Fourier series, the Power Spectral Density (PSD) function that has been widely accepted and used as the description of road roughness can be obtained. Although the PSD functions that has been widely accepted and used as the description of the road roughness can be obtained. The PSD functions for any two road sections are almost impossibly the same, some studies proposed functional representations for the average property of road roughness. One qualitative expression of the spectral density that is simple and frequently used is [22-24,28]

$$S(f) = CF^{-w} \qquad (2.1)$$

where f is the spatial frequency, C is the roughness coefficient of the road surface, w is the frequency taper parameter and $w \approx 2$. Gillespie proposed another function as [25,26]

$$S(f) = S_0 \ [1+(f_0/f)^2] \ / \ (2\pi f)^2 \qquad (2.2)$$

where S_0 is the roughness magnitude parameter indicating the roughness level and f_0 is the cutoff spatial frequency. Combined with the random number sequence, both equations provide a description of road roughness for the study of vehicle dynamics. By setting w=2.5 and the roughness coefficient to be values given by Robson [28], figure 2.2

is the PSD of various terrains based in equation (2.1), equation (2.2) also generates a similar plot that reflects the essential of any general surface. The deviation in elevation of a road surface diminishes with frequency to about the second power. Experiments verified that

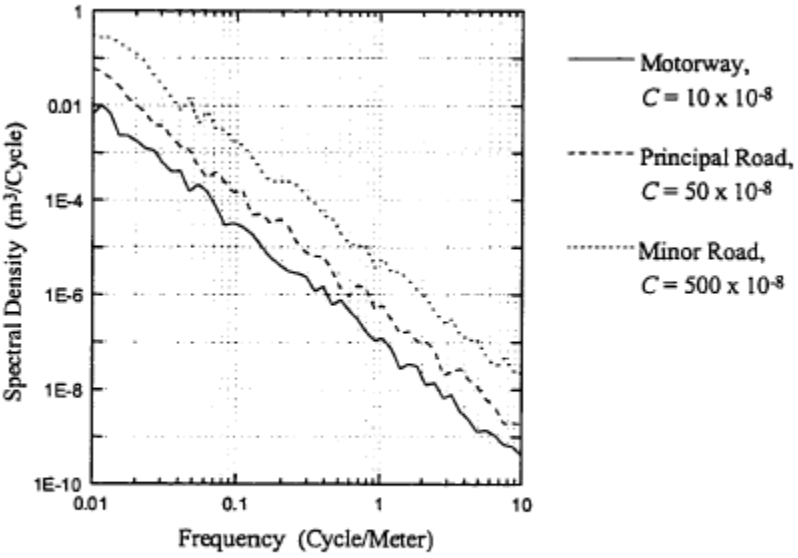


Figure 2.2 power Spectral Density of various terrains given by equation (2.1) from"Design of active Suspension with a controllable damper" by Chang[40]

both (2.1) and (2.2) given appropriate coefficient values can physically represent road surfaces [26,28].

2.2.2 Dynamic Loads

When a vehicle is static on the ground, the body weight is evenly supported by all the wheels. When it travels, the forces on the vehicle include gravity, tractive force, wheel rolling resistance force and aerodynamic force. These forces as well as acceleration, braking and cornering make the load to shift among the wheel axes. Responding to these load shifts, the flexible components in a vehicle such as springs and tires make the vehicle body vibrate. This section introduces the load shift caused by

travelling behaviors except cornering that will be discussed later. The effects of these forces to the vehicle motion have critical implications for the design of a suspension system. Figure 2.3 illustrates these forces on a vehicle from the side view.

In this figure, the symbols used stand for as follows:

- M is the mass of the vehicle located at its centre of mass.
- H_m is the height from the center of mass to the ground.
- F_d and F_l are the sums of aerodynamic drag and lift forces respectively acting on the vehicle body.

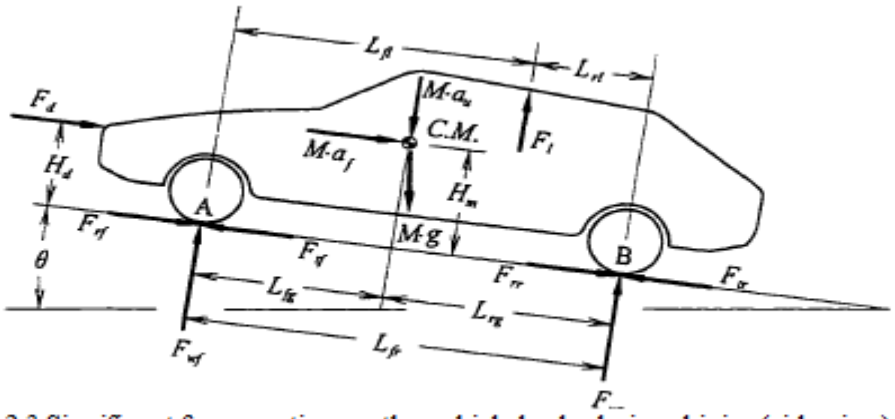


Figure 2.3 Significant forces acting on the vehicle body during driving(side view) "Design of Active Suspension with a controllable damper"by Chang[40]

- H_d is the height from the point on which the aerodynamic drag force, F_d acts to the ground.
- F_{wf} and F_{wr} are the dynamic reacting forces from the ground at the front and rear wheel axles respectively.
- F_{rf} and F_{rr} are the rolling resistance forces at the front and rear wheels respectively.
- F_{tf} and F_{tr} are the tractive forces at the front and rear wheels respectively.
- L_{fr} is the longitudinal distance from the front wheel to the rear wheel.

- L_{fg} and L_{rg} are longitudinal distances from the front and rear wheels respectively to the center of the mass.
- L_{fl} and L_{rl} are longitudinal distances from the front and rear wheels respectively to the point where the aerodynamic lift force acts on.
- $M.a_f$ and $M.a_u$ are the d'Alembert forces added for convenience to express the dynamic equilibrium equations a_f and a_u are the forward and upward accelerations respectively of the vehicle.

From fig. 2.3, we have the force equilibrium equations as

$$F_l + F_{wf} + F_{wr} - M.g\cos\boldsymbol{\theta} - M.a_u = 0 \quad (2.3)$$

$$F_{tf} + F_{tr} - F_{rf} - F_{rr} - F_d - M.g\cos\boldsymbol{\theta} - M.a_f = 0 \quad (2.4)$$

The torque, M_A , at A about an axis parallel to the y-axis is

$$M_A = F_d H_d + (Mg\cos\boldsymbol{\theta} + M a_u).L_{fg} + (Mg\sin\boldsymbol{\theta} + M a_f).H_m - F_l L_{rl} - F_{wr} L_{fr} \quad (2.5)$$

The torque, M , at B about an axis parallel to the y-axis is

$$M = F_d H_d - (Mg\cos\boldsymbol{\theta} + M a_u).L_{fg} + (Mg\sin\boldsymbol{\theta} + M a_f).H_m - F_l L_{rl} - F_{wr} L_{fr} \quad (2.6)$$

In equations (2.4), the rolling resistance forces can be expressed as

$$F_{rf} = \mu_f F_{wf} \quad (2.7)$$

$$F_{rr} = \mu_r F_{wr} \quad (2.8)$$

That is, the magnitude of the rolling resistance force is equal to a rolling resistance coefficient, μ times the vertical force acting on the ground. However, the function of the rolling resistance coefficient is very complicated if it is possible to be calculated [25]. The value of rolling resistance coefficient depends on the structure of ground material, composition of the rubber, design of the tire, temperature, tire pressure, vehicle speed, etc. Usually, it has a range from 0.01 to 0.025 for most passenger cars when the vehicles

are running on concrete surface [27]. As to the aerodynamic forces, the drag force is mainly caused by the action of viscous friction of the air in the boundary layer on the surface of the car. Besides the friction force, the gross flow of air governed by the Bernoulli's equation makes the air pressure to vary over the vehicle body. Such pressure distribution affected by the shape of the vehicle body generates a drag force along the longitudinal direction (x-axis), side force along the lateral direction (y-axis), and lift force along the vertical direction (z-axis). The distribution of the aerodynamic force over the vehicle body is determined by the relative speed of the air to the vehicle, the body shape and even the layout of the engine compartment [25]. In the qualitative analysis of the vehicle dynamics for suspension design, the aerodynamic forces and the acting points are assumed to have been derived through experiments.

Under a stable condition without acceleration in pitch, M_A and M_B must be zero. Equations (2.5) and (2.6) yield the dynamic axle loads as

$$F_{wf} = (1/L_{fr}) [(Mg \cos\boldsymbol{\theta} + M a_u).L_{fg} - F_d H_d + (Mg \sin \boldsymbol{\theta} + M a_f). H_m - F_l L_{rl}] \quad (2.9)$$

$$F_{wr} = (1/L_{fr}) [(Mg \cos\boldsymbol{\theta} + M a_u).L_{fg} + F_d H_d + (Mg \sin \boldsymbol{\theta} + M a_f). H_m - F_l L_{rl}] \quad (2.10)$$

Equations (2.9) and (2.10) express the dynamic variation of axle load under different conditions.

2.2.3 Acceleration and Braking

Assume the vehicle is running on a level ground at a low speed, $\boldsymbol{\theta}$ is zero and the aerodynamic forces are negligible, (2.9) and (2.10) can be simplified as

$$F_{wf} = (L_{rg} / L_{fr)}) Mg - (H_m / L_{fr}).M a_f \quad (2.11)$$

$$F_{wr} = (L_{fg} / L_{fr)}) Mg - (H_m / L_{fr}).M a_f \quad (2.12)$$

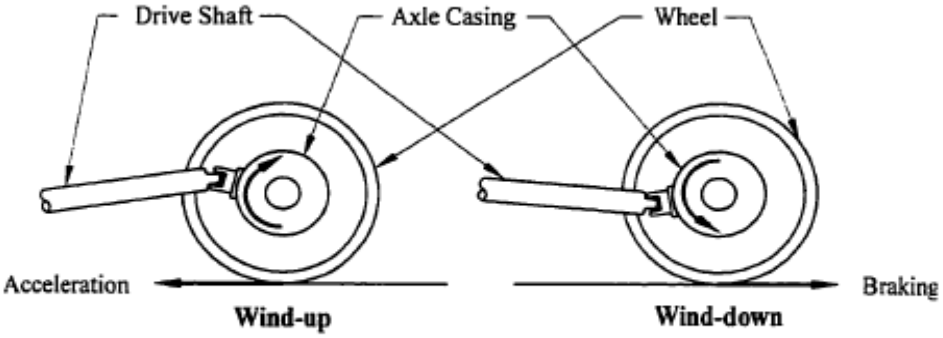


Figure 2.4 Axle wind-up and axle wind-down
from" Design of active suspension witha a controllable damper" by Chang[40]

That is, the load will be transferred from the front axle to the rear axle during acceleration. Conversely, during braking the load is transferred from the rear axle to the front axle. The load shift will cause nose-down pitch (dive) during and reward pitch (squat) during acceleration.

Another motion caused by acceleration (braking) is axle wind-up (axle wind-down). Conventionally, the term wind-up is used for both axle wind-up and axle wind-down. Wind-up is an undesired rotation that suspension designers tried to handle. It happens at the connection of a drive shaft and the axle casing as shown in fig. 2.4. The axle casing rotates upward or downward due to sudden application of brakes. Without any flexible device, this motion will directly transmit an impact throughout the power train.

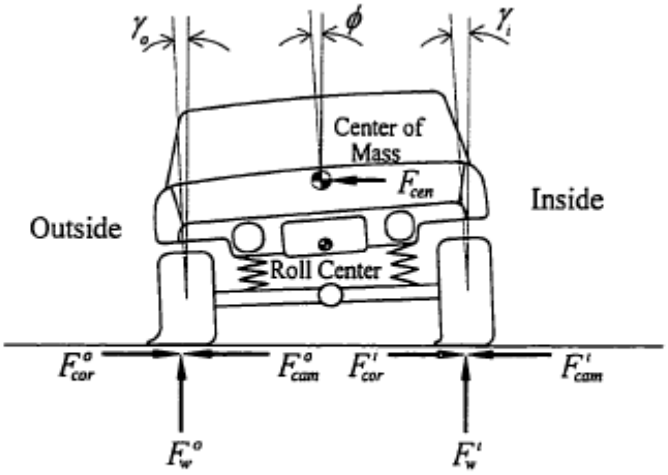


Figure 2.5 Response of a vehicle during cornering
from"Design of active suspension with a controllable damper" by Chang[40]

2.2.4 Cornering

In the above sections, the dynamic load at each wheel of a vehicle was introduced. Equations (2.11) and (2.12) explained the load shift during acceleration and braking. One of the most important vehicle behaviors involved in the consideration of a suspension design is cornering. Fig. 2.5 illustrates the response of a vehicle body during cornering. To balance the centrifugal force F_{cen} exerting on the center of mass and keep the vehicle on the track, a cornering force F_{cor} must be developed at the contact point of the tire and ground. The tires also lean out. The inclination of a wheel is defined as the camber angle $\boldsymbol{\Upsilon}$. Camber on a wheel will produce a force called camber thrust (or camber force) $F_{cam.}$ The requirement of cornering force, the size of camber angle, and the position of roll center are three variables in a suspension design to handle cornering behavior. The interaction of these forces generates a roll moment about the roll center and makes the z-axis of the vehicle coordinate turn a roll angle $\boldsymbol{\Phi}$.

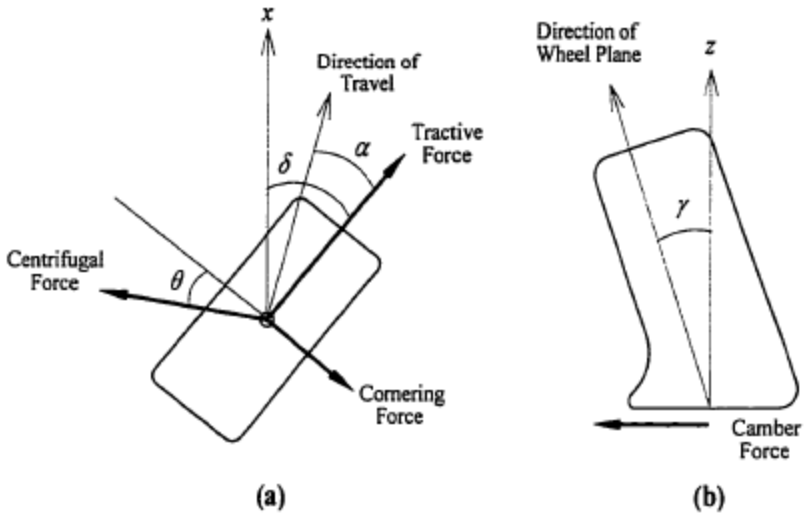


Figure 2.6 Significant forces involved in cornering at tire
from "Design of active suspension with a controllable damper" by Chang[40]

The forces developed on a tire during cornering are shown in figure 2.6 in which (a) is the projection on the ground and (b) views the tire along the longitudinal direction. During cornering, the driver changes the heading direction of the wheel. The angle between x direction of the vehicle and the heading direction of the wheel is defined as the steer angle, $\boldsymbol{\delta}$. A centrifugal force will act on the tire and cause the tire to slip outwardly. To control the vehicle into following the track and completing the turning, the tire must develop a cornering force, F_{cor}, against the ground. The deviation between the heading direction of the wheel and the direction the vehicle travels is defined as the slip angle, $\boldsymbol{\alpha}$. The larger the centrifugal force is, the larger is the required slip angle to generate an adequate counteractive cornering force. When the slip angle is smaller than 5 degrees, a linear relationship exists between the slip angle and the cornering force [25]. The cornering force can be expressed as

$$F_{cor} = C_{cor}.\boldsymbol{\alpha} \tag{2.13}$$

where the constant C_{cor} is the cornering stiffness relating the cornering force to the slip angle [21]. In figure 2.6(b), the angle between plane of the wheel and the vertical line is

defined as the camber angle. The camber will cause a tread area distortion at the contact point of the tire and the ground. This distortion results in a camber force that is proportional to the camber angle. Thus the camber force can be expressed as

$$F_{cam} = C_{cam}.\boldsymbol{\alpha} \tag{2.14}$$

where C_{cam} is the camber stiffness. The camber stiffness is decided by the property of tire stiffness and the vertical force acting on the wheel. Equation (2.14) indicates that a larger camber angle will produce a larger camber force. Figure 2.6 demonstrates that at a given turning, a larger slip angle is necessary to generate the required cornering force to counter a large camber force. To increase the slip angle, the driver needs to increase the steer angle. Thus for the handling reason, one criterion of a modern suspension design is the minimization of the camber change during cornering.

The location of the roll center shown in figure 2.5 is decided by the geometry of a suspension system. As mentioned earlier, the cornering force and camber force are related to the vertical load exerted on the wheel. However, the vertical load is transfers during cornering. With a determined roll center, the roll angle and the load transfer at each of the front and rear axle can be easily calculated. The importance of roll center is that it indicates the location where the force is transmitted from the wheel to the sprung mass, and then decides the vibration of sprung mass. Generally, the roll angle is reduced if the roll center is high. However, high roll center produces more scrub in bump [27]. Since the roll center is decided by the geometry of a suspension, the roll center moves as soon as the suspension designer needs to incorporate the factors of height as well as the position deviation of the roll center.

2.3 Passive Suspension System

Before the idea of automobile was even proposed, suspensions have been used on horse-drawn carriages. The following section is a rephrase from "Design of an active suspension with a controllable damper" by Chang [40]. In the earlier days, the purpose of a suspension was to reduce fatigue or improve ride comforts of the driver or passengers. Due to the limited availability of manufacturing tools, the main component of an early suspension was leaf-spring, commonly used in 18^{th}-century carriages. After decades of improvements in automotive technology, the functions of vehicle suspension systems expanded beyond the improvement of ride comforts to the enhancement of vehicle control and stability. Modern suspensions allow the wheel to comply with an uneven surface and maintain the road surface contact of the wheel. The shock absorbers dissipate the energy created by impacts and thus improve the ride comforts and vehicle handling. Through advanced study of vehicle dynamics, the designers adopted different mechanisms for suspensions to react to various dynamic forces and to resist undesired roll motion of the chassis. The ride comfort, driving safety, and vehicle controllability are more ensured. In this section, the properties of the two main components – springs and shock absorbers will be introduced. Immediately following will be the discussion of the suspension designs used in mass-production vehicles.

2.3.1 Springs

The springs used in transports exist in any of the gas, liquid or solid type. The gas springs, typically using nitrogen, have the advantages of high compliance, high energy capacity, low weight, and low fatigue rate. However, to support a heavy weight, both the stiffness and the required size of the spring, proportional to the energy capacity, become

very high. Therefore, air springs are usually used in vehicles like trains or heavy trucks where a large energy capacity is required and a large accommodation space is available. A liquid spring has the advantages of a gas spring but is heavier. The liquid used must be relatively compressible. Oil is typically adopted. Liquid springs are unfavored by normal ground vehicles due to their slow response. However, the high energy capacity and low possibility of sudden damage make the liquid springs widely applied on the oleo legs of aircraft.

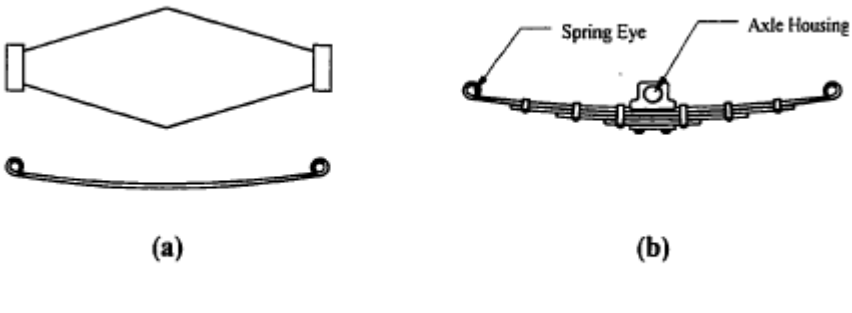


Figure 2.7 (a) Single-beam leaf Spring (b) Semi-elliptical leaf spring
from"Design of active suspension with a controllable damper" by Chang[40]

Solid springs have been used on suspensions for more than one hundred years. As mentioned above, the leaf springs that are easy to manufacture have been in use for a long time. A leaf spring can be as simple as a single thin beam. Its two ends attached to the vehicle frames while the wheel axle attached to its center. Because the bending moment is strongest at the center, two concepts have conventionally been applied to the design of a leaf spring to generate the required compliance. First, the width of the beam is proportional to the strength of the bending moment, the upper face is like diamond shape illustrated in fig. 2.7(a). Secondly, since the width is restricted by the space available, the thin beam is stacked on top of one another as shown in fig. 2.7(b) to produce the same bending stiffness in the normal direction. The leaf spring is obsolete today due to its

heavy weight and the inherent friction between thin beams. Recently the development of material science gives the leaf spring a new life. A leaf spring made of glass-reinforced plastic has been installed on a production sport car in favor of its light weight.

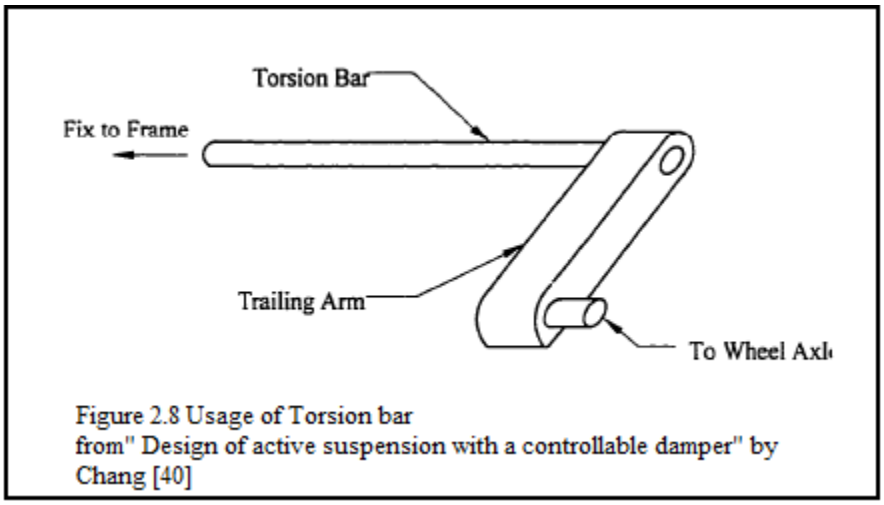


Figure 2.8 Usage of Torsion bar
from" Design of active suspension with a controllable damper" by Chang [40]

Some vehicles use torsion bars as springs. The usage of a torsion bar is shown in figure 2.8. The front end of a torsion bar is attached to a beam called trailing arm and the rear end is attached to the vehicle frame. The irregularities on the road surface cause the trailing arm to pivot up and down. By this action, the torsion bar has a higher stiffness that makes it more suitable for heavy vehicles like military tanks. A torsion bar is sometimes installed in addition to coil springs to increase the roll stiffness in resisting undesired cornering roll and enhancing driving safety. Named for this rolled stiffness enhancement function, this bar is called a “stabilizer” (or “anti-roll bar”). Unlike a torsion bar that connects a trailing arm and the vehicle chassis, a stabilizer attaches its two ends to the tips of two opposite trailing arms. When a car is cornering on a tilt curve, the effect of centrifugal force puts more weight on the outer wheels than on inner wheels as explained in the last section. Therefore, the outer spring is compressed and the inner spring is expanded. The increased slip angle forces the car to lean outward. With a

stabilizer, compression of the outer spring twists the stabilizer bar in one direction while the expansion of inner spring twists the stabilizer bar in the opposite direction. While restoring to its original shape, the stabilizer reduces the roll motion.

Today most suspensions use a coil spring at each wheel. Essentially a coil spring is a coiled torsion bar. The superior advantage of a coil spring is that is can offer a progression of stiffness by the design of varying coil spacing and varying coil diameter. Thus a single spring can handle a wide range of loads from tiny vibrations to heavy impacts.

2.3.2 Shock Absorbers

A shock absorber is essentially a damper that is responsible for dissipating the energy that causes vehicle vibrations. Before 1900, the friction disc was first used as suspension damper. The energy was dissipated through the friction between bronze and leather. Soon people realized that it was not a good design. First, unless the disturbance force exceeds the static friction of the suspension, all the disturbance force is transmitted to the sprung mass. Secondly, the damping force is constant. Thirdly, the friction effect is very sensitive to water or oil contamination. This concept was finally abandoned. In 1901, the first hydraulic damper was patented. The inherent advantage of hydraulic damper is proportional to the vertical velocity between the unsprung mass and the sprung mass. After 1950, the telescopic hydraulic shock absorbers were nearly the exclusive damping device in automotive industry.

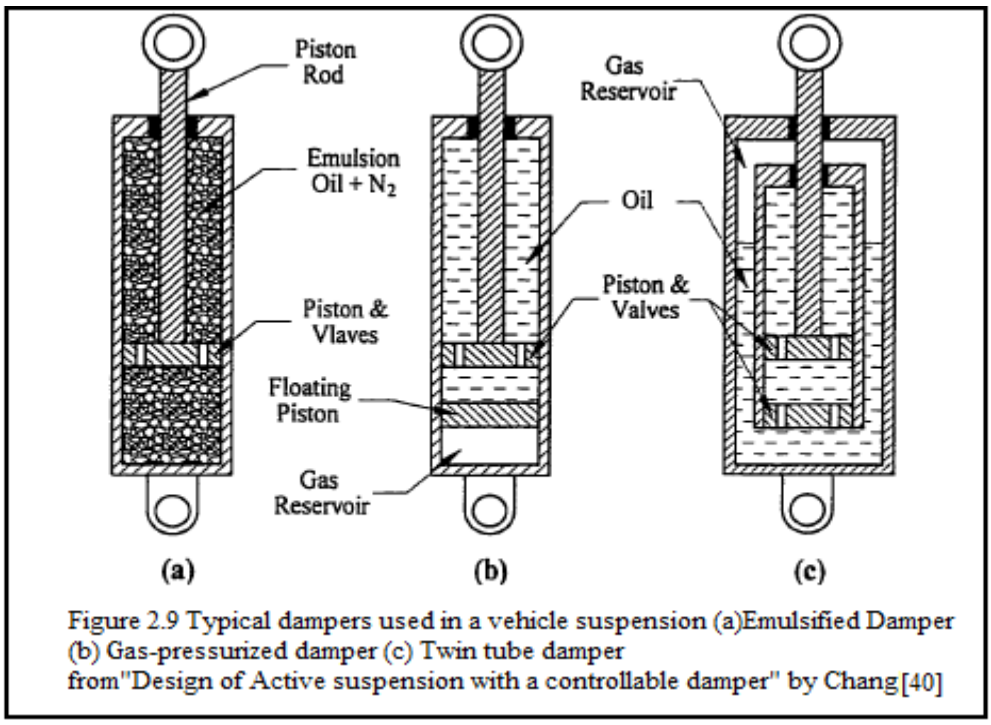


Figure 2.9 Typical dampers used in a vehicle suspension (a)Emulsified Damper (b) Gas-pressurized damper (c) Twin tube damper from"Design of Active suspension with a controllable damper" by Chang[40]

Currently there are two types of telescopic dampers. One is called single-tube emulsified shock absorber [25,40] as shown in fig 2.09(a) and its improved version called anti-emulsion (or gas pressurized) damper [25,40] as shown in fig 2.09(b). The other type is the twin-tube damper [25,40] as shown in fig 2.09(c). The emulsified type is simply a fluid-filled cylinder with a valved piston that will move up and down with the relative movement of the wheel chassis. The fluid is an emulsion of oil and gas, typically nitrogen. When encountering a bump, the piston is forced through the valves to balance the pressure differential. Later the pressurized fluid exerts a force on the piston-rod area trying to expand the damper. The emulsion is better than pure oil because the gas in the emulsion increases the compressibility that produces a large pressure differential across the piston within a short time. As the piston is moving, the fluid flows from one side of the piston to the other through valves, which restrains the velocity of the flow. Thus, the kinetic energy of the motion is dissipated by releasing heat into the fluid and so is the

vibration shock dampened. In the anti-emulsion damper, pure oil is used and the gas is separated at one end by an additional floating piston. Apart from this, its function is similar to that of an emulsified damper. Some dampers place the gas reservoir at remote site connecting the cylinder through a pipe. The design helps to dissipate the heat generated. Instead of placing the gas reservoir at one end, the twin-tube damper arranges the gas reservoir as an annulus around the inner cylinder. The single tube type is good for dissipating heat, but its exposure of the working cylinder makes the cylinder prone to damage from flying stone. Although the twin-tube damper keeps the working cylinder inside the design is inferior in cooling.

2.3.3 Development of Passive Suspension

The commercial passive suspensions have been categorized into two groups based on the linkage structure. One is called the solid-state suspension (or dependent suspension) and the other is the independent suspension. Ina solid-axle suspension, the right wheel and the left wheel are mounted at either end of a rigid beam axle. The movements of a wheel and the opposite wheel relate to each other. Contrary to a solid-axle suspension, an independent suspension allows each wheel to be geometrically independent. The hopping motion of one wheel is not transmitted to the opposite wheel.

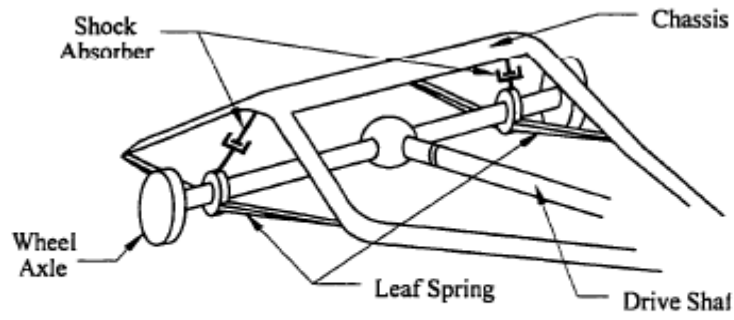


Figure 2.10 One Example of Solid-Axle Suspensions - the Hotchkiss rear suspension
from"Design of active suspension with a controllable damper" by Chang[40]

One example of solid – axle suspensions is the Hotchkiss suspension [40] briefly sketched in fig 2.10. The wheel axle is connected to each side of the chassis by the two ends of a leaf spring and a shock absorber. Because of the easy manufacture of the leaf spring, the Hotchkiss suspension was commonly used until 1960's. Due to the high longitudinal stiffness of the leaf spring, an additional trailing arm (ref. to Fig 2.09) is often used to reduce the axle wind- up caused by sudden braking or acceleration. To improve the Hotchkiss suspension, the four-link suspension [25,40] briefly sketched in figure 2.11 is the currently widely used solid-axle suspension. The leaf spring suspension is replaced with a coil spring. Two lower links support the springs and shock absorbers. The other two upper links absorb braking and acceleration torque to avoid axle wind-up. In general, the solid axle suspension has several advantages. First, the camber angle of a wheel is not affected by the roll motion. Secondly, the wheel alignment is readily maintained [25]. However, the hopping motion at the wheel will transmit to opposite wheel. Tramp motion in which a pair of wheel hops in opposite phase often happens, leading to roll vibration.

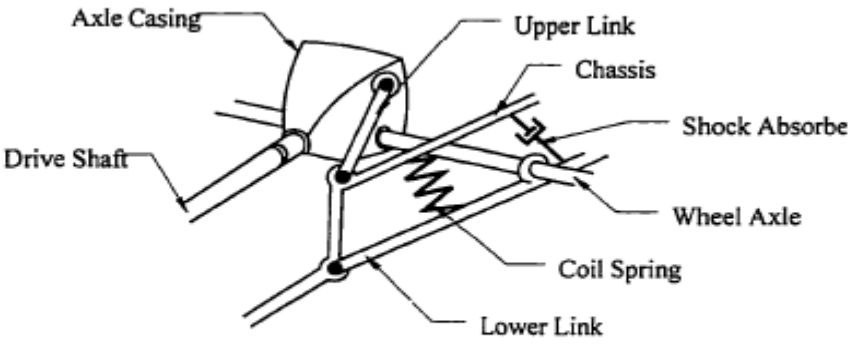


Figure 2.11 Left side of a four- link rear suspension
from"Design of active suspension with a controllable damper" by Chang[40]

The independent suspension is currently used by almost all passenger cars. Because the hopping motion at one wheel will not affect the other wheels, the advantages of independent suspension are reduced steering vibration and enhanced roll stiffness. Compared to a solid-axle suspension, an independent suspension has larger deflection. Consequently, the independent suspension substantially improves ride comforts and handling. In addition, its design which is free of the linkage rigid beam axle spares room for the engine and lowers the front end of the body, further reducing the aerodynamic drag force. Through the geometry design, the roll center is also easily controlled.

The trailing arm suspension [25,40] shown in fig 2.07 was the first the first independent suspension commercially used by German manufacturers. It used a torsion bar as the spring. Later in the United States, the Short-long arm suspension [25,40] shown in fig 2.11 was designed and used. As the name says, the Short-long arm suspension has unequal lengths for its upper and lower arms, a design that improves the chamber of outside wheel while maintaining a favorable camber of inside wheel during cornering. MacPherson adopted a different composition of a shock absorber and a spring [25,27,40] as shown in fig. 2.13.

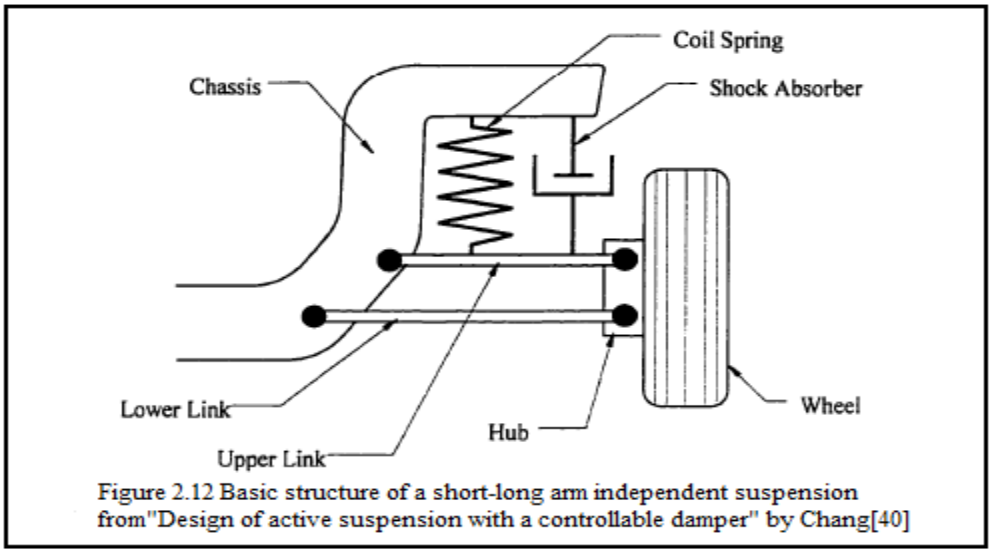


Figure 2.12 Basic structure of a short-long arm independent suspension from"Design of active suspension with a controllable damper" by Chang[40]

The upper end of the MacPherson strut was fixed to the chassis and the lower end is rigidly fixed to the wheel hub. This arrangement sustains the wheel in camber direction. The MacPherson strut also saves space and can easily link with an anti-roll bar. Most of the suspensions currently used by production cars are extension of the SLA suspension of the MacPherson strut. To meet the unique requirements of different vehicles, designers created various layouts of the upper and lower links or added additional two or three links to react to longitudinal and lateral forces [25,27] . The progress of passive suspensions has transcended the initial function of isolating road roughness to overcome the camber change during cornering, dive during braking and braking and squat during acceleration as well as to lower the roll center in avoidance of rollover.

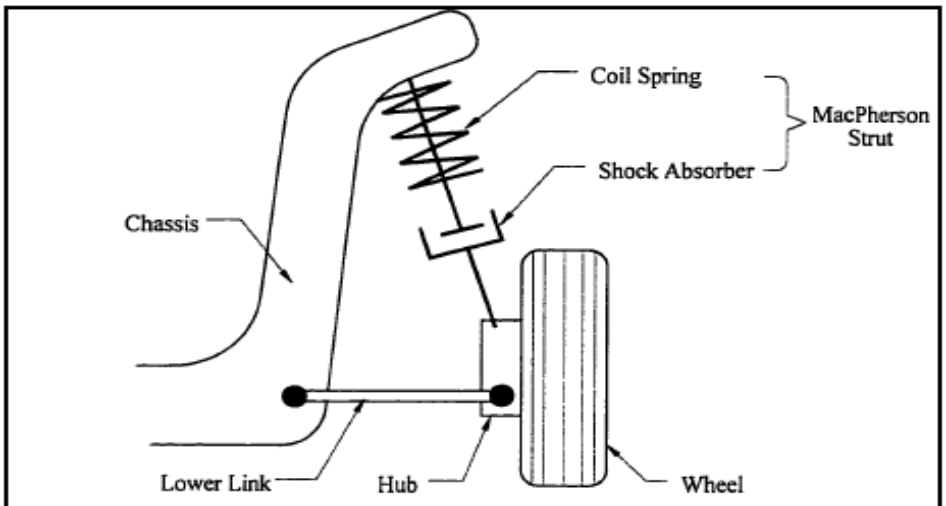


Figure 2.13 Basic structure of a Macpherson strut independent suspension from"Desgin of Active suspensoion witha controllable damper" by Chang [40]

2.4 Active Suspension

In above sections, the passive suspension was introduced starting from the understanding of major excitation sources to the various mechanisms for reducing the effects caused by those excitations. Over the past few decades, suspension performance has been greatly improved with new material, ingenious mechanical design, and advanced analysis techniques. Recently, the advances in control algorithm and electronics launch the suspension design into another direction, that is, the active suspension.

2.4.1 Active Suspension versus Passive Suspension

As mentioned in Chapter 1, a passive suspension has two inherent drawbacks. First, it always needs to tradeoff between ride quality and handling property. Because the vibration of a vehicle is generated by the combination of multiple sources such as acceleration, braking and cornering, it is difficult for a passive suspension with a mechanical design to improve the performance in one aspect without sacrificing the others. Second, resonance occurs at the natural frequency of each passive component used. Fig 2.14 is a typical mechanical model of a passive suspension [40] and the

frequency responses of the sprung mass to disturbance input. In this figure, M_1 is the sprung mass representing the portion of the mass supported by the suspension. M_2 is the unsprung representing the wheel. It is commonly known that the deviation of the sprung mass is proportional to the disturbance frequency until the natural frequency is passed. At high frequency, the sprung mass is isolated from the disturbance by the spring. The amplitude of the resonant peak may be isolated from the disturbance by the spring. The amplitude of

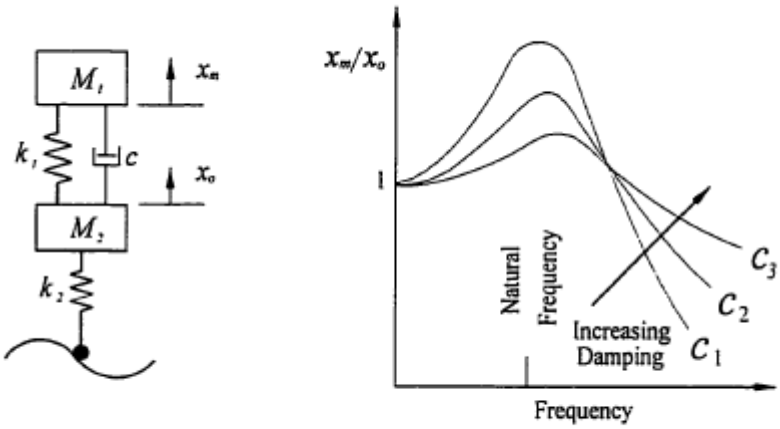


Figure 2.14 A typical passive suspension model and the effects of various damping rates from"Design of Active Suspension with a controllable damper" by Chang[40]

the resonant peak may be reduced by increasing the damping rate, but at high frequency a larger damping rate produces larger deviation. Currently, besides adopting new mechanism, the designers choose the spring and damper according to the function of the vehicle.

By adding an actuator that can inject or extract energy into a suspension system, an active suspension solves the two problems of a passive suspension mentioned in the last paragraph. First, the actuator can respond to various dynamic forces independently. In this way, the compromise between ride and handling qualities can be reduced.

Secondly, the integration of an actuator changes the characteristics of the suspension so that the disturbance response will no longer directly relate to the natural frequency. Thus, the actuator can moderate the motion of the vehicle body to significantly improve both ride and handling performance. The actuator used may be electromechanical, hydraulic, pneumatic or magnetic. Due to the bulky size of the power source, the active suspension was introduced for rail vehicles in early studies. Recently, the advances in control theory and power electronics have attracted academic institutes and industry to focus on applying this technique to passenger cars. In summary, the special features of an active suspension may be described as follows:

1. Unlike the passive suspension which can only temporarily store and dissipate the energy from disturbance, an active suspension can generate force to moderate vehicle body motion.
2. In a passive suspension, the spring and damper can only respond to the local disturbance that exerts force on that suspension. However, an active system can use remote sensors to coordinate the functions of several actuators. In this way, a control function may contain many variables to achieve optimal performance. For examples, the control law may combine information such as vehicle velocity, acceleration, suspension deflection, roll rate gyro, etc.

Along with the active suspension and the passive suspension, the adaptive suspension is also under recent extensive studies. An adaptive suspension carries the body weight by a passive suspension like a passive suspension, but the damping rate can be selected from several preset levels upon the detected chassis motion. Another variation of a passive suspension is the adaptive height control suspension also called self-leveling suspension.

The self leveling suspension also uses a coil spring and a damper like any other passive suspension, but a gas spring is added to adjust the height of the sprung mass. Therefore, the suspension always has maximum working space for deflection despite the changes in load. The gas spring can also lower the vehicle body at high speed to reduce aerodynamic drag force or lift the vehicle body on rough road to increase the ground clearance.

2.4.2 Control Methodologies

The advantages of active suspension are well known. The remaining question is how to control the actuators to achieve the optimal control. Over the years many researchers and car companies are trying to improve on the control of the actuators. Many different techniques have been developed, Linear Quadratic Regulator, Fuzzy Logic, Sliding mode control, Adaptive-Network-based Fuzzy Inference Systems, etc. In the next section of the chapter many different techniques would be explained and compared.

2.5 Literature Review

2.5.1 Introduction

The various applications on active suspensions on vehicles have been researched extensively. Many of the papers focus on Quarter Car Suspension Systems. Some of them use Half Car Suspension systems and some use Full Car suspension systems. Many researchers focused on Passive Suspension Systems, some on Semi-active Suspensions systems and others on Active Suspension systems. A few literatures are considered [29, 31, 32, 37] with Quarter and Half suspension systems and reviewed to analyze the results. Below is a summary of some of the work published on Vehicle Suspension Systems.

2.5.2 Brief Review of Significant Papers

Many papers focus on Active Car Suspension Systems using different control methods and different suspensions models. One of the papers from Yahaya Md, Sam [29] focuses on 'LQR (Linear Quadratic Regulator) Controller for Active Car Suspension' for a Quarter Car model. The Objectives for control scheme in the paper was to improve the ride quality and handling performance within a given suspension stroke limitation. It considers the ride quality to be measured by the vertical acceleration of the vehicle body i.e. Sprung Mass. The handling performance is determined by the tire deflection, which is the difference of position between the wheel and the road surface input. The paper considers a two degree of freedom quarter car model and its state space equations are given as follows:

$$\dot{X} = Ax + Bu + G\delta \qquad (1)$$

$$\begin{bmatrix} \dot{(z2 - zr)} \\ \ddot{z2} \\ \dot{(z1 - z2)} \\ \ddot{z1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k2}{m2} & -\frac{b1}{m2} & \frac{k1}{m2} & \frac{b1}{m2} \\ \frac{b1}{m2} & \frac{k1}{m2} & -\left(\frac{b1+b2}{m2}\right) & -\frac{k2}{m2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z2 - zr \\ \dot{z2} \\ z1 - z2 \\ \dot{z1} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ \frac{m1}{m2} \\ 0 \\ -1 \end{bmatrix} [u] + \begin{bmatrix} -1 \\ \frac{b2}{m2} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \dot{zr} \end{bmatrix} \qquad (2)$$

The Controller design is done with state variable feedback regulator

$$u = - K x$$

where K is the feedback gain matrix.

The optimization procedure consists of determining the control input u, which minimizes the performance index. The performance index is given as

$$J = \int_0^{\infty} (x'Qx + u'Ru)dt$$

The matrix gain K is represented by,

$$K = R^{-1}B'P$$

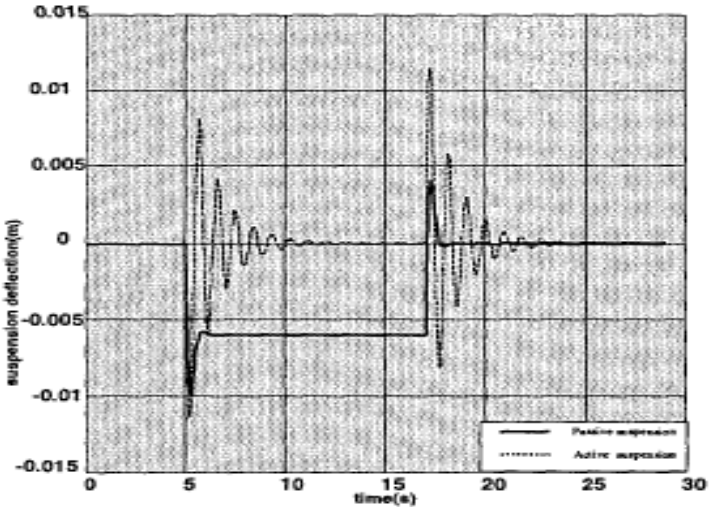
And the matrix P is to satisfy the Riccati equation is

$$A'P + PA - PBR^{-1}BP + Q = 0$$

And then the feedback regulator

$$u = -(R^{-1}BP')\,x$$

Simulation results were used to show Relative suspension displacement, Relative suspension velocity and Relative body velo. The simulation was carried according to the paper in Matlab and the simulation results for the step input are as shown below:



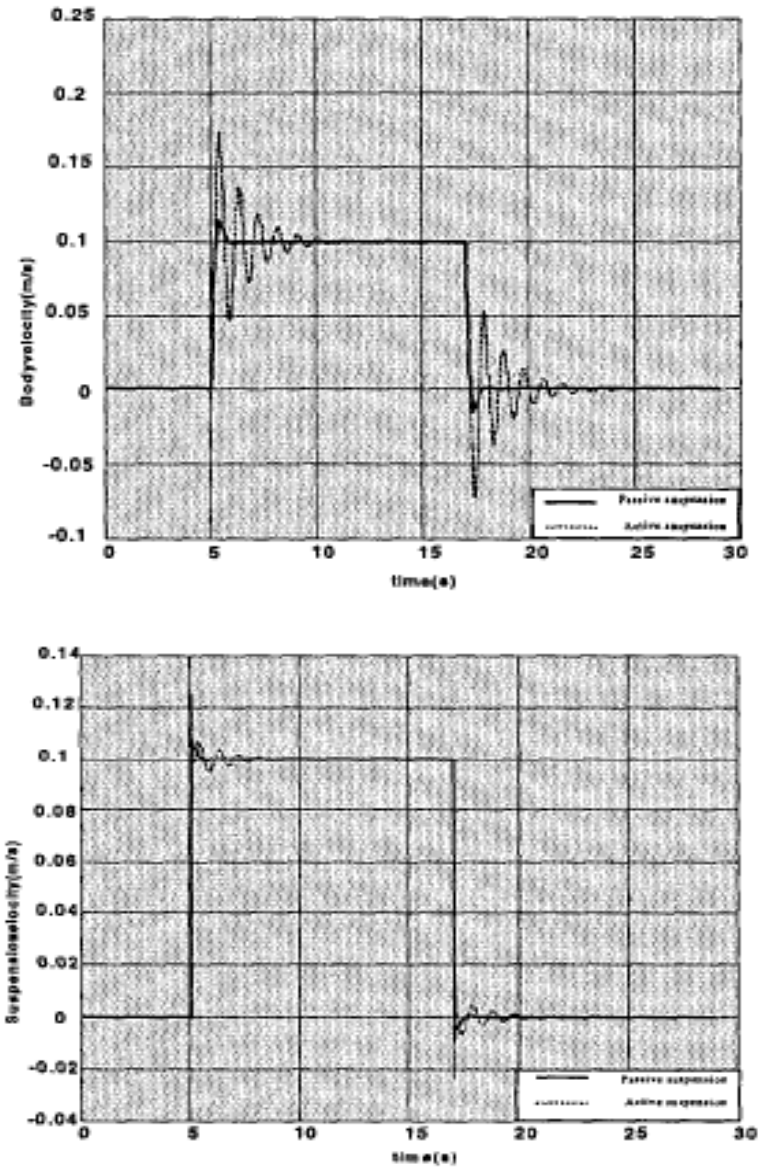


Figure 2.15: Suspension Deflection, Body Velocity and Suspension velocity for the paper by Yahaha[29]

It clearly shows that the control action was not enough to ensure good ride quality as well as good handling.

A paper from form Yahaha, Sam, Halim Shah [31] 'Sliding Mode Control Design for Active Suspension on a Half-Car model' describes sliding mode control for a half car suspension model. The strategy uses proportional integral sliding mode control scheme

and compares it with LQR method. The additional Integral in the proposed sliding mode control provides on more degree of freedom and also reduces the steady state error. According to the paper sliding mode control has relatively simpler structure and it guarantees the system stability. It describes a Half-car model with a Variable structure control design. Variable Structure Control (VSC) utilizes a high speed switching control law to drive the state trajectory on this surface in the state space called the sliding surface and to maintain the state trajectory on this surface for all subsequent time. It requires proper design of the sliding surface for VSC to attain conventional goals of control such as stabilization, tracking and regulation. The paper explains VSC in two steps:

i. The design of the switching surface so that the plant restricted to the sliding surface has desired system response.

ii. The construction of the control scheme to drive the plant's state trajectory to the sliding surface.

The PI sliding surface is defined as follows:

$$\sigma(t) = Cx(t) - \int_0^t (CA + CBK)x(\tau)d\tau$$

where K satisfies $\lambda(A + BK) < 0$ and C is chosen so that CB is nonsingular. If the matrix C is chosen such that CB is nonsingular, this yields the equivalent control of

$$u_{eq}(t) = Kx(t) - (CB)^{-1}Cf(t)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^n$ is the control input, and f(t) the continuous function represents the uncertainties with the mismatched condition.

Using the above equation the system equation is given below:

$$\dot{x}(t) = (A + BK)x(t) + \{I_n - B(CB)^{-1}C)\}f(t)$$

The system is boundedly stable on the sliding surface at σ(t) = 0. The simulation results for two small bumps are as shown below:

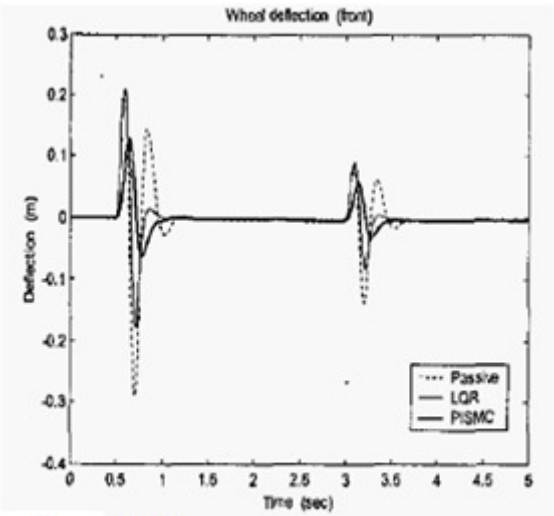


Figure 2.16 (a) The deflection of front wheel

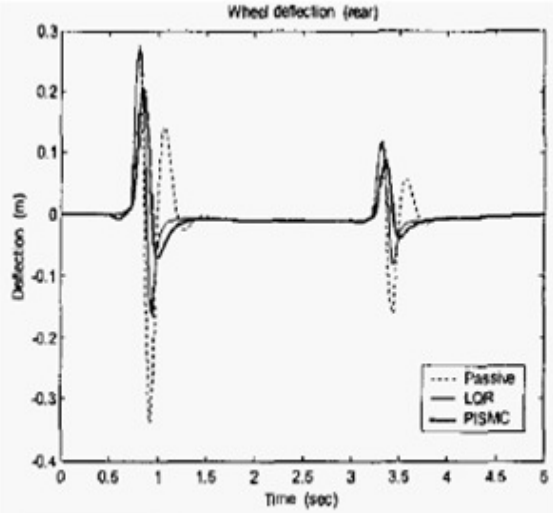


Figure 2.16 (b) . The deflection of rear wheel

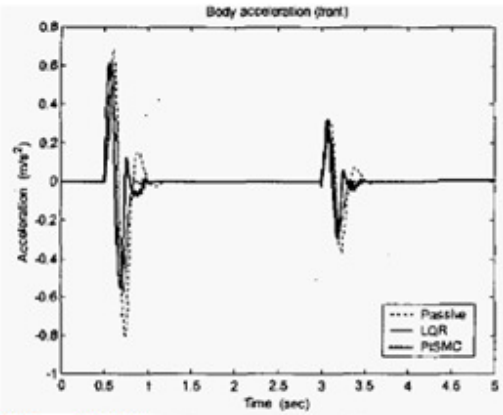


Figure 2.16(c) . The acceleration of front body

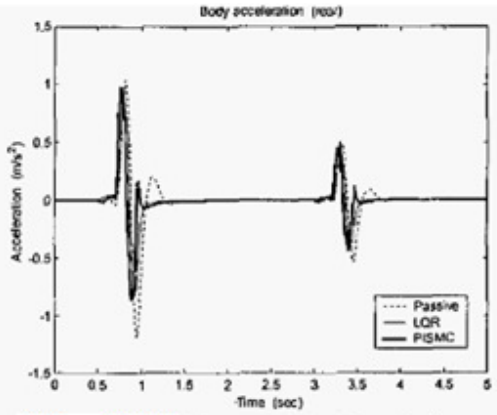


Figure 2.16(d) The acceleration of rear body

The results clearly shows that the vibration caused due to the bumps makes the car unstable as the suspension deflection and acceleration varies a huge amount after the use of sliding mode control. So a better control effort is required to ensure better ride quality and handling quality.

A paper by Chiou and Jung [32],'Nonlinear Active Suspension Control Design applied to a Half-Car Model' describes another method for active car suspension system.

This paper gives a nonlinear backstepping design for the control of Half Suspension system to improve the trade-off between ride quality and suspension travel. The design with two additional nonlinear filters shows potentials to achieve these conflicting control objectives. Two filters are used for both the front and rear wheels. The effective bandwidths of these two filters depend on the magnitudes of the suspensions individually. A suspension model with Front and Rear car suspension model is considered in the approach. The paper focuses not to improve ride quality but to prevent suspension travels from hitting their travel limits. The regulated variables are chosen as $z_1=x_1 - \bar{x_3}$ and $z_3 = x_5 - \bar{x_7}$ where $\bar{x_3}$ & $\bar{x_7}$ are two filtered versions of x_3 & x_7:

$$\dot{\bar{x}}_3 = -(\varepsilon_{01} + \kappa_1\varphi(\zeta_1))(\bar{x}_3 - x_3)$$

$$\dot{\bar{x}}_7 = -(\varepsilon_{02} + \kappa_2\varphi(\zeta_2))(\bar{x}_7 - x_7)$$

where $\zeta_1 = x_1 - x_3$ and $\zeta_2 = x_5 - x_7$. The nonlinear functions $\varphi(\zeta_1)$ and $\varphi(\zeta_2)$ are defined as follows:

$$\varphi(\zeta_1) = \begin{cases} (\frac{\zeta_1 - m_1}{m_2})^2, & \zeta_1 > m_1 \\ 0, & |\zeta_1| \le m_1 \\ (\frac{\zeta_1 + m_1}{m_2})^2, & \zeta_1 < -m_1 \end{cases}$$

$$\varphi(\zeta_2) = \begin{cases} (\frac{\zeta_2 - m_3}{m_4})^2, & \zeta_2 > m_3 \\ 0, & |\zeta_2| \le m_3 \\ (\frac{\zeta_2 + m_3}{m_4})^2, & \zeta_2 < -m_3 \end{cases}$$

It then calculates derivatives of z_1 and z_2 for front wheel and derivatives of z_3 and z_4 for rear wheel. The simulation results are as shown below:

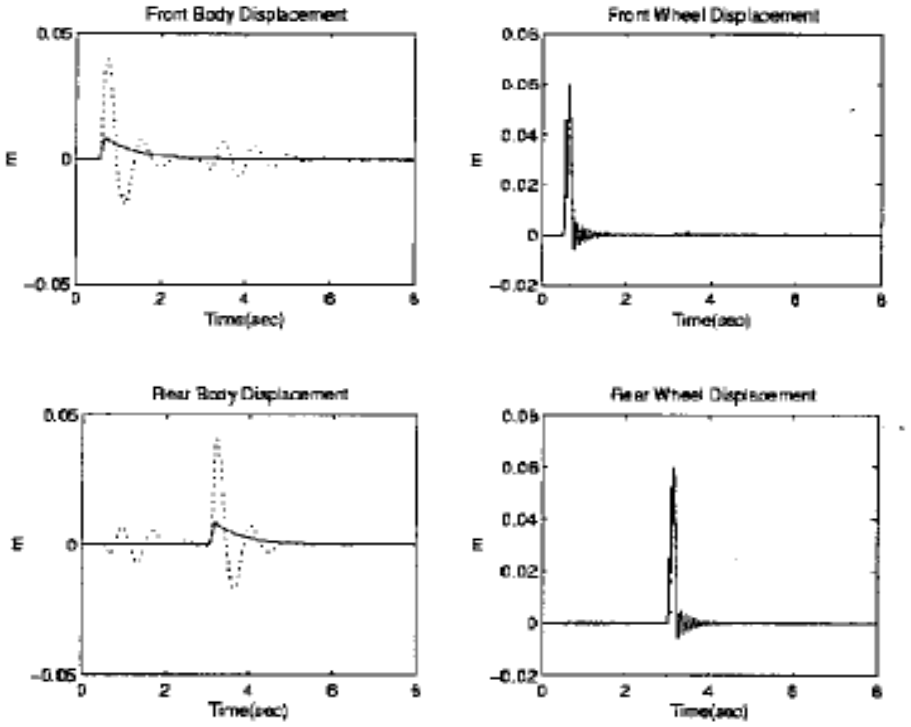


Figure 2.17 Front & Rear Body Displacement for a paper by Chiou[32]

The results clearly show the restriction on the travel of suspension, but the ride quality and handling is affected in an effort to restrict the travel.

A paper by Medhi and team [37] 'Design of Fuzzy Logic and Optimal Control to an Automotive Active Suspension System' shows Fuzzy Logic control implemented for Quarter car model. The paper considers fuzzy logic control as an option for active suspension control and compares it with LQR (Linear Quadratic Regulator) method. It divides the fuzzy logic control method into two parts. The first part is using x_1(relative displacement between two mass) as feedback signal to generate control force for balancing the force generated by passive elements (spring and damper). The second part is using fuzzy inference rules to regulate the sprung mass trajectory. The fuzzy sets are as shown below:

	Variable	Fuzzy set
Input	z_s: Sprung mass position	$Z_S=\{N,Z,P\}$
	$\dot{z}_s$: Sprung mass velocity	$\dot{z}_s=\{NB,NM,NS,Z,PS,PM,PB\}$
output	f_{zs} :The control force	$f_{zs}=\{NB,NM,NS,Z,PS,PM,PB\}$

where N – negative, Z - Zero, P - Positive, NB - Negative Big, NM - Negative Medium, NS - Negative Small, PS - Positive Small, PM - Positive Medium, PB - Positive Big. The fuzzy rules were set according to the expert's knowledge and suspension knowledge and are shown as follows:

Z_S	$\dot{z}_s$	f_{zs}	Z_S	$\dot{z}_s$	f_{zs}
P	NB	NB	P	PS	PB
Z	NB	NB	Z	PS	PS
N	NB	NB	N	PS	NB
P	NM	NS	P	PM	PB
Z	NM	NM	Z	PM	PM
N	NM	NB	N	PM	PS
P	NS	PB	P	PB	PB
Z	NS	NS	Z	PB	PB
N	NS	NB	N	PB	PB
P	Z	PB			
Z	Z	Z			
N	Z	NB			

Table 2.1: Fuzzy Rules

The figure below shows the simulation results in the paper which compares tire deflection between fuzzy control and optimal control.

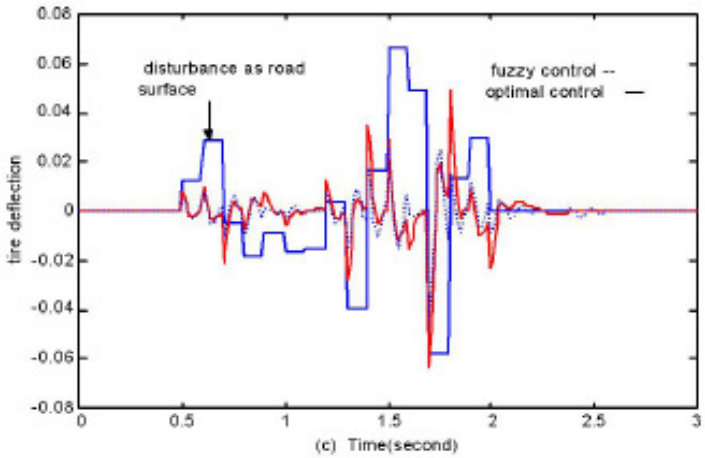


Fig. . Tire deflection with fuzzy control and optimal control for (a): step response. (b): bump response. (c):white noise as irregular road surface.

The result clearly shows that the control effort was not enough to achieve good ride quality and better handling. The main problem with fuzzy logic control was the input variables for fuzzy logic, which ignores the other state variables. It does not generalize for all road irregularities.

Chapter 3: Control Methodologies

Last chapter explained the suspension systems and development over the years in detail. This chapter will explain two different control methodologies in detail, dealt with in the research work.

3.1 Linear Quadratic Regulator (LQR)

The theory of optimal control is concerned with operating a dynamic system at minimum performance measure and optimizing the results. The case where the system dynamics are described by a set of linear differential equations and the performance measure is described by a quadratic functional is called the Linear Quadratic problem. This technique is appropriate when one is given some desired transient performance specifications which may be directly translated into locations for the dominant closed-loop poles. For multi-input, multi-output systems, however, it is not always obvious how to interpret performance specifications in terms of pole locations. The term "linear-quadratic" refers to the linear system dynamics and the quadratic cost function.

For a continuous-time linear system described by

$$\dot{x} = Ax + Bu$$

with a performance measure is defined as

$$J = \frac{1}{2}x^T(T)F(T)x(T) + \int_0^T \left(x^T Q x + u^T R u\right) dt$$

The matrices Q and R are called the state and control penalty matrices, respectively. If the components of Q are chosen large relative to those of R, then deviations of x from zero will be penalized heavily relative to deviations of u from zero. On the other hand, if

the components of R are large relative to those of Q, then control effort will be more costly and the state will not converge to zero as quickly.

The feedback control law that minimizes the value of the cost is

$$u = -Fx$$

where *F* is given by

$$F = R^{-1}B^{T}P$$

and *P* is found by solving the continuous time algebraic Riccati equation.

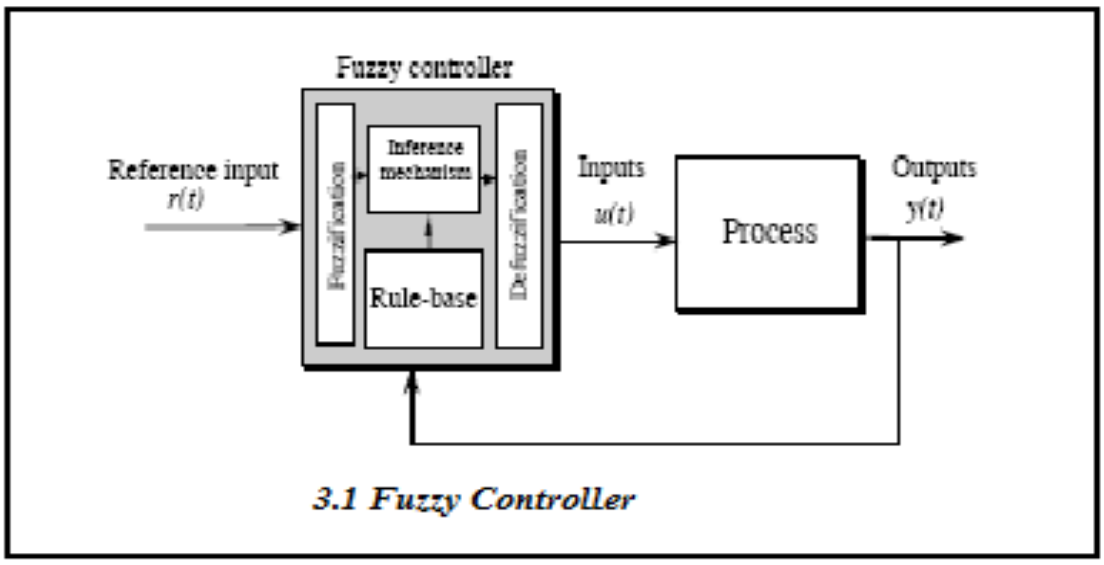
In spite of optimality, there are some problems in practical implementation. The fixed feedback gains that are deduced by LQR cannot accommodate the parameter variations of the process. The road irregularities keeps on changing for a vehicle, a linear quadratic regulator doesn't compensate for the changes. With the constant feedback, it doesn't provide an optimal control for all the road irregularities. Hence a Fuzzy Logic Controller is proposed for the active car suspension system, which can compensate for the constantly changing road conditions as the feedback gain is not fixed with the method.

3.2 Fuzzy Logic

Fuzzy logic technology is one of the AI (Artificial Intelligence) techniques first developed by Lotfi Zadeh in 1965. This technology solves control problems in fuzziness and linguistic way. Fuzzy Logic has rapidly become one of the most successful of today's technologies for developing sophisticated control systems. With its aid, complex requirements may be implemented in amazingly simple, easy maintained and inexpensive controllers. Fuzzy logic has been successfully applied to a large number of diverse applications such as nuclear reactor control, chemical process control, anti-lock braking system for vehicles and many other applications.

The primary goal of control engineering is to distill and apply knowledge about how to control a process so that the resulting control system will reliably and safely achieve high performance operation. Fuzzy logic provides a methodology for representing and implementing our knowledge about how best to control a process.

A block diagram of a fuzzy control system is shown in figure 3.1.



3.1 Fuzzy Controller

The fuzzy controller consists of the following four elements:

1. A rule-base, which contains a fuzzy logic quantification of the expert's linguistic description of how to achieve good control.
2. An inference mechanism, which emulates the expert's decision making in interpreting and applying knowledge about how best to control the plant.
3. A Fuzzification interface, which converts controller inputs into information that the inference mechanism can easily use to activate and apply rules.
4. A defuzzification interface, which converts the conclusions of the inference mechanism into actual inputs for the process.

3.2.1 Choosing Fuzzy Controller inputs and Outputs

The fuzzy controller is to be designed to automate how a human expert who is successful at this task would control the system. First, we need to decide what is to be used as inputs to the decision-making process. We also need to define the output for the system.

3.2.2 Linguistic Variables, Values and Rules

A fuzzy system is static nonlinear mapping between its inputs and outputs. It is assumed that the fuzzy systems has inputs $u_i \in U_i$ where i=1,2,3,……,n and outputs $y_i \in Y_i$ where i=1,2 3,……,n. The inputs and output are "crisps" that is, they are real numbers, not fuzzy sets. The fuzzification block converts the crisp inputs into fuzzy sets, the inference mechanism uses the fuzzy rules in the rule base to produce fuzzy conclusions and the defuzzification block converts these fuzzy conclusions into crisp outputs.

3.2.2.1 Universe of Discourse

The ordinary sets U_i and Y_i are called "Universe of Discourse" for u_i and y_i respectively. Most often universe of discourse is simply the set of real numbers or some interval or subset of real numbers. Universe of discourse is mostly referred to the outermost membership functions saturate for input universe of discourse, or the points beyond which the outputs will not move for the output universe of discourse.

3.2.2.2 Linguistic Variables

To specify rules for the rule-base, the expert will use a "linguistic description"; hence, linguistic expressions are needed for the inputs and outputs and the characteristics of input and outputs. Examples for linguistic variables are Position error, voltage in, etc.

3.2.2.3 Linguistic Values

Linguistic variables take on linguistic values that are used to describe characteristics of the variables. The linguistic value of the linguistic variable is defined over the universe of discourse.

3.2.2.4 Linguistic Rules

The mapping of the inputs to the outputs for a fuzzy system is in part characterized by a set of condition → action rules, or in modus ponens (if-then) form,

If premise **Then** consequent

Usually, the inputs of the fuzzy system are associated with the premise, and the outputs are associated with the consequent. These If-Then rules can be represented in many forms. Two standard forms, multi-input multi-output and multi-input single-output are considered here. The MISO form of linguistic rule is

If u_1 is A_1 and u_2 is A_2 and ……and u_n is A_n Then y is B. (2.1)

It is an entire set of linguistic rules of this form that the expert specifies on how to control the system. We assume that there are total of R rules in the rule-base numbered 1,2,3,….,R and we naturally assume that the rules in the rule base are distinct; however this does not in general need to be the case. For simplicity we have tuples

$(j,k,\ldots\ldots\ldots\ldots l;p,q)_i$

To denote the i^{th} MISO rule of the form given in equation (2.1)

3.2.3 Fuzzy Sets, Fuzzy logic and the Rule - Base

Fuzzy sets and fuzzy logic are used to heuristically quantity the meaning of linguistic variables, linguistic values, and linguistic rules that are required to be specified. The concept of a fuzzy set is introduced by first defining a "membership function".

3.2.3.1 Membership Functions

Let U_i denote a universe of discourse and $A_i^j \epsilon A_i$ denote a specific linguistic value for the linguistic variable u_i^. The function $\mu(u_i)$ associated with A_i^j that maps U_i to [0,1] is called a "membership function". This membership function describes the "certainty" that an element of U_i denoted u_i, with a linguistic description $u_{i'}$ may be classified linguistically as A_i^j . There are many choices of membership function possible such as Triangular, Bell, Gaussian, etc. The most common is Gaussian Membership function as

Table 3.1 Mathematical Characteristic of Gaussian Membership Function

	Gaussian membership functions
Left	$\mu^L(u) = \begin{cases} 1 & \text{if } u \le c^L \\ \exp\left(-\frac{1}{2}\left(\frac{u-c^L}{\sigma^L}\right)^2\right) & \text{otherwise} \end{cases}$
Centers	$\mu(u) = \exp\left(-\frac{1}{2}\left(\frac{u-c}{\sigma}\right)^2\right)$
Right	$\mu^R(u) = \begin{cases} \exp\left(-\frac{1}{2}\left(\frac{u-c^R}{\sigma^R}\right)^2\right) & \text{if } u \le c^R \\ 1 & \text{otherwise} \end{cases}$

3.2.3.2 Fuzzy Sets

Given a linguistic variable u_i with a linguistic value A_i^j defined on the universe of discourse U_i and a membership function $\mu(u_i)$ that maps U_i to [0,1], a fuzzy set denoted with A_i^j is defined as

$$A_i^j = \{(u_i, \mu_{A_i^j}(u_i)) : u_i \in U_i\}$$

3.2.3.3 Fuzzy Logic

We specify some set theory and logical operations on fuzzy sets. Fuzzy sets results from the following operations

3.2.3.3.1 Fuzzy Subset

Given fuzzy sets A_i^1 and A_i^2 associated with the universe of discourse U_i with membership functions denoted $\mu(u_1)$ and $\mu(u_2)$ respectively, A_i^1 is defined to a fuzzy subset of A_i^2 denoted by $A_i^1 \subset A_i^2$ for all $u_i \in U_i$

3.2.3.3.2 Fuzzy Complement

The complement of a fuzzy set A_i^j with a membership function $\mu(u_i)$ has a membership function given by $1-\mu(u_i)$.

3.2.3.3.3 Fuzzy Intersection

The intersection of fuzzy sets A_i^1 and A_i^2 associated with the universe of discourse U_i with membership functions denoted $\mu(u_1)$ and $\mu(u_2)$ respectively, A_i^1 is defined to a fuzzy subset of A_i^2 denoted by $A_i^1 \cap A_i^2$ for all $u_i \in U_i$.

3.2.3.3.4 Fuzzy Union (OR)

The intersection of fuzzy sets A_i^1 and A_i^2 associated with the universe of discourse U_i with membership functions denoted $\mu(u_1)$ and $\mu(u_2)$ respectively, A_i^1 is defined to a fuzzy subset of A_i^2 denoted by $A_i^1 \cup A_i^2$ with membership function defined by either one of the following methods:

1. Maximum: Here, we find the maximum of the membership values as in

$$\mu_{Ai1 \cup Ai2}(u_i) = \max \{ \mu_{Ai}^1(u_i), \mu_{Ai}^2(u_i),: u_i \epsilon U_i \}$$

2. Average: Here, we find the algebraic sum of the membership value as in

$$\mu_{Ai1 \cup Ai2}(u_i) = \{ \mu_{Ai}^1(u_i) + \mu_{Ai}^2(u_i) - \mu_{Ai}^1(u_i)\, \mu_{Ai}^2(u_i): u_i \epsilon U_i \}$$

3.2.3.3.5 Fuzzy Cartesian product

The intersection and union above are both defined for fuzzy sets that lie on the same universe of discourse. The fuzzy Cartesian product is used to quantify operations on many universes of discourse. If A_1^i, A_2^j,..... An^1 are fuzzy sets defined on the universe of discourse U_1, U_2,……U_n respectively, their Cartesian product is a fuzzy set denoted by

$A_1^i x\ A_2 x.....x\ An^1$ with a membership function defined by

$$\mu_{Ai1*Ai2\ldots*Ain}(u_i) = \{\mu_{A1}^1(u_i) * \mu_{A2}^2(u_i) * ……..*\mu_n^1(u_i)\}$$

'*' arises from the interpretation of a standard Cartesian product, which is formed by taking an element from the first element of the product "and' the second element from the first element of the product "and" so on.

3.2.3.4 Fuzzy Quantification of rules: Fuzzy Implications

We consider the If-Then rules for the MISO system in equation 2.1. The fuzzy sets are defined as follows:

$A_1^j = \{(u_1, \mu_{A1}^j(u_1):u_1 \epsilon U_1\}$

$A_2^k = \{(u_2, \mu_{A2}^k(u_2):u_2 \epsilon U_2\}$

.

.

.

$An^l = \{(u_n, \mu_{An}^l(u_n):u_n \epsilon U_n\}$

$B_q^p = \{(y_q, \mu_{Bq}^p(u_q):u_q \epsilon U_q\}$

These fuzzy sets quantify the terms in the premise and the consequent of the given. If-Then rule, to make a "fuzzy implication"

If A_1^j and A_2^k and …….A_n^1 Then B_q^p

where the fuzzy sets are defined above. Each rule forms a rule base, which we denote by $(j,k,……..,l,p,q)_I$,i=1,2,3,……,R, is represented with such a fuzzy implication.

3.2.4 Fuzzification

Fuzzy sets are used to quantify the information in the rule-base and the inference mechanism operates on fuzzy sets to produce fuzzy sets; hence, we must satisfy how the fuzzy system will convert its numeric inputs $u_i \epsilon U_i$ into fuzzy sets so that they can be used by the fuzzy system. The following section is a rephrase from "Fuzzy Control" by Passino, K and Yurkovich.

Let U_i^* denote the set of all possible fuzzy sets that can be defined on U_i. Given $u_i \in U_i$, fuzzification transforms u_i to a fuzzy set denoted by $A_i^{\wedge fuz}$ defined on the universe of discourse U_i. This transformation is produced by the fuzzification operator F defined by

$$F: U_i \rightarrow U_i^*$$

where

$$F(u_i) = A_i^{\wedge fuz}$$

Quite often "singleton fuzzification" is used, which produces a fuzzy set $A_i^{\wedge fuz} \in U_i^*$ with a membership function defined by

$$\mu_{A_i^{fuz}}(x) = \begin{cases} 1 & x = u_i \\ 0 & \text{otherwise} \end{cases}$$

Any fuzzy set with this form for its membership function is called a "singleton". Singleton fuzzification is generally used in implication since, without the presence of noise, we are absolutely certain that u_i takes on its measured value and since it provides certain savings in the computations needed to implement a fuzzy system. The reasons other fuzzification methods have not been used very much are (1) they add computational complexity to the inference process and (2) the need for them has not been that well justified. This is partly due to the fact that very good functional capabilities can be achieved with the fuzzy system when only singleton fuzzification is used.

3.2.5 The Inference Mechanism

The inference mechanism has two basic tasks: (1) determining the extent to which each rule is relevant to the current situation as characterized by the inputs u_i, i=0,1,2,……..,n and (2) drawing conclusions using the current inputs u_i and the

information in the rule-base. For matching note that $A_1^i x\ A_2 x.....x\ An^l$ is the fuzzy set representing the premise of the i^{th} rule $(j,k,\ldots\ldots,l;p,q)_i$

3.2.5.1 Matching

Suppose that at some time we get inputs u_i , i = 1,2,….n and fuzzification procedures

$$A_1^{\wedge fuz}, A_2^{\wedge fuz} \ldots A_n^{\wedge fuz}$$

the fuzzy sets representing the inputs. There are then two basic steps to matching.

Step 1: Combine Inputs with Rule Premises: The first step in matching involves finding fuzzy sets $A_1^i, A_2^j,.....\ An^l$ with membership functions

$$\mu_{A1^i}(u_1) = \mu_{A1^i}(u_1) * \mu_{A1^{\wedge fuz}}(u_1)$$

$$\mu_{A2^j}(u_2) = \mu_{A2^j}(u_2) * \mu_{A2^{\wedge fuz}}(u_2)$$

.

.

$$\mu_{An^l}(u_n) = \mu_{An^l}(u_n) * \mu_{An^{\wedge fuz}}(u_n)$$

that combine the fuzzy sets from fuzzification with fuzzy sets used in each of the terms in the premises of the rules. If singleton fuzzification is used, then each of these fuzzy sets is a singleton that is scaled by premise membership function $\mu_{A1^i}(\underline{u}_1) = \mu_{A1^i}(u_1)$ for $\underline{u}_1 = u_1$ and $\mu_{A1^i}(\underline{u}_1) = 0$ for $\underline{u}_1 \neq u_1$. That is, with singleton fuzzification we have $\mu_{A1^{\wedge fuz}}(u_i) = 1$ for all i=1,2,….,n for the given u_i inputs so that

$$\mu_{A1^i}(u_1) = \mu_{A1^i}(u_1)$$

$$\mu_{A2^j}(u_2) = \mu_{A2^j}(u_2)$$

.

.

$$\mu_{An^l}(u_n) = \mu_{An^l}(u_n)$$

We see that when singleton fuzzification is used, combining the fuzzy sets that were created by the fuzzification process to represent the inputs with the premise membership functions for the rules is particularly simple.

Step 2: Determine which rules are on: In second step, we form membership values $\mu_{i\,(}u_1, u_2, u_3, \ldots\ldots, u_n)$ for i^{th} rules premise that represent the certainty that each rule premise holds for the given inputs. Define

$$\mu_{i\,(}u_1, u_2, u_3, \ldots\ldots, u_n) = \mu_{A1^i}(u_1) * \mu_{A2^j}(u_2) * \ldots\ldots * \mu_{An^l}(u_n)$$

this is simply a function of the function of the inputs u1. When singleton fuzzification is used we have

$$\mu_{i\,(}u_1, u_2, u_3, \ldots\ldots, u_n) = \mu_{A1^i}(u_1) * \mu_{A2^j}(u_2) * \ldots\ldots * \mu_{An^l}(u_n)$$

We use $\mu_{i\,(}u_1, u_2, u_3, \ldots\ldots, u_n)$ to represent the certainty that the premise of rule i matches the input information when we use singleton fuzzification. This $\mu_{i\,(}u_1, u_2, u_3, \ldots\ldots, u_n)$ is simply a multidimensional certainty surface. This concludes the process of matching input information with the premises of the rules.

3.2.5.2 Inference Step

There are two standards alternatives to performing the inference step, one that involves the use of implied fuzzy sets and the other that uses the overall implied fuzzy set.

Alternative 1: Determine implied fuzzy sets

Next, the inference step is taken by computing, for the i^{th} rule $(j,k,\ldots\ldots,l;p,q)_i$, the implied fuzzy sets B_q^p with membership

$$\mu_{Bq}^{p}(y_q) = \mu\,(u_1, u_{2,\ldots\ldots\ldots\ldots}, u_n) * \mu_{Bq}^{p}(y_q)$$

The implied fuzzy set B_q^p specifies the certainty level that the output should be a specific crisp output y_q within the universe of discourse Y_q, taking into consideration only rule i. Note that since $\mu_i(u_1, u_{2,\ldots\ldots\ldots\ldots}, u_n)$ will vary with time, so will the shape of the membership functions $\mu_{Bq}^{p}(y_q)$ for each rule.

Alternative 2: Determine the overall implied Fuzzy Set

Alternatively, the inference mechanism could, in addition compute the overall implied fuzzy set B_q with membership function

$$\mu_{Bq}^{p}(y_q) = \mu_{Bq}^{1}(y_q)\ \Theta\ \mu_{Bq}^{2}(y_q)\ \Theta\ \ldots\ldots\ldots\Theta\ \mu_{Bq}^{n}(y_q)$$

that represents the conclusion reached considering all the rules on the rule base at the same time. Center-average defuzzification method performed the aggregation of the conclusions of all the rules that are represented by the implied fuzzy sets.

3.2.6 Defuzzification

A number of defuzzification strategies exist and it is not hard to invent more. Each provides a means to choose a single based on either the implied fuzzy sets or the overall implied fuzzy set.

3.2.6.1 Implied Fuzzy Sets

As they are common, we first specify typical defuzzification techniques for the implied fuzzy sets B_q^i:

3.2.6.1.1 Center of Gravity (COG)

A crisp output y_q^{crisp} is chosen using the center of area and area of each implied f fuzzy set and is given by

$$y_q^{\text{crisp}} = \frac{\sum_{i=1}^{R} b_i^q \int_{y_q} \mu_{\hat{B}_q^i}(y_q) dy_q}{\sum_{i=1}^{R} \int_{y_q} \mu_{\hat{B}_q^i}(y_q) dy_q}$$

Where R is the number of rules, b_i^q is the center of area of the membership function of B_q^p associated with the implied fuzzy set B_q^i for i^{th} rule $(j,k,\ldots..,l;p,q)_i$ and

$$\int_{y_q} \mu_{\hat{B}_q^i}(y_q) dy_q$$

denotes the area under $\mu_{Bq}^{i}(y_q)$. Notice that COG can be easy to compute since it is often easy to find closed-form expressions for $\int_{yq} \mu_{Bq}^{i}(y_q)\, dy_q$, which is the area under a membership function.

3.2.6.1.2 Center of Average

A crisp output y_q^{crisp} is chosen using centers of each of the output membership functions and the maximum certainty of each of the conclusion represented with the implied fuzzy sets, and is given by

$$y_q^{\text{crisp}} = \frac{\sum_{i=1}^{R} b_i^q \sup_{y_q}\{\mu_{\hat{B}_q^i}(y_q)\}}{\sum_{i=1}^{R} \sup_{y_q}\{\mu_{\hat{B}_q^i}(y_q)\}}$$

where "sup" denotes the 'supremum" (i.e. the least upper bound which can often be thought of as the minimum value). Hence $\sup_x\{\mu(x)\}$ can simply be thought of as the highest value of $\mu(x)$. Also, b_i^q is the center of area of the membership function of B_q^p associated with the implied fuzzy set B_q^i for the i^{th} rule $(j,k,\ldots..,l;p,q)_i$. Notice that a the fuzzy system must be defined so that

$$\sum_{i=1}^{R} \sup_{y_q} \{\mu_{B_q^i}(y_q)\} \neq 0$$

for all u_i. Also note that $\sup_{yq}\{\ \mu_{\ Bq}^{\ i}(y_q)\}$ is often very easy to compute since if $\mu_{Bq}^{\ i}(y_q)=1$ for at least one y_q, then for many inference strategies, we have

$$\sup_{y_q} \{\mu_{B_q^i}(y_q)\} = \mu_i(u_1, u_2, \ldots, u_n)$$

which has already been computed in the matching process. Moreover, the formula for defuzzification is then given by

$$y_q^{\text{crisp}} = \frac{\sum_{i=1}^{R} b_i^q \mu_i(u_1, u_2, \ldots, u_n)}{\sum_{i=1}^{R} \mu_i(u_1, u_2, \ldots, u_n)}$$

where we must ensure that the denominator is not equal to zero for all u_i. Also note that this implies that the shape of the membership functions for the output fuzzy sets does not matter; hence you simply use singleton centered at the appropriate positions.

3.2.6.2 The Overall Implied Fuzzy Set

We present typical defuzzification techniques for overall implied fuzzy set B_q:

3.2.6.2.1Max Criterion

A Crisp output y_q^{crisp} is chosen as the point on the output universe of discourse Y_q foe which the overall implied fuzzy set B_q achieves a maximum-that is

$$y_q^{\text{crisp}} \in \left\{ \arg\sup_{y_q} \left\{ \mu_{\hat{B}_q}(y_q) \right\} \right\}$$

Here, "$\text{argsup}_x\{\mu(x)\}$" returns the value of x that results in the supremum of the function $\mu(x)$being achieved. Sometimes the supremum can occur at more than one point in Y_q. In this case you also need to specify a strategy on how to pick only one point for y_q^{crisp}.

3.2.6.2.2 Mean of Maximum

A crisp output y_q^{crisp} is chosen to represent the mean value of all elements whose membership in B_q is a maximum. We define b_q^{max} as the supremum of the membership function of B_q is a maximum. We define b_q^{max} as the supremum of the membership function of B_q over the universe of discourse Y_q. Moreover, we define a fuzzy set $B_q\hat{}$ ϵ Y_q with a membership function defined as

$$\mu_{\hat{B}_q}(y_q) = \begin{cases} 1 & \mu_{\hat{B}_q}(y_q) = b_q^{\max} \\ 0 & \text{otherwise} \end{cases}$$

The a crisp output, using the mean of maximum method, is defined as

$$y_q^{\text{crisp}} = \frac{\int_{y_q} y_q \mu_{\hat{B}_q}(y_q) dy_q}{\int_{y_q} \mu_{\hat{B}_q}(y_q) dy_q}$$

where the fuzzy system must be defined so that denominator is not equal to zero for all u_i.

Note that the integrals in the above equations must be computed at each time instant since they depend on B_q, which changes with time. This can require excessive computational resources for continuous universe of discourse.

3.2.6.2.3 Center of Area

A crisp output y_q^{crisp} is chosen as the center of area for the membership function of the overall implied fuzzy set B_q. For a continuous output universe of discourse Y_q, the center of area output id denoted by

$$y_q^{crisp} = \frac{\int_{y_q} z_q \mu_{B_q}(z_q) dz_q}{\int_{y_q} \mu_{B_q}(z_q) dz_q}$$

The fuzzy system must be defined so that the denominator is not equal to zero for all u_i. Note that, similar to the mean of the maximum method, this defuzzification approach can be computationally expensive. For instance, we leave it to reader to compute the area of the overall implied fuzzy set $\mu_{overall}(u) = \max_u\{\mu_1(u), \mu_2(u)\}$. Notice that in this case the computation is not easy as just adding the areas of the two chopped –off triangles that represent the implied fuzzy sets. Computation of the area of the overall implied fuzzy set does b\not count the area that the implied fuzzy sets overlap twice; hence the area of the overall implied fuzzy set can in general be much more difficult to compute in real time.

3.2.7 Mathematical Representations of Fuzzy Systems

Each formula for defuzzification in the previous section provides a mathematical description of a fuzzy system. There are many ways to represent the operations of a fuzzy system with mathematical formulas. Center-average defuzzification is used for MISO fuzzy systems, and similar defuzzification strategies are used for MIMO fuzzy systems.

Assume that center-average defuzzification so that the formula describing computation of the output is

$$y = \frac{\sum_{i=1}^{R} b_i \mu_i}{\sum_{i=1}^{R} \mu_i}$$

3.2.8 Mamdani's Method

Mamdani's fuzzy inference method is the most commonly seen fuzzy methodology. The following section is a rephrase from "Introduction to Fuzzy Logic using Matlab" by Deepa, Sivanandam, Sumathi. Mamdani's method was among the first control system built using fuzzy set theory. It was proposed by Mamdani in 1975 as an attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators.

Mamdani type inference, as defined it for the Fuzzy logic toolbox, expects the output membership functions to be fuzzy sets. After the aggregation process, there is a fuzzy set for each output variable that needs defuzzification. An example of a Mamdani inference system is shown in figure 3.2.

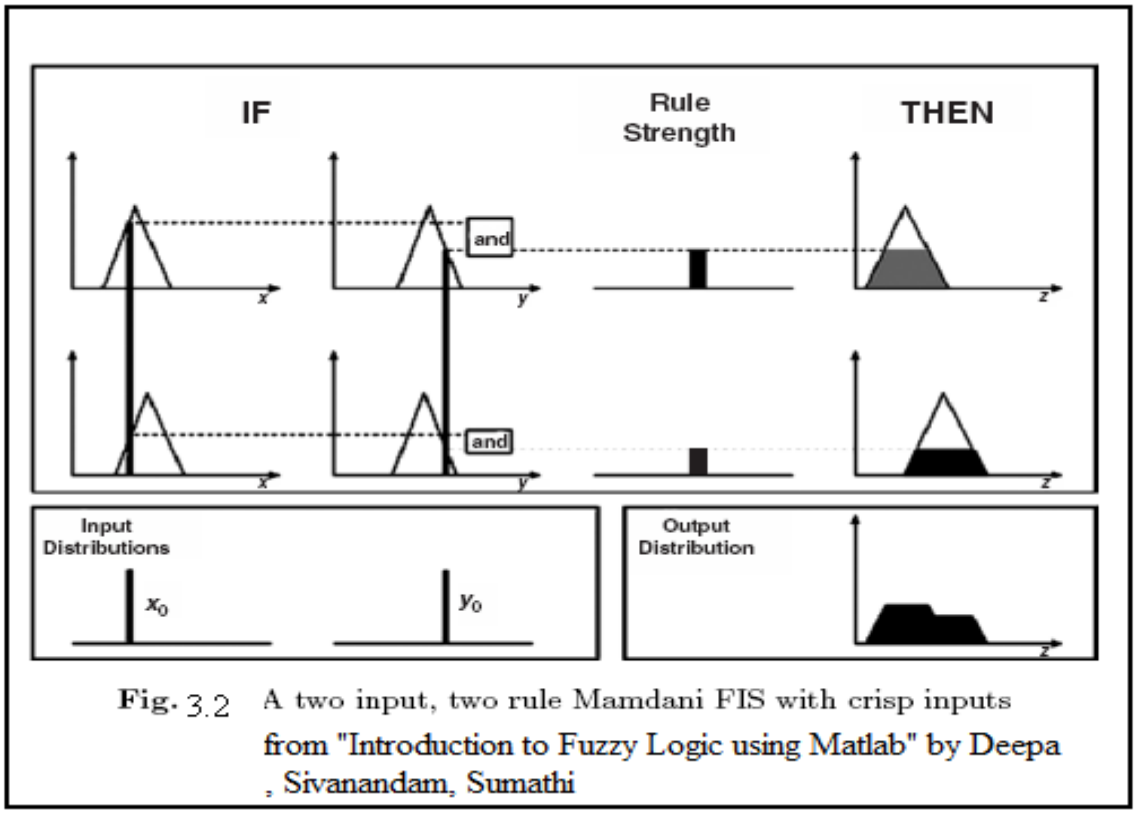


Fig. 3.2 A two input, two rule Mamdani FIS with crisp inputs from "Introduction to Fuzzy Logic using Matlab" by Deepa , Sivanandam, Sumathi

To compute the output of this FIS given the input, six steps have to be followed:

1. Determining a set of fuzzy rules.
2. Fuzzyfying the input membership functions
3. Combining the fuzzified inputs according to the rule strength and the output membership function.
4. Finding the consequence of the rule by combining the rule strength and the output membership function.
5. Combining the consequences to get an output distribution.
6. Defuzzifying the output distribution.

3.2.9 Takagi-Sugeno Fuzzy Model

The basic of Sugeno fuzzy model is implemented into the neural fuzzy system. The Sugeno fuzzy model was proposed by Takagi Sugeno and Kang in an effort to formalize a system approach to generating fuzzy rules from an input-output data set. Sugeno fuzzy model is also known as Sugeno-Takagi model. A typical fuzzy rule in a Sugeno fuzzy model has the format

$$\text{If x is A and y is B THEN } z = f(x,y)$$

where A, B are fuzzy sets in the antecedent; Z=f(x,y) is a crisp function in the consequent. Usually f(x,y) is a polynomial in the input variables x and y, but it can be any other functions that can appropriately describe the output pf the output of the system within the fuzzy region specified by the antecedent of the rule. When f(x,y) is a first order polynomial, we have the first order Sugeno fuzzy model. When f is a constant, we have the zero-order Sugeno fuzzy model, which can be viewed either as a special case of the Mamdani FIS where each rule's consequent is specified by a fuzzy singleton, or a special case of Tsukamoto's fuzzy model where each rule's consequent is specified by a membership function of a step function centered at the constant. Moreover, a zero-order Sugeno fuzzy model is functionally equivalent to a radial basis function network under certain minor constraints.

The first two parts of the fuzzy inference process, fuzzyfying the inputs and applying the fuzzy operator, are exactly the same. The main difference between Mamdani and Sugeno is that the Sugeno output membership functions are either linear or constant. A typical rule in a Sugeno fuzzy model has the form

If input 1 = x AND 2 = y, THEN output is z = ax+by+c

For a Zero order Sugeno model, the output level is a constant. The output level z_i of each rule is weighted by firing strength w_i of the rule. The final output of the system is the weighted average of all rule outputs, computed as

$$\text{Final output} = \frac{\sum_{i=1}^{N} w_i z_i}{\sum_{i=1}^{N} w_i}.$$

A Sugeno rule operates as shown in figure 3.3

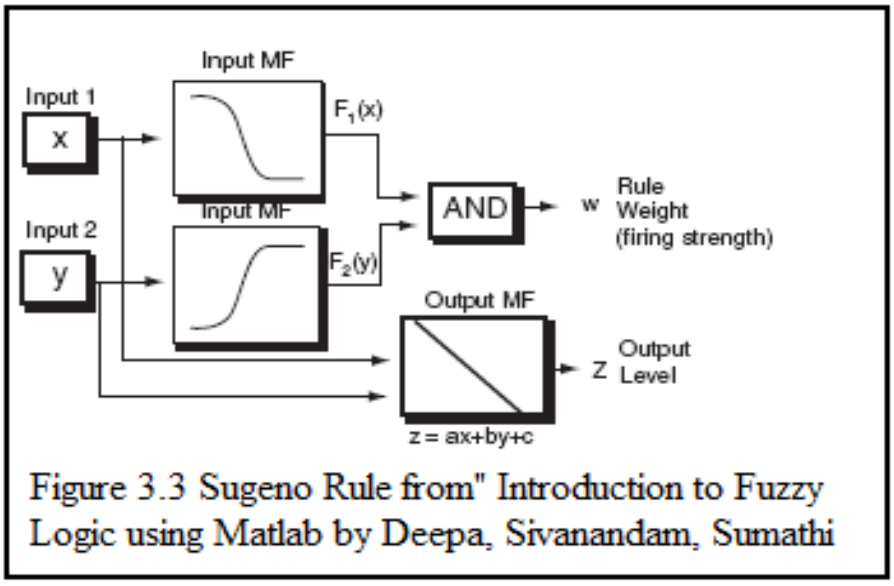


Figure 3.3 Sugeno Rule from" Introduction to Fuzzy Logic using Matlab by Deepa, Sivanandam, Sumathi

Higher-order Sugeno fuzzy models are possible, but they introduce significant complexity with little obvious merit. Sugeno fuzzy models whose output membership functions are greater than first-order are not supported by the Fuzzy Logic Toolbox. Because of the linear dependence of each rule on the input variables of a system, the Sugeno method is ideal for acting as an interpolating supervisor of multiple linear controllers that are applied, respectively, to different operating conditions of a dynamic nonlinear system. A Sugeno FIS is extremely well suited to the task of smoothly

interpolating the linear gains that would be applied across the input space; it is natural and efficient gain scheduler. Similarly a Sugeno system is suited for modeling non linear systems by interpolating between multiple linear models.

3.2.10 Comparison between Sugeno and Mamdani Method

The main difference between Mamdani and Sugeno is that the Sugeno output membership functions are either linear or constant. Also the difference lies in the consequents of their fuzzy rules, and thus their aggregation and defuzzification procedure differs suitably. The number of the input fuzzy sets and fuzzy rules needed by the Sugeno fuzzy systems depend on the number and locations of the extrema of the function to be approximated. In Sugeno method a large number of fuzzy rules must be employed to approximate periodic or highly oscillatory functions. The minimal configuration of the TS fuzzy systems can be reduced and becomes smaller than that of the Mamdani fuzzy systems in non trapezoidal or non triangular input fuzzy sets are used. Far fewer mathematical results exist for TS fuzzy controllers than do for Mamdani fuzzy controllers, notable those on TS fuzzy control system stability.

3.2.11 Advantages of Mamdani Method

- It is intuitive.
- It has widespread acceptance.
- It is well suited to human input.

3.2.12 Advantages of Sugeno Model

- It is computationally efficient.

- It works well with linear techniques.
- It works well with optimization and adaptive techniques.
- It has guaranteed continuity of the output surface.
- It is well suited to mathematical analysis.

Chapter 4: Design of Front Axle Car Suspension Model

In this chapter we will consider the front axle car suspension model represent in figure 4.1. A front axle car suspension model is considered to investigate the behavior of car suspension for different disturbances on left and right side of the car. The design variables are the stiffness and damping coefficient of the suspension modules.

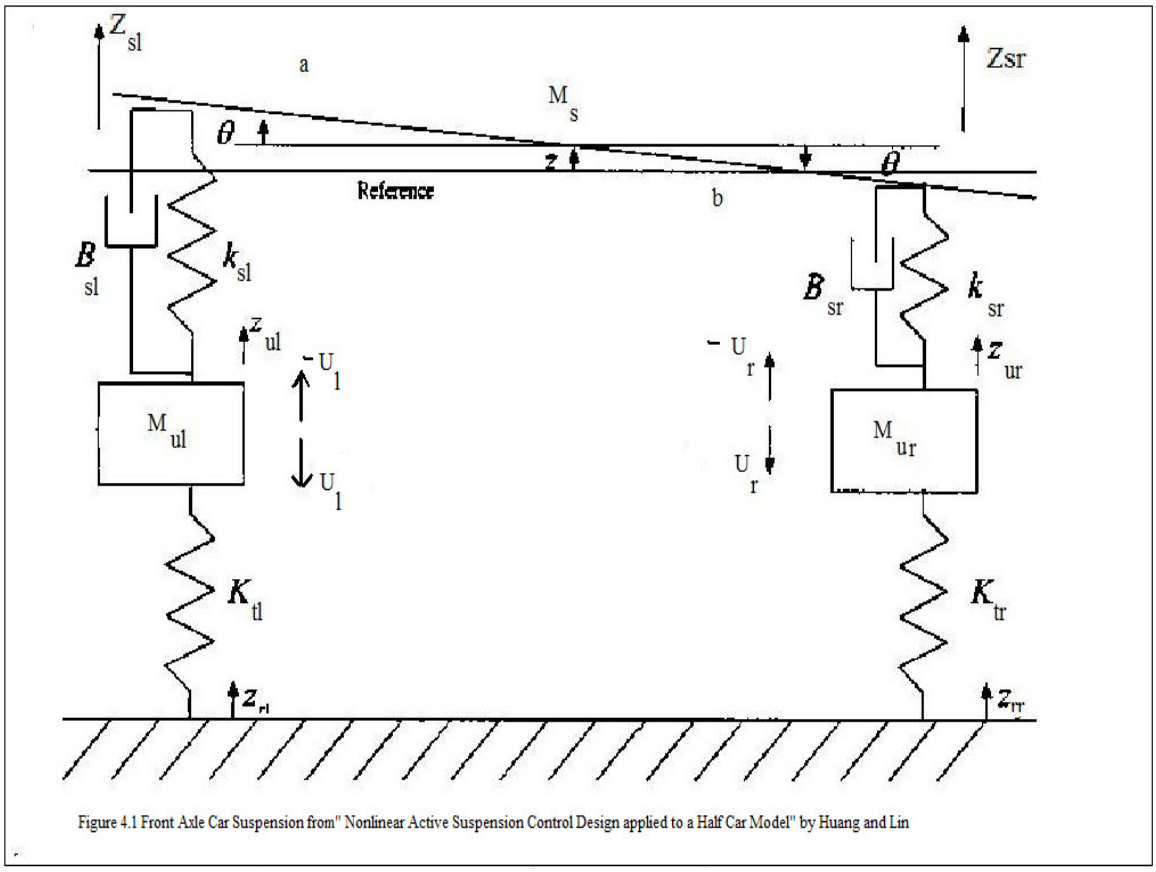


Figure 4.1 Front Axle Car Suspension from" Nonlinear Active Suspension Control Design applied to a Half Car Model" by Huang and Lin

4.1 Model Description

The model of a front axle car suspension [32,33] is shown in figure 4.1. The front axle suspension model is represented as a nonlinear four degree of freedom system, which has heave, pitch and the motion of the left and right wheels of the front body of the car. The suspension system is considered to be Independent Suspension System. It

consists of a sprung mass (car body) connected to two unsprung masses (left and right front wheels) at each corner. The sprung mass is free to heave and pitch, while the unsprung masses are free to bounce vertically with respect to the sprung mass. The suspensions between the sprung mass and unsprung masses are modeled as linear viscous dampers and spring elements, while the tires are modeled as simple springs without damping components. It is assumed that the tires don't leave the ground.

It is considered that the model of car is subjected irregular excitation from the road surface as shown in figure 4.1. The sprung mass of the vehicle is denoted by M_s, unsprung masses are denoted by M_{ul} & M_{ur} for left and right respectively. The road irregularities are denoted as $\dot{Z}_{rl}$ & $\dot{Z}_{rr}$ respectively. J_x is the half pitch of Moment of Inertia. The actuators which are used for the control action are connected in parallel with the springs and dampers between the unsprung mass (M_{ul} & M_{ur}) and sprung mass (M_s). The force generated by the actuators is considered to be acting towards the road i.e. acting downward from the sprung mass.

From figure 4.1, the displacement for the sprung mass are given by

Left Wheel

$$Z_{sl} = z - a \sin\boldsymbol{\theta} \approx z - a\boldsymbol{\theta} \tag{4.1}$$

Right Wheel

$$Z_{sr} = z - b \sin\boldsymbol{\theta} \approx z - b\boldsymbol{\theta} \tag{4.2}$$

where Z_{sl} is the left body sprung mass displacement, Z_{sr} is the right body sprung mass displacement, a is the distance between the left suspension and center of gravity, b is the distance between the right suspension and center of gravity, $\boldsymbol{\theta}$ is the pitch angle and z is

the displacement of the center of gravity. Both z_{sl} & z_{sr} are measured with respect to the road level. The displacements z_{sl} & z_{sr} are considered to be acting upwards i.e moving away from the road level. The equivalent forces in both wheels are given by

Left Wheel

$$U_l = -k_{sl} (z_{sl} – z_{ul}) – B_{sl} (\dot{z}_{sl} – \dot{z}_{ul}) - u_l \quad (4.3)$$

Right Wheel

$$U_r = -k_{sr} (z_{sr} – z_{ur}) – B_{sr} (\dot{z}_{sr} – \dot{z}_{ur}) - u_r \quad (4.4)$$

where u_l and u_r are the left and right force inputs, B_{sl} and B_{sr} are the left and right damping coefficients, k_{sl} and k_{sr} are the left and right spring coefficient and z_{ul} and z_{ur} are the left and right unsprung mass displacements. The displacements z_{ul} & z_{ur} are measured with respect to the road level. Both the displacements z_{ul} & z_{ur} are considered to be acting upwards i.e moving away from the road level.

By applying Newton's second law and using the static equilibrium position as the origin for both the displacement of the center of gravity and angular displacement of the vehicle body, the equations of the motion for the system can be formulated. The equations of motion for heave is

Left Body

$$M_s \ddot{z}_{sl} = -k_{sl} (z_{sl} – z_{ul}) – B_{sl} (\dot{z}_{sl} – \dot{z}_{ul}) -k_{sr} (z_{sr} – z_{ur}) – B_{sr} (\dot{z}_{sr} – \dot{z}_{ur}) - u_r - u_l + a M_s \ddot{\theta} \quad (4.5)$$

Right Body

$$M_s \ddot{z}_{sr} = -k_{sl} (z_{sl} – z_{ul}) – B_{sl} (\dot{z}_{sl} – \dot{z}_{ul}) -k_{sr} (z_{sr} – z_{ur}) – B_{sr} (\dot{z}_{sr} – \dot{z}_{ur}) - u_r - u_l - b M_s \ddot{\theta} \quad (4.6)$$

And the equation of motion for pitch is

$J_x \ddot{\theta} = - U_l a + U_r b$

$$= ak_{sl} (z_{sl} - z_{ul}) + aB_{sl} (\dot{z}_{sl} - \dot{z}_{ul}) - au_l - bk_{sr} (z_{sr} - z_{ur}) - bB_{sr} (\dot{z}_{sr} - \dot{z}_{ur}) + bu_r \quad (4.7)$$

where M_s is the mass of the car body and J_x is the centroidal moment of Inertia.

Also centroidal moment of Inertia [32] is given as $J_x = M_s r_x^2$ (4.8)

where r_x is the radius of gyration,

Applying Newton's second law again on the front and rear wheel unsprung masses, the equation of motion can also be formulated as follows:

Left Wheel

$$M_{ul} \ddot{z}_{ul} = k_{sl} (z_{sl} - z_{ul}) + B_{sl} (\dot{z}_{sl} - \dot{z}_{ul}) - k_{tl}(z_{ul} - z_{rl}) + u_l \quad (4.9)$$

Right Wheel

$$M_{ur} \ddot{z}_{ur} = k_{sr} (z_{sr} - z_{ur}) + B_{sr} (\dot{z}_{sr} - \dot{z}_{ur}) - k_{tr}(z_{ur} - z_{rr}) + u_r \quad (4.10)$$

where M_{ul} and M_{ur} are the unsprung masses on the left and right wheels and k_{tl} and k_{tr} are the left and right tire spring coefficients. z_{rl} & z_{rr} denotes the road roughness for both left wheel and right wheel respectively. To model the road input, the vehicle is moving with constant forward speed. Then the vertical velocities z_{rl} & z_{rr} can be modeled as white noise, which is true for most real roadways.

4.2 State Space Representation

A state space representation of the front axle car suspension is more desirable for control analysis and design purposes.

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

where $x \in R^n$ is the state vector, $\dot{x}$ its derivative & $u \in R^n$ is the input vector, A, B,C, D are the state space matrices and $y \in R^n$ is the output vector.

It is important to note that the state space representation is not unique. There are several state vectors and A,B,C,D matrix representations that yield the same input-output relationship may not change, some representations have useful physical interpretations while others are more suitable for analysis and design.

Let the state variables be defined as

- $x_1 = z_{sl} - z_{ul}$, Left Body Sprung Mass Displacement
- $x_2 = \dot{z}_{sl}$, Absolute Velocity of the Sprung Mass Left Side
- $x_3 = z_{ul} - z_{rl}$, Left Body Unsprung Mass Displacement
- $x_4 = \dot{z}_{ul}$, Left Body Unsprung Mass Absolute Velocity
- $x_5 = z_{sr} - z_{ur}$, Right Body Sprung Mass Displacement
- $x_6 = \dot{z}_{sr}$, Absolute Velocity of the Sprung Mass Right Side
- $x_7 = z_{ur} - z_{rr}$, Right Body Unsprung Mass Displacement
- $x_8 = \dot{z}_{ur}$, Right Body Unsprung Mass Absolute Velocity

The state equation of motion can be written as follows:

$$\dot{x} = Ax + Bu + D\dot{z}_r$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ -\boldsymbol{\alpha}\, k_{sl} & -\boldsymbol{\alpha}\, B_{sl} & 0 & \boldsymbol{\alpha}\, B_{sl} & \boldsymbol{\beta}\, k_{sr} & \boldsymbol{\beta}\, B_{sr} & 0 & -\boldsymbol{\beta}\, B_{sr} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ K_{sl}/M_{ul} & B_{sl}/M_{ul} & -K_{tl}/M_{ul} & -B_{sl}/M_{ul} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ \boldsymbol{\beta}\, k_{sl} & \boldsymbol{\beta}\, B_{sl} & 0 & -\boldsymbol{\beta}\, B_{sl} & -\boldsymbol{\gamma} k_{sr} & -\boldsymbol{\gamma} B_{sr} & 0 & \boldsymbol{\gamma} B_{sr} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & k_{sr}/M_{ur} & B_{sr}/M_{ur} & -K_{tr}/M_{ur} & -B_{sr}/M_{ur} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ -\alpha & \beta \\ 0 & 0 \\ 1/M_{ul} & 0 \\ 0 & 0 \\ \beta & -\gamma \\ 0 & 0 \\ 0 & 1/M_{ur} \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}$$

where

$\alpha = [(a^2/ J_x) + (1/M_s)]$

$\beta = [(ab/J_x) - (1/M_s)]$

$\gamma = [(b^2/J_x) + (1/M_s)]$

Chapter 5: Fuzzy Logic Controller Design

Chapter 3 explains fuzzy logic in detail. In this chapter we would discuss how we are using two Fuzzy Logic Controllers for Front Axle suspension control. The following section is a rephrase from "Fuzzy and Adaptive fuzzy control for the automobiles active suspension system" by A.B.Sharkawy. As universal function approximators, fuzzy systems can be implemented in varieties of ways for any control problem. Essentially, the designer should make sure that the controller would have the proper information available to make good decisions. Therefore there are many choices for the proper control inputs so that the controller is able to steer the system in directions needed to achieve good performance.

As opposed to optimal control theory, fuzzy logic control has been considered by many authors [1,37] as an alternative control methodology to active suspension control system. The kernel of the fuzzy logic controller is a set of linguistic control rules which capture human thinking and organize the general reasoning to determine the control rules such satisfactory performance under different road profiles can be achieved. The procedure of this fuzzy control process can be described as shown in figure 5.1

5.1 Step I: Fuzzify the non-fuzzy variables into the fuzzy variables

For the non-fuzzy variables (Suspension Deflections, Suspension Velocities & Control Forces), linguistic values are encoded into fuzzy sets. These values are used to describe control procedure. We denote the non-fuzzy variables by the 5 fuzzy sets, that is Negative Large (NL), Negative Small (NS), Zero (Z), Positive Small (PS) and Positive Large (PL). They are denoted in the finite universes of discourse, Left Side Suspension

Deflection $z_{sl} - z_{ul}$, Left Side Suspension Velocity $\dot{z}_{sl} - \dot{z}_{ul}$, Left Suspension Control u_l, Right Side Suspension Deflection $z_{sr} - z_{ur}$, Left Side Suspension Velocity $\dot{z}_{sr} - \dot{z}_{ur}$ and

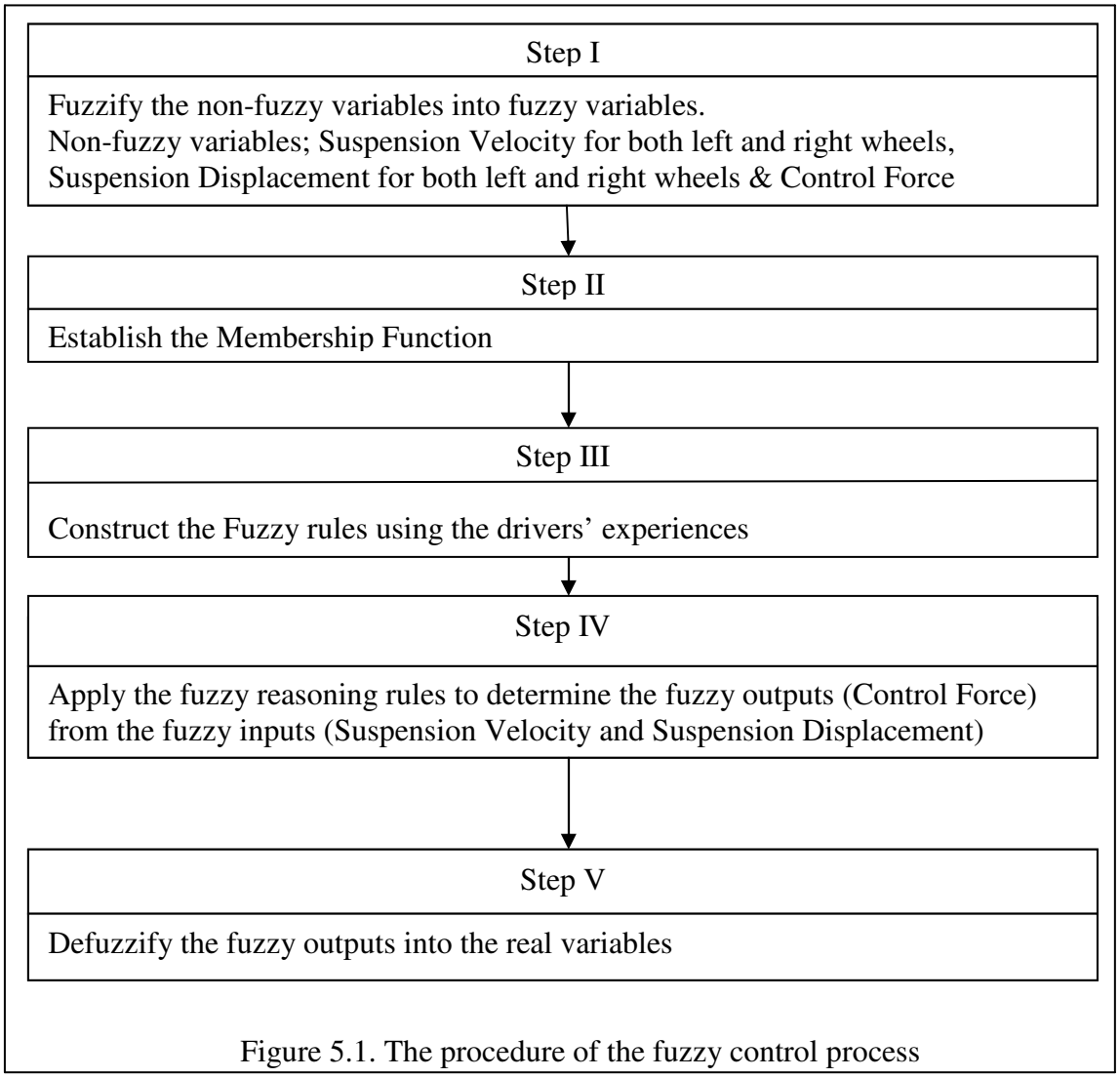


Figure 5.1. The procedure of the fuzzy control process

Right Suspension Control u_r. Most of the papers on fuzzy logic control for vehicle suspension, example Kuo and Li [32] use inputs as sprung mass velocity $\dot{z}_s$ and unsprung mass velocity $\dot{z}_u$ and the actuating force is the output. Their selection, which ignores the other state variables, was based on some characteristics of the system dynamics, like frequency response, which cannot be generalized to all road irregularities.

5.2 Step II: Establish the membership functions of the fuzzy variables

To determine the membership functions of the fuzzy variables, we can use the fuzzy statistical approach by the aid of a computer. According to the experts and some study on the vehicle suspension control fuzzy sets are given membership functions. Counting the fraction of positive responses found in the total number of responses, we obtain the value of the membership function of this element of the universe of discourse. Gaussian membership function with σ =1 is selected for inputs of Suspension deflections and Suspension velocities. Suspension deflections are equally space in the universe of discourse [-2.5, 2.5] as shown in figure 5.2. Suspension velocities are equally spaced in the universe of discourse [-5.0, 5.0] as shown in figure 5.3. The Gaussian membership function is specified by two parameters {c,σ}:

$$\mu_{A_j^i}(x_j) = \text{gaussian}(x_j; c, \sigma) = \exp\left[-\frac{1}{2}\left(\frac{x_j - c}{\sigma}\right)^2\right]$$

where c represents the membership function's centre and σ determines its width.

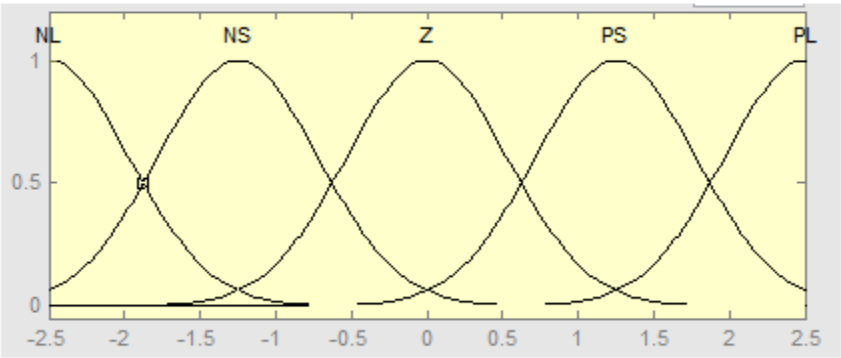


Figure 5.2 Suspesnsion Displacement

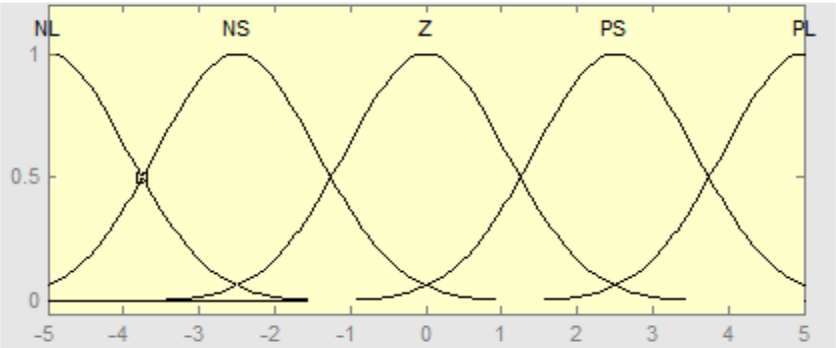


Figure 5.3 Suspension Velocity

For the Output Control forces a membership function with single spike is used. In many cases it is much more efficient to use a single spike as the output membership functions rather than a distributed fuzzy set. This is sometimes known as a singleton output membership function, and it can be thought as pre-defuzzified fuzzy set. It enhances the efficiency of defuzzification process because it greatly simplifies the computation required by the more general Mamdani method, which finds the centroid of a two-dimensional function. Rather than integrating across the two-dimensional function to find the centroid, the weighted average of a few data points is used in this method. The output is divide in a universe of discourse [-150 150] as shown in figure 5.4.

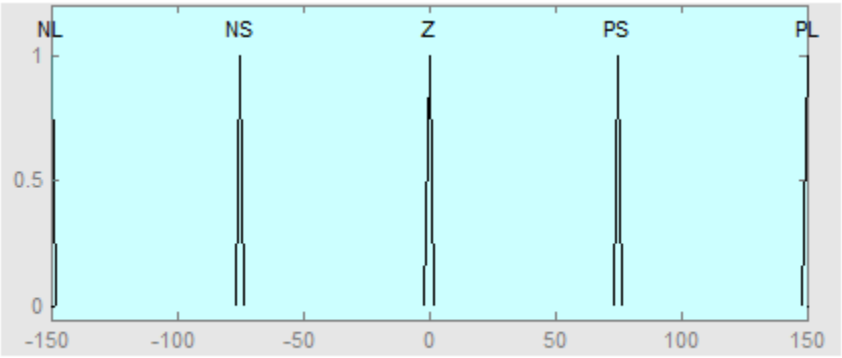


Figure 5.4 Output Control Force

5.3 Step III: Establish the Fuzzy Reasoning Rules

The most important advantage of the fuzzy basis functions is that a linguistic IF-THEN rule is directly related to a fuzzy basis function expansion providing a natural frame work to combine both numerical information and linguistic information in a uniform fashion.

A Fuzzy Logic Controller consists of a collection of L fuzzy IF-THEN rules in the following form:

Rule^{l} = IF x_1 is $A_1{}^{l}{}_{1}$ and ……and x_m is $A^{l}{}_{m}$ THEN u is $\boldsymbol{\theta}^{l}$

where l =1,2,……,l is the rule number, x_j (j = 1,2……,m) ϵ U and u:U $\subset R^n \rightarrow R$ are respectively, the input and output variables $A_j{}^{i}$s are the antecedent linguistic terms in the rule l; $\boldsymbol{\theta}^{l}$s are labels of the rule conclusion being, for that type of rule, a real number called 'fuzzy singleton'. The conclusion, a numerical value not a fuzzy set, can be considered as 'pre-defuzzified' output. According to the suspension design, the knowledge of the behavior of the vehicle suspension and from Sharkawy [1], the fuzzy rules for reasoning of Control force consist of 25 rules. The rules are described as follows:

Rule 1: IF ($z_s - z_u$ = NL) AND ($\dot{z}_s - \dot{z}_u$ = NL) THEN u = NL

Rule 2: IF ($z_s - z_u$ = NL) AND ($\dot{z}_s - \dot{z}_u$ = NS) THEN u = NL

..

..

Rule 25: IF ($z_s - z_u$ = PL) AND ($\dot{z}_s - \dot{z}_u$ = PL) THEN u = PL

Table 5.1 shows the fuzzy reasoning rules for Vehicle Suspension System. Same rules will be used for both Left and Right Side for Car Suspension System.

Control U		**Suspension Deflection ($z_s - z_u$)**				
		NL	NS	Z	PS	PL
Suspension Velocity ($\dot{z}_s - \dot{z}_u$)	NL	NL	NL	NL	NS	PL
	NS	NL	NL	NS	Z	PS
	Z	NL	NS	Z	PS	PL
	PS	NS	Z	PS	PL	PL
	PL	Z	PS	PL	PL	PL

Table 5.1 Fuzzy Reasoning Rule Base

After the rules are set for the fuzzy reasoning the plot for the output surface as per the rules can be given as shown in figure 5.5

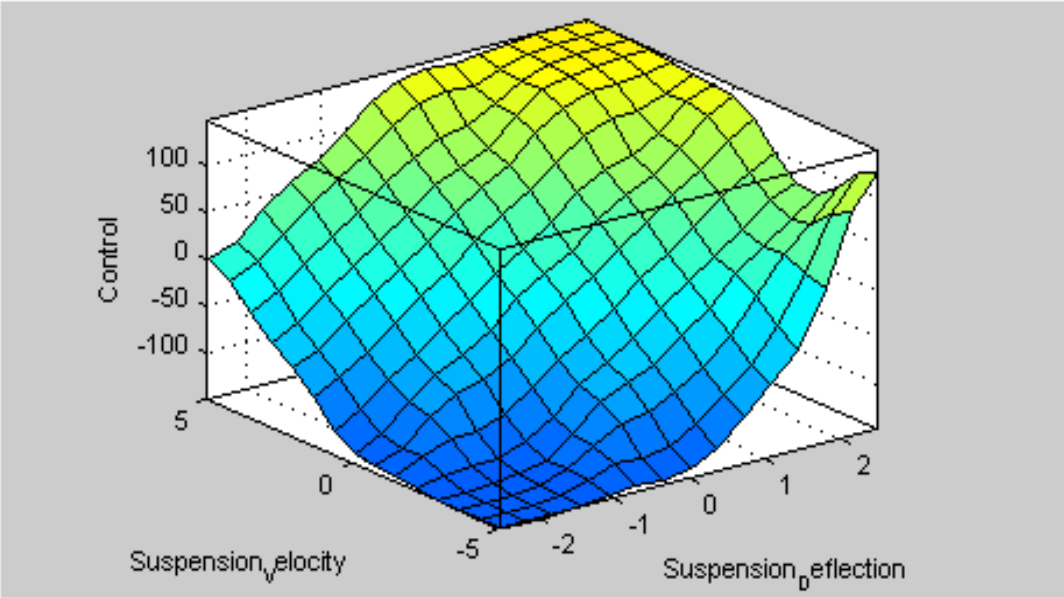


Figure 5.5 Output Surface

5.4 Step IV: Apply Fuzzy Reasoning Process

In this study, Mamdani's method is used for the fuzzy reasoning process since it is better suited to human input as compared to Sugeno's method. Then, the output control force $\mu_1(u), \mu_2(u),\ldots\ldots, \mu_{25}(u)$ is calculated for each rules.

5.5 Step V: Defuzzify the fuzzy outputs into the real variable

Defuzzification maps output fuzzy sets defined over an output universe of discourse to crisp outputs. After the fuzzy reasoning process, we obtain a group of $\mu(u)$. Center- average defuzzifier is used for the output i.e. control force. The control force generated after the defuzzification is used by the actuator. The fuzzy rules are reduced into the following fuzzy logic system:

$$u(x) = h_o \frac{\sum_{l=1}^{L} \theta^l \prod_{j=1}^{m} \mu_{A_j^l}(x_j)}{\sum_{l=1}^{L} \prod_{j=1}^{m} \mu_{A_j^l}(x_j)}$$

where $h_0 > 0$ is a tuning gain and $\mu_{Aj}{}^{i}(x_j)$ is the membership grade of the input x_j in the membership functions A^i_j of rule l and $\boldsymbol{\theta}^l$'s free parameters.

Chapter 6: Simulations and Results

In the last chapter fuzzy logic control scheme for active car suspension system is explained in details. This chapter considers the simulation results and is used for comparisons. Simulations are generated using Matlab Simulink software. The performance of Fuzzy Logic Controller is compared to optimal LQR method. Active car suspension system using fuzzy logic is compared to passive suspension systems. The simulations will be carried under different road irregularities.

The numerical values for the model parameters [32] are as follows:

$M_s = 575$ kg

$M_{ul} = M_{ur} = 60$ kg

$J_x = 531$ kg/m^2

$K_{sl} = k_{sr} = 16812$ N/m

$a = 0.765$ m; $b = 0.77$ m

$k_{tl} = k_{tr} = 190000$ N/m

$B_{sl} = B_{sr} = 1000$ N/m

6.1 Proposed Front Axle Car Suspension System using Fuzzy Logic Controllers

The proposed model in the research is as shown in figure 6.0. Using the values for all the parameters it was simulated in Matlab-Simulink. Different fuzzy logic controllers are proposed for both left and right wheels of the front axle suspension control system. Control signal was amplified using a gain of 100 to increase the range of control signal. The basic goal for the research work proposed is to design a suspension which can compensate for different road disturbances on both the side of the vehicle as well.

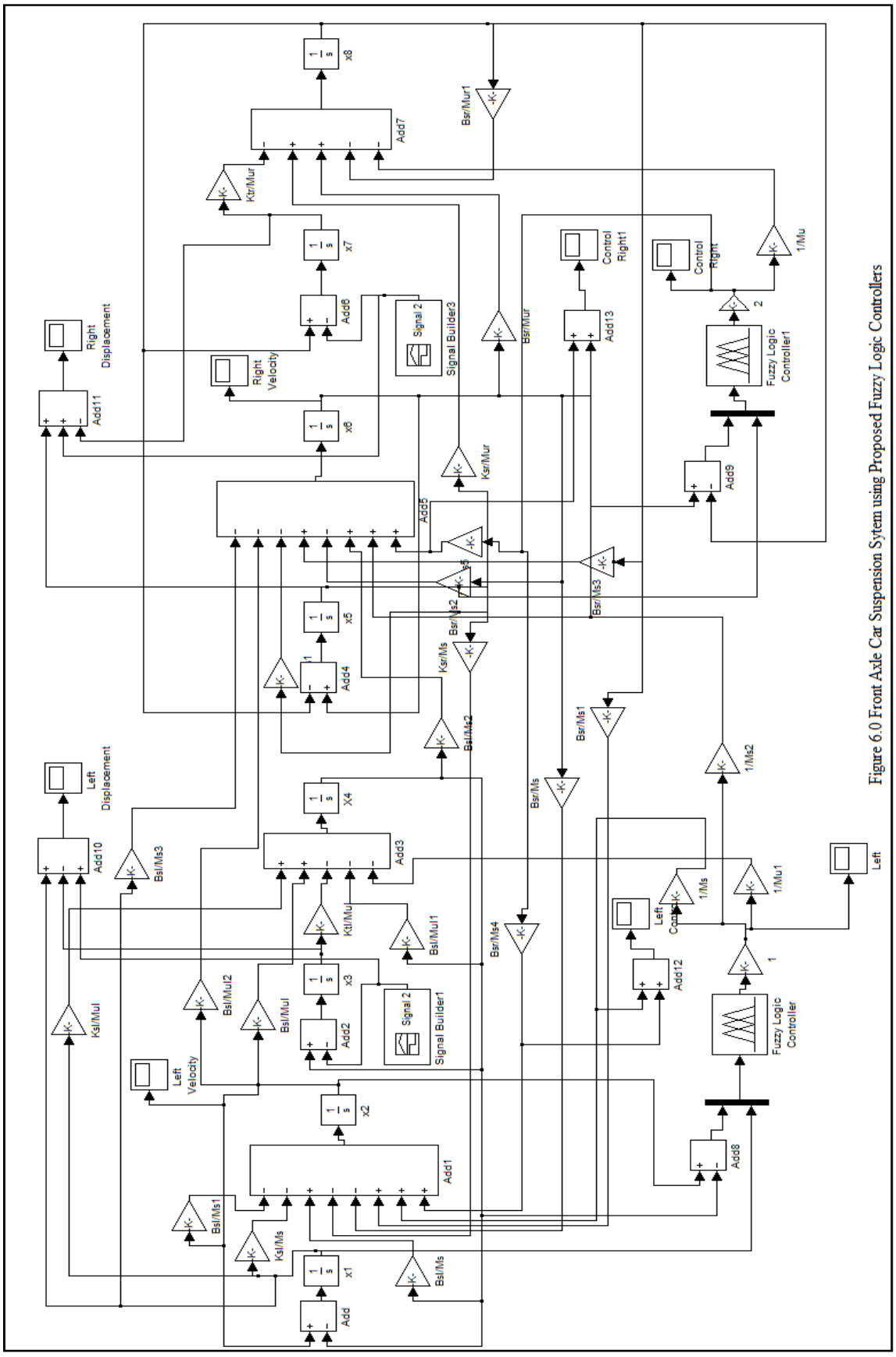


Figure 6.0 Front Axle Car Suspension Sytem using Proposed Fuzzy Logic Controllers

For the purpose front axle car suspension is used to investigate different road disturbances on both side of the vehicle. Car Suspension is an area of research for many years and researchers have focused on many different aspects of the car suspension system but there is no research which investigates the problem of different road disturbances on both sides of the car. The research proposed in this thesis investigates the problem of different road disturbances on both sides of the car and proposes a model which can compensate for such disturbances without compromising much on ride quality and handling. Fuzzy Logic controllers are proposed in the research to be used as a control element to control the actuator used for generating the control force. The focus is to minimize sprung mass travel which ensures much better ride quality and better handling. In order to achieve good ride quality and handling the proposed system is very useful.

6.2 Simulation I: Active Car Suspension Systems vs Passive Car Suspension Systems

The major difference between an active suspension and a passive suspension is the additional actuator. Due to the long history of hydraulic applications in automotive industry, researches tend to adopt the hydraulic actuators in developing their active-suspension systems. Hydraulic actuators have the ability to generate large force. However, a hydraulic system is bulky in size and adds maintenance problems. Pneumatic systems offer an alternative potential. With the advantages of pneumatic actuator, the cost and its availability due to the systems such as air bags and brake system, pneumatic actuator is selected.

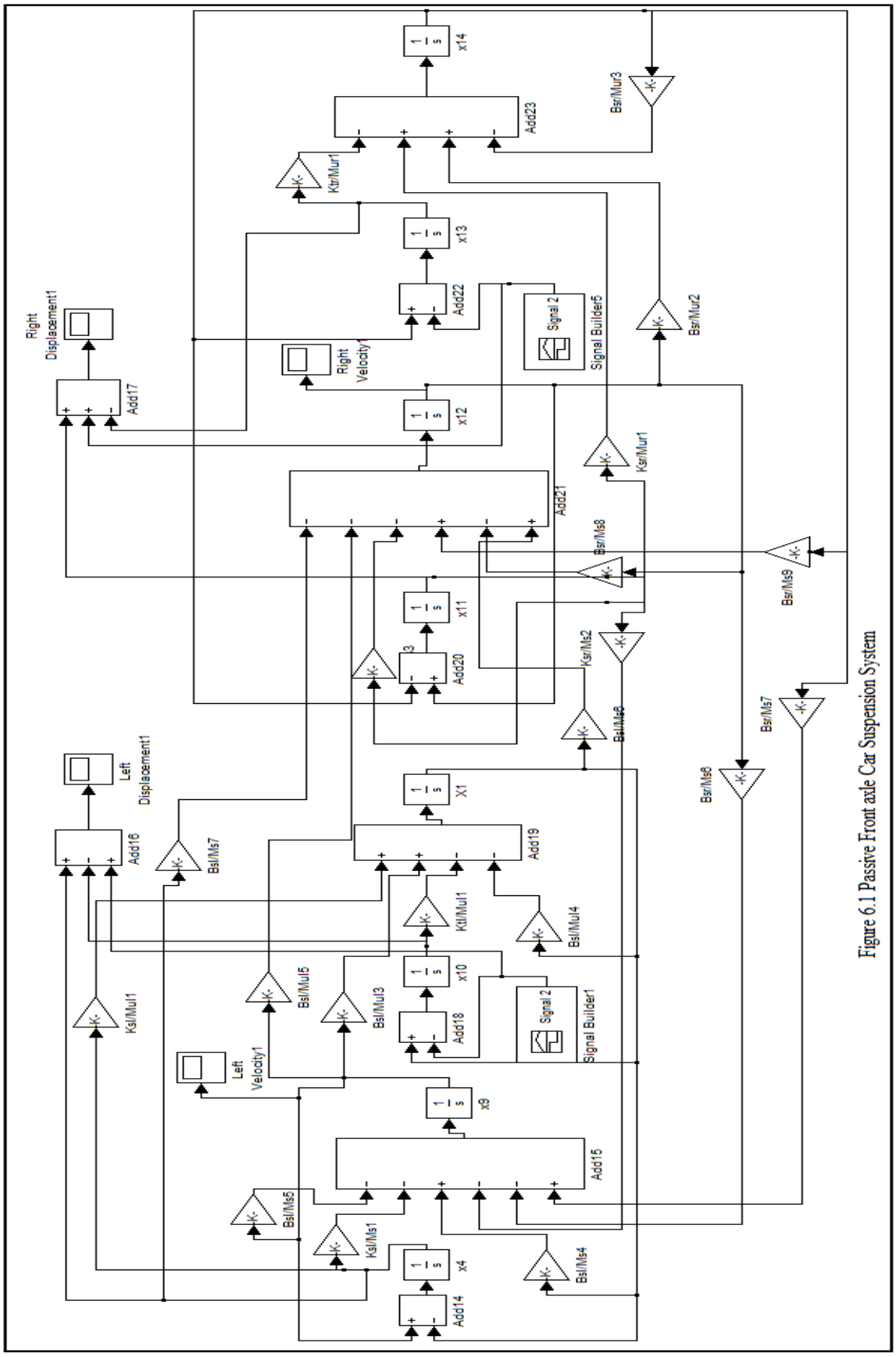


Figure 6.1 Passive Front axle Car Suspension System

Active Suspension Systems with Fuzzy Logic Controllers and Passive Suspension Systems are compared to show how an active suspension improves the system performance compared to passive suspension systems.

The results can be used to as a reason to explore the requirement of active car suspension systems in cars. Figure 6.1 shows the passive front axle car suspension system which is used for comparison with the proposed active car suspension system using fuzzy logic controllers. The simulation results can be divided as follows:

i. Same bumps on both wheels:

Let both z_{rl} and z_{rr} road disturbances for left and right wheels for front axle car suspension respectively take place in two consequent step input. Figure 6.2(a) and figure 6.2(b) shows the road disturbances which were introduced on both left wheel and right wheel of the front axle suspension system respectively.

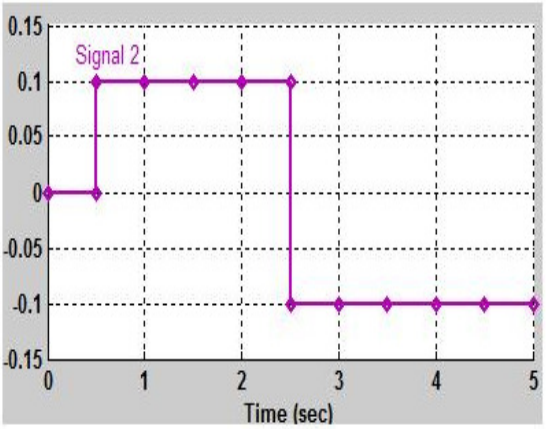


Fig 6.1 (a) Left Wheel Input

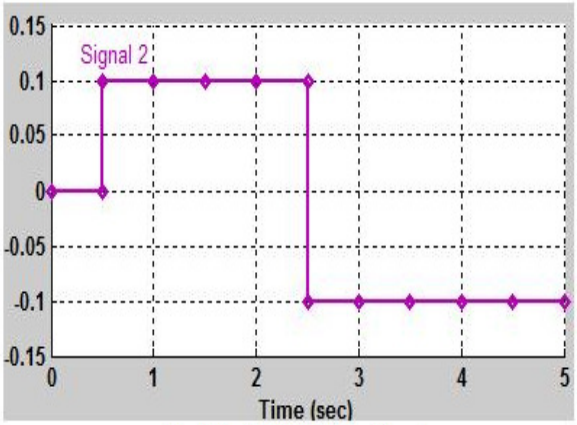


Fig 6.2 (b) Right Wheel Input

Figure 6.2(c) and figure 6.2(d) shows the relative sprung mass displacement in m with respect to time for both left and right side of the front axle respectively for both active suspension system proposed & passive suspension system. Figure 6.2(e) and figure 6.2(f) shows the absolute sprung mass velocity in m/s with respect to time for both left

and right side of the front axle car suspension system respectively for both active suspension system proposed & passive suspension system.

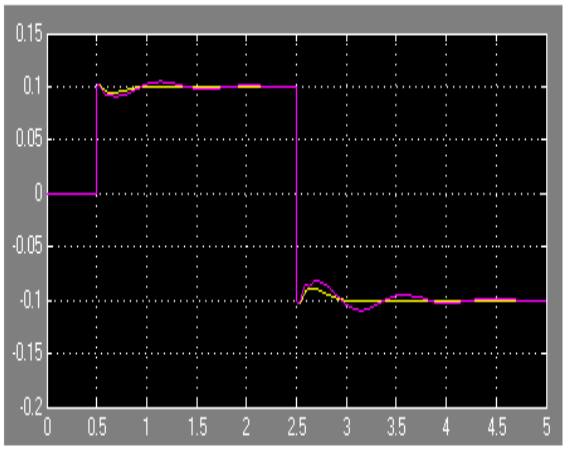


Fig 6.2 (c) Left Side Sprung Mass Displacement

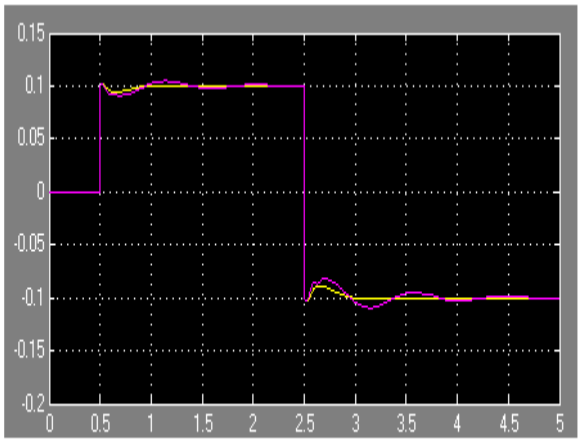


Fig 6.2(d) Right Side Sprung Mass Displacement

Legends: Yellow: Active Suspension using Fuzzy Controller
Magenta: Passive Suspension system response

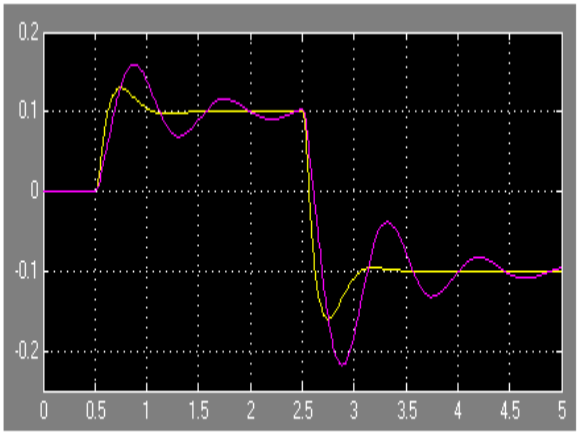


Fig 6.2(e) Left Side Sprung Mass Velocity

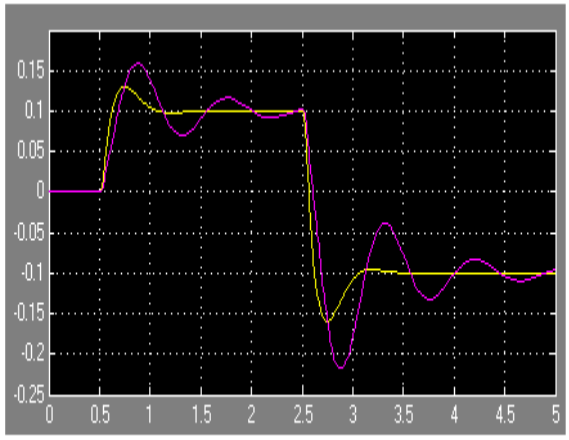


Fig 6.2 (f) Right Side Sprung Mass Velocity

Legends: Yellow: Active Suspension using Fuzzy Controller
Magenta: Passive Suspension system response
Figure 6.2 Sprung Mass Displacement & Velocity for Active vs Passive Suspensions

The figure clearly shows that a car with active suspension system with the proposed system responds very quickly and gets the system to settle quickly. The passive car suspension system has very slow and poor response compared to the active suspension system proposed in the research.

ii. Different bumps on both wheels:

Let both z_{rl} and z_{rr} road disturbances for left and right wheels for front axle car suspension respectively. We consider the case when there's no bump on Left wheel and a step variation is introduced on the Right wheel & vice versa. Figure 6.3(a) and figure 6.3(b) shows the road disturbances which were introduced on both left wheel and right wheel of the front axle suspension system respectively.

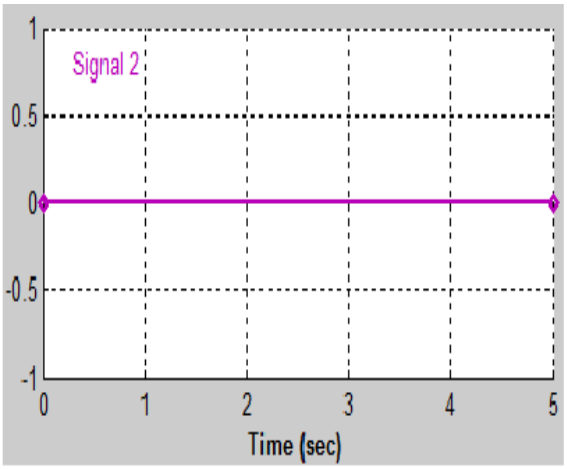


Figure 6.3(a) Left Wheel Input

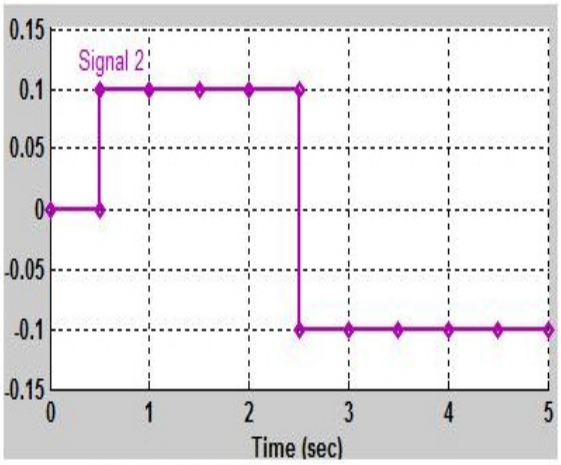


Fig 6.3 (b) Right Wheel Input

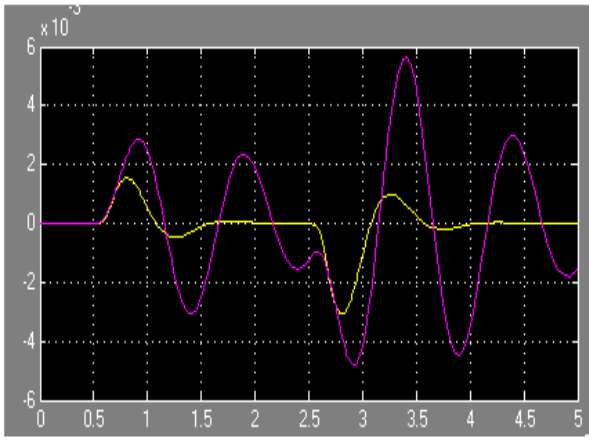


Fig 6.3 (c) Left Side Sprung Mass Displacement

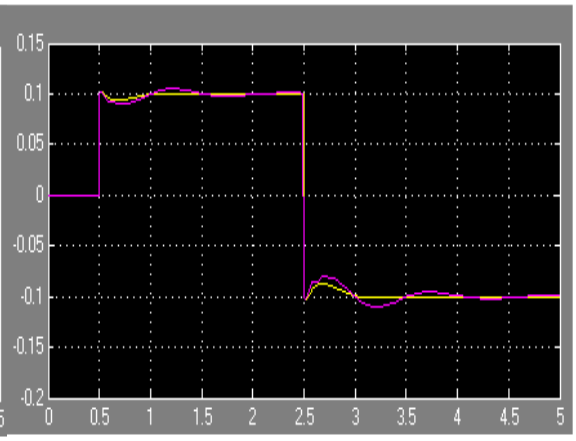


Fig 6.3 (d) Right Side Sprung Mass Displacement

Legends: Yellow – Active Suspension using Fuzzy Logic Controller Output
Magenta – Passive Suspension

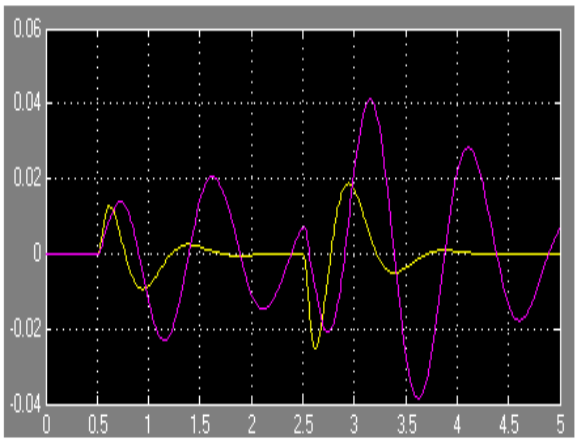


Fig 6.3 (e) Left Side Sprung Mass Velocity

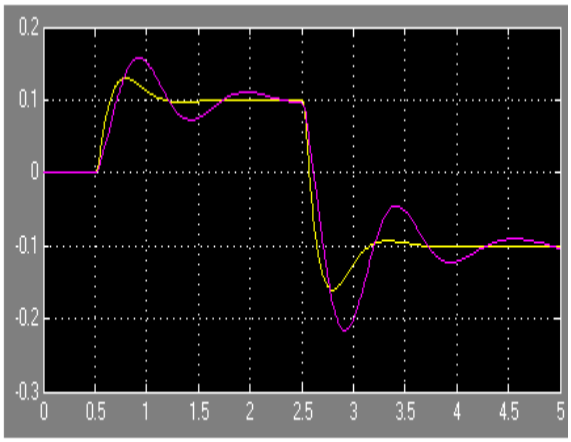


Fig 6.3 (f) Right Side Sprung Mass Velocity

Legends: Yellow – Active Suspension using Fuzzy Logic Controller Output
Magenta – Passive Suspension

Figure 6.3 Sprung Mass Displacement & Velocity for Active vs Passive Suspensions with only Right wheel displacement

Figure 6.3(c) and figure 6.3(d) shows the relative sprung mass displacement in m with respect to time for both left and right side of the front axle respectively for both active suspension system proposed & passive suspension system. Figure 6.3(e) and figure 6.3(f) shows the absolute sprung mass velocity in m/s with respect to time for both left and right side of the front axle car suspension system respectively for both active suspension system proposed & passive suspension system.

The figures clearly show that the proposed system responds really nicely to different road disturbances on both sides of the front wheels. Passive suspension really shows poor response in compensating for different disturbances of both sides compared to the proposed active car suspension system using fuzzy logic controllers.

For Right Side no bump the graphs are shown below:

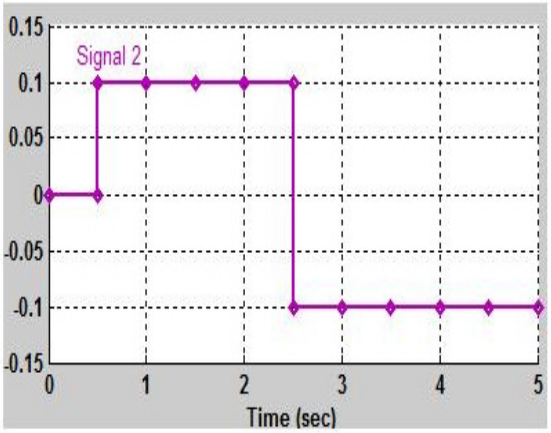


Figure 6.4(a) Left Wheel Input

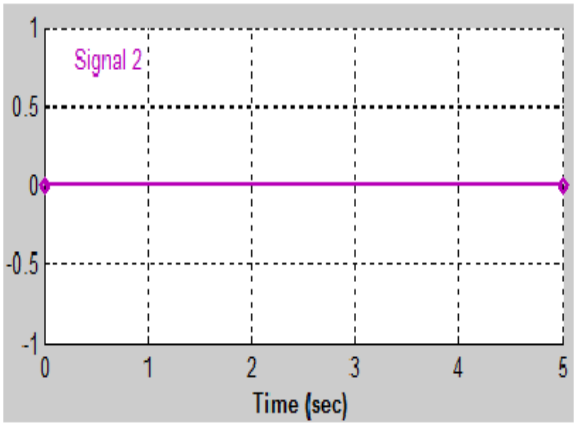


Fig 6.4 (b) Right Wheel Input

Figure 6.4(a) and figure 6.4(b) shows the road disturbances $\dot{z}_{rl}$ and $\dot{z}_{rr}$ which were introduced on both left wheel and right wheel of the front axle suspension system respectively.

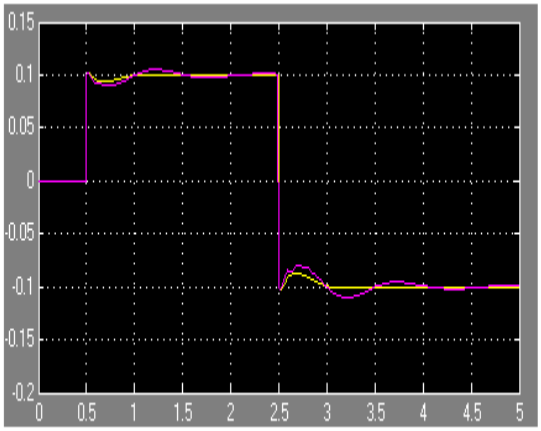


Fig 6.4 (c) Left Side Sprung Mass Displacement

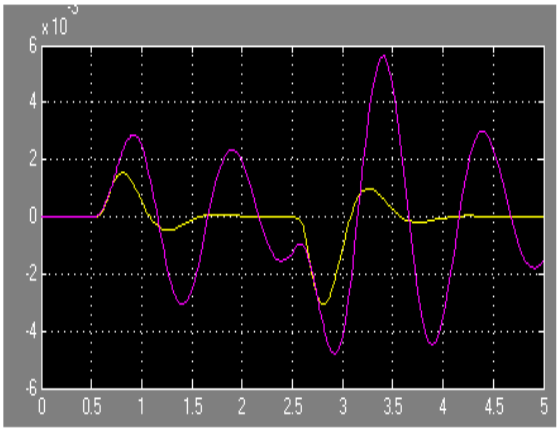


Fig 6.4(d) Right Side Sprung Mass Displacement

Legends: Yellow – Active Suspension using Fuzzy Logic Controller Output
Magenta – Passive Suspension

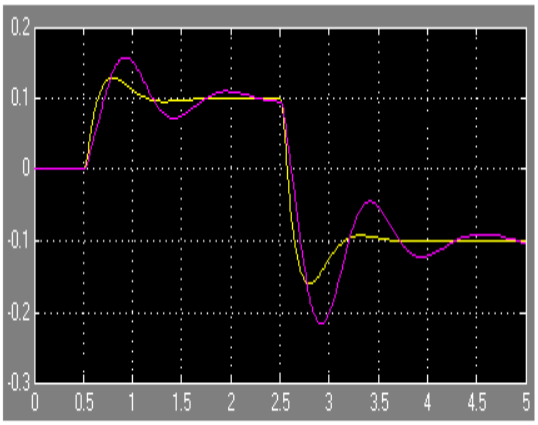


Fig 6.4 (e) Left Side Sprung Mass Velocity

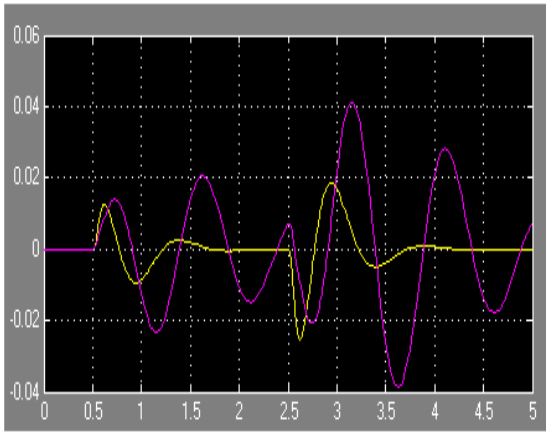


Fig 6.4 (f) Right Side Sprung Mass Velocity

Legends: Yellow – Active Suspension using Fuzzy Logic Controller Output
Magenta – Passive Suspension

Figure 6.4 Sprung Mass Displacement & Velocity for Active vs Passive Suspensions with only Left wheel displacement

Figure 6.4(c) and figure 6.4(d) shows the relative sprung mass displacement in m with respect to time for both left and right side of the front axle respectively for both active suspension system proposed & passive suspension system. Figure 6.4(e) and figure 6.4(f) shows the absolute sprung mass velocity in m/s with respect to time for both left and right side of the front axle car suspension system respectively for both active suspension system proposed & passive suspension system. The figures clearly show that the proposed system responds really nicely to different road disturbances on both sides of the front wheels. Passive suspension really shows poor response in compensating for different disturbances of both sides compared to the proposed active car suspension system using fuzzy logic controllers.

All the results in this section help in proving that passive car suspension system is not very effective when a vehicle faces different road disturbances on both sides of the vehicle. It makes the vehicle very vulnerable during such different road disturbances on both sides of the vehicle, whereas the proposed system seems to be very effective.

6.3 Simulation II: Fuzzy Logic Controller vs. Linear Quadratic Regulator

Major difference between Fuzzy Logic Control and Linear Quadratic Regulator is the variation of control efforts with fuzzy logic controller. Linear quadratic regulator approach is considered as an option for active suspension system by many researchers. Responses of fuzzy logic controller and linear quadratic regulator with different road disturbances are plotted to compare both systems and to prove the drawback of a linear quadratic regulator for active car suspension systems. The basic reason of comparing is to prove the method proposed in the research is very effective compared to a linear quadratic regulator. Figure 6.5 shows the Matlab simulink model used for Front axle suspension control using Linear Quadratic Regulator method.

The feedback gain matrix is found using the state space obtained in chapter 4 and using the 'care' function in Matlab. Function 'care' computes the unique solution X of the continuous-time algebraic Riccati equation.

$A^TX + XA - XBB^TX + Q = 0$

where Q=[20 0 0 0 0 0 0 0;0 20 0 0 0 0 0 0;0 0 20 0 0 0 0 0;0 0 0 20 0 0 0 0;0 0 0 0 20 0 0 0;0 0 0 0 0 20 0 0;0 0 0 0 0 0 20 0;0 0 0 0 0 0 0 20]

and $K= X B R^{-1}$

where R= [1]

Many different values of gain matrix were tried to optimize the performance of the car suspension system. The feedback gain matrix to optimize the performance of front axle car suspension control is given as

K= [0.0049 0.0200 -0.1137 -0.0096 0.0049 0.0200 -0.1141 -0.0096]'

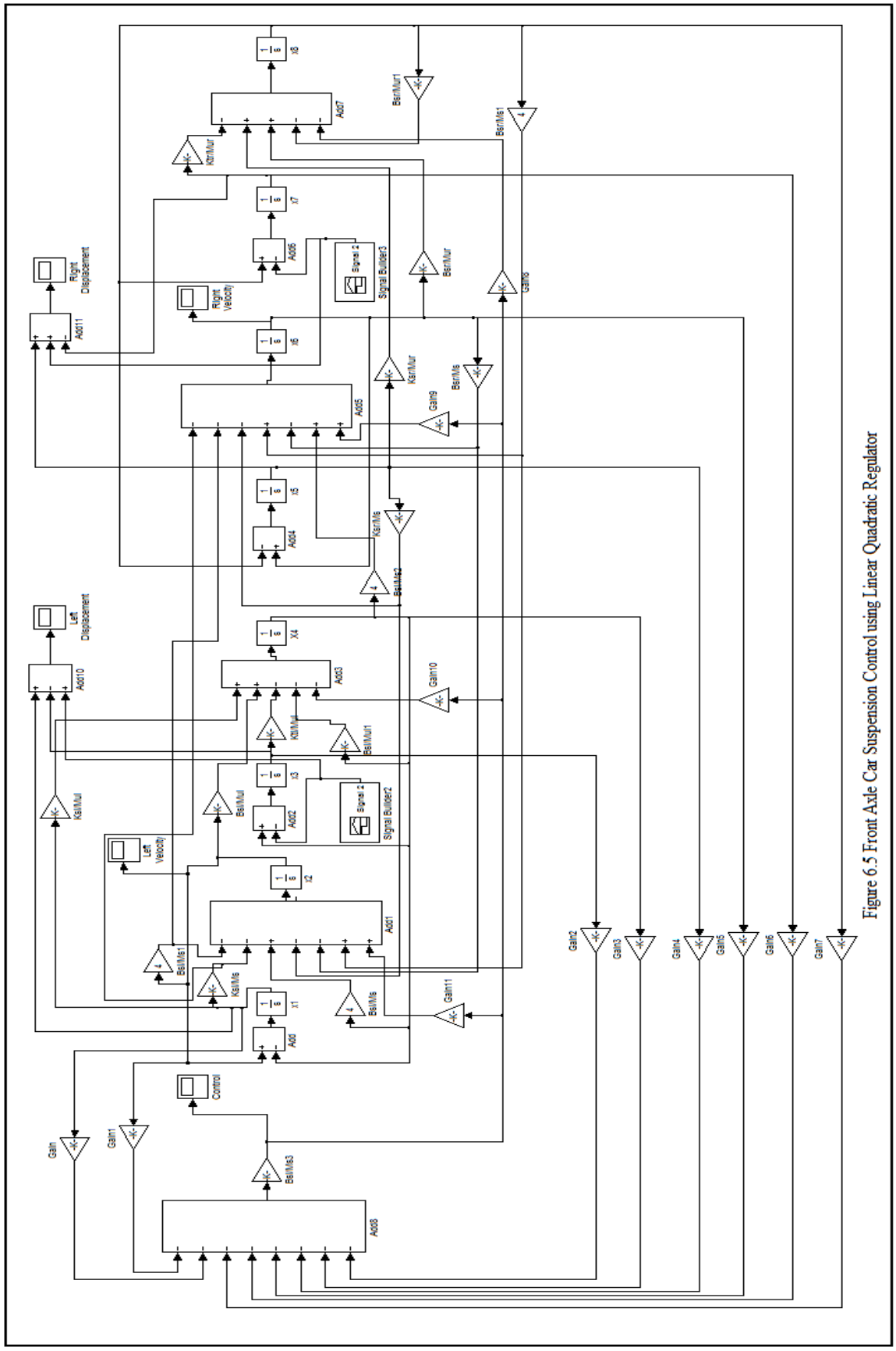


Figure 6.5 Front Axle Car Suspension Control using Linear Quadratic Regulator

i. Same bumps on both wheels:

Let both z_{rl} and z_{rr} road disturbances for left and right wheels for front axle car suspension respectively take place in two consequent step input. Figure 6.6(a) and figure 6.6(b) shows the road disturbances which were introduced on both left wheel and right wheel of the front axle suspension system respectively.

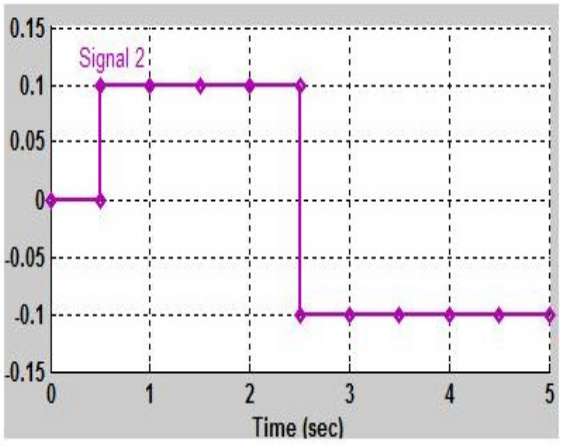


Fig 6.6 (a) Left Wheel Input

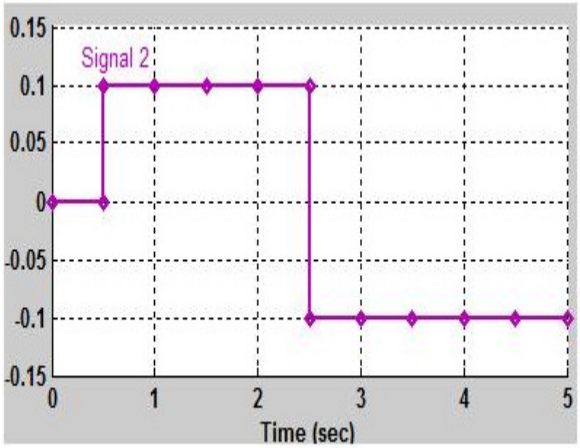


Fig 6.6 (b) Right Wheel Input

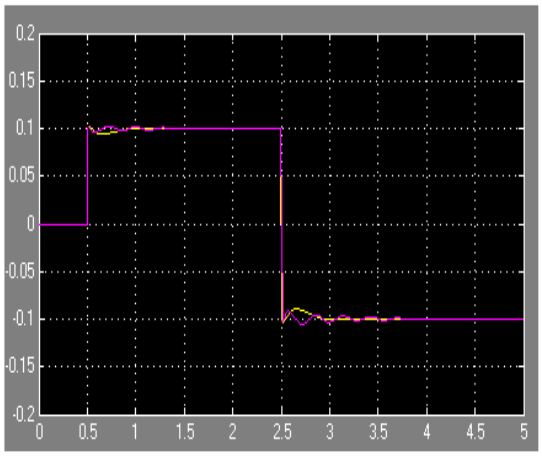


Fig 6.6 (c) Left Side Sprung Mass Displacement

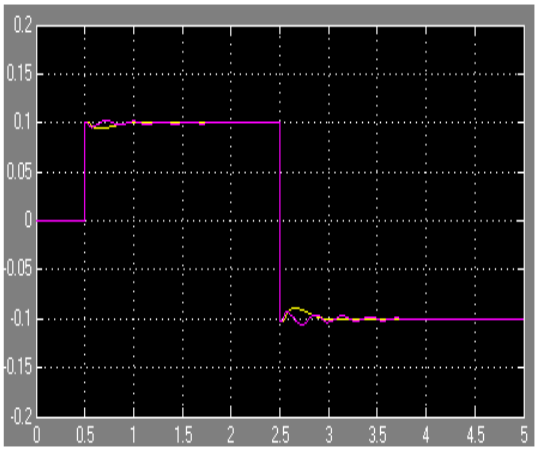


Fig 6.6 (d) Right Side Sprung Mass Displacement

Legends: Yellow: Active Suspension using Fuzzy Logic Controller
Magenta: Active Suspension using LQR

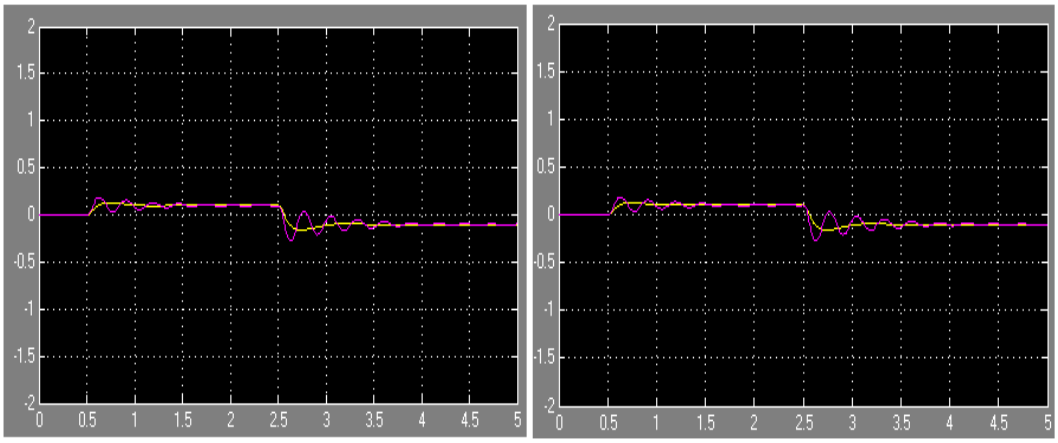


Fig 6.6 (e) Left Side Sprung Mass Velocity

Fig 6.6 (f) Right Side Sprung Mass Velocity

Legends: Yellow: Active Suspension using Fuzzy Logic Controller
Magenta: Active Suspension using LQR.

Figure 6.6(c) and figure 6.6(d) shows the relative sprung mass displacement in m with respect to time for both left and right side of the front axle respectively for both active suspension system proposed & linear quadratic regulator active suspension system. Figure 6.6(e) and figure 6.6(f) shows the absolute sprung mass velocity in m/s with respect to time for both left and right side of the front axle car suspension system respectively for both active suspension system proposed & linear quadratic regulator active suspension system.

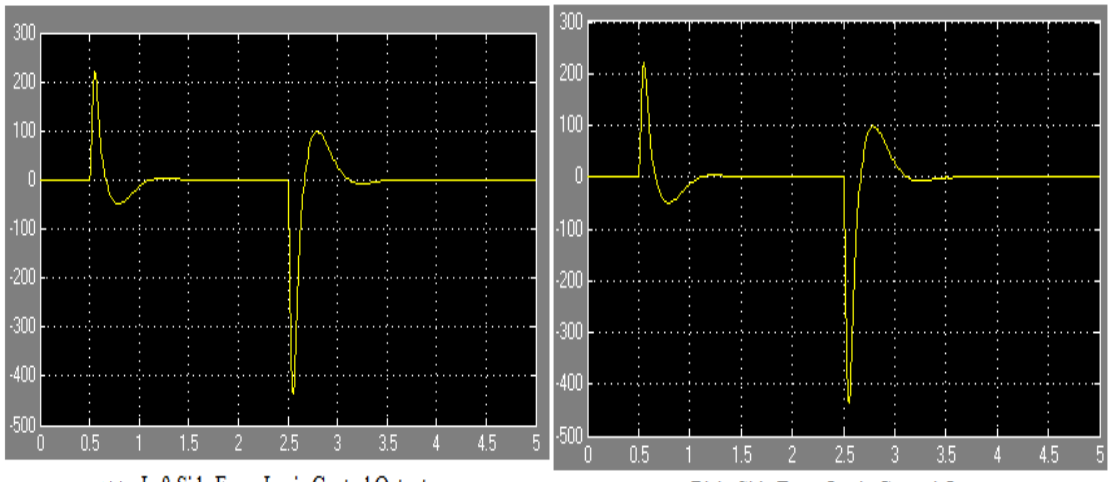


Fig 6.6 (g) Left Side Fuzzy Logic Control Output

Fig 6.6 (h) Right Side Fuzzy Logic Control Output

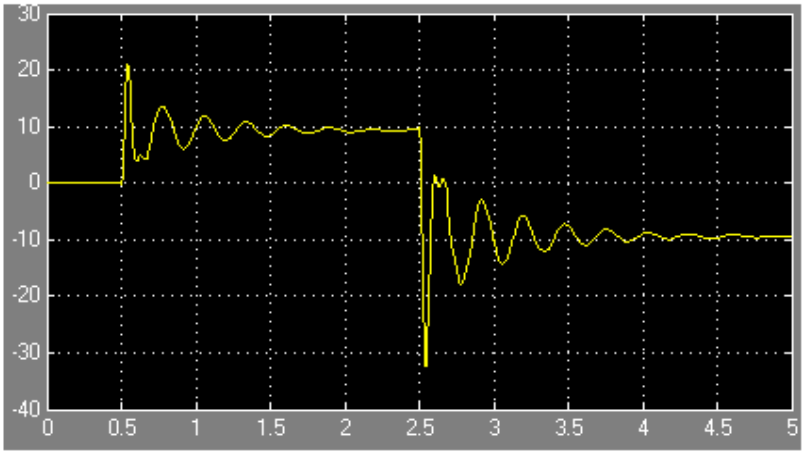


Fig 6.6 (i) Control Ouput with LQR

Figure 6.6 Sprung Mass Displacement & Velocity for FLC vs LQR and Control Outputs for both the System

Figure 6.6(g) and figure 6.6(h) shows the control signal generated by fuzzy logic controller in Newton with respect to time for left side and right side respectively. Figure 6.6(i) shows the control signal generated by the linear quadratic regulator in Newton with respect to time.

It can be easily seen that linear quadratic regulator method is not as effective as the proposed fuzzy logic controller method. The control generated by linear quadratic method is inadequate to decrease the vibrations quickly. The response of linear quadratic method is much slower compared to the proposed fuzzy logic controller method.

ii. Different bumps on both wheels:

Let both z_{rl} and z_{rr} road disturbances for left and right wheels for front axle car suspension respectively. We consider the case when there's no bump on Left wheel and a step variation is introduced on the Right wheel & vice versa. Figure 6.7(a) and figure 6.7(b) shows the road disturbances which were introduced on both left wheel and right wheel of the front axle suspension system respectively.

When left wheel has no bump and right wheel has the step variation, the results are as follows:

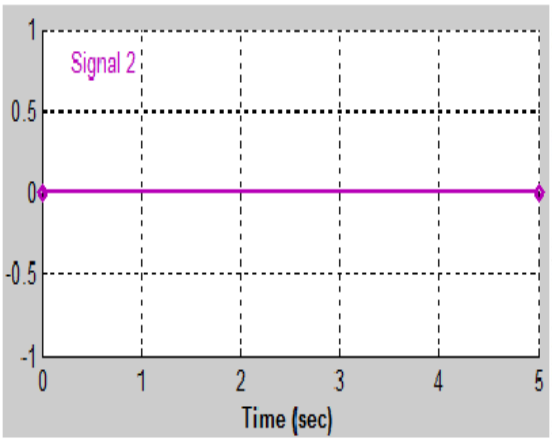


Fig 6.7 (a) Left Wheel Input

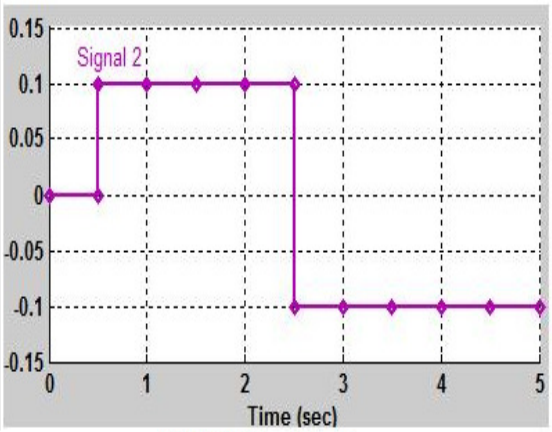


Fig 6.7 (b) Right Wheel Input

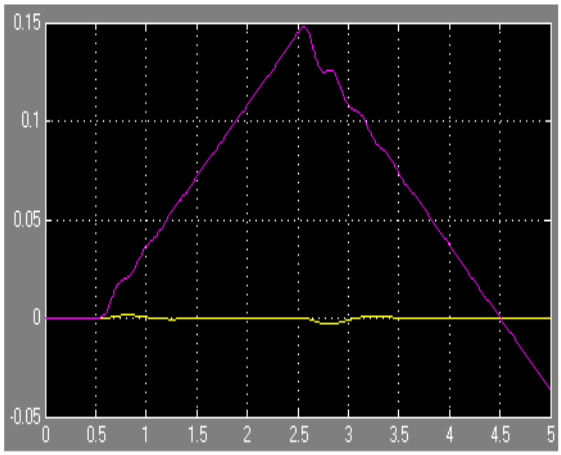


Fig 6.7(c) Left Side Sprung Mass Displacement

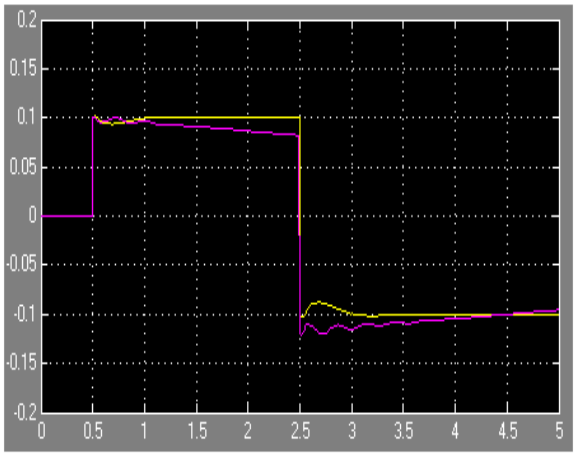


Fig 6.7 (d) Right Side Sprung Mass Displacement

Legends: Yellow: Active Suspension using Fuzzy Logic Controller
Magenta: Active Suspension using LQR

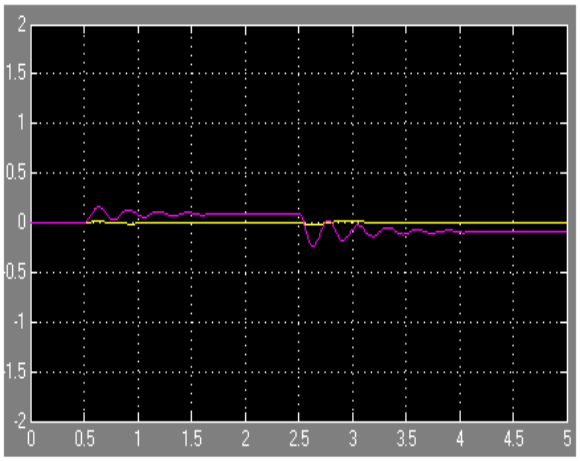


Fig 6.7 (e) Left Side Sprung Mass Velocity

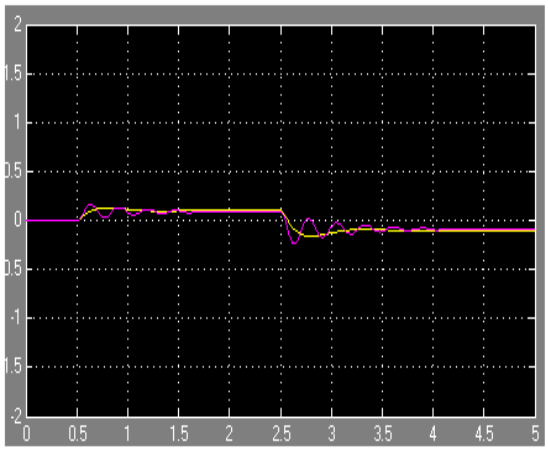


Fig 6.7;(f) Right Side Sprung Mass Velocity

Legends: Yellow: Active Suspension using Fuzzy Logic Controller
Magenta: Active Suspension using LQR

Figure 6.7(c) and figure 6.7(d) shows the relative sprung mass displacement in m with respect to time for both left and right side of the front axle respectively for both active suspension system proposed & linear quadratic regulator active suspension system. Figure 6.7(e) and figure 6.7(f) shows the absolute sprung mass velocity in m/s with respect to time for both left and right side of the front axle car suspension system respectively for both active suspension system proposed & linear quadratic regulator active suspension system. The triangular shape for the linear Quadratic method for left side sprung mass displacement shows that the linear quadratic method tries to compensate for the raise on the right side of the car to get the car suspension travel to the same level, but is very slow in the process. The linear quadratic method never settles down due to the compensation and decreases the ride comfort as well as provides poor handling for the car.

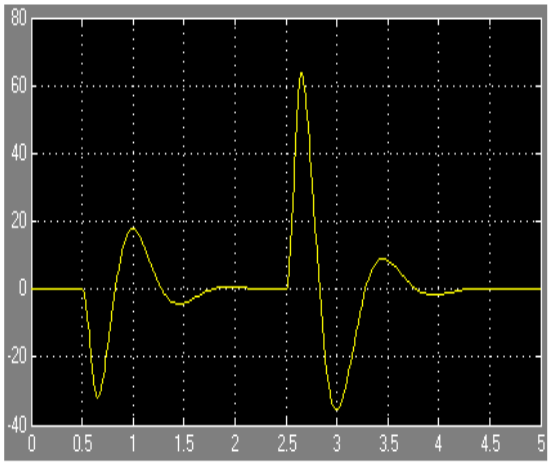


Fig 6.7 (g) Left Side FLC Control Output

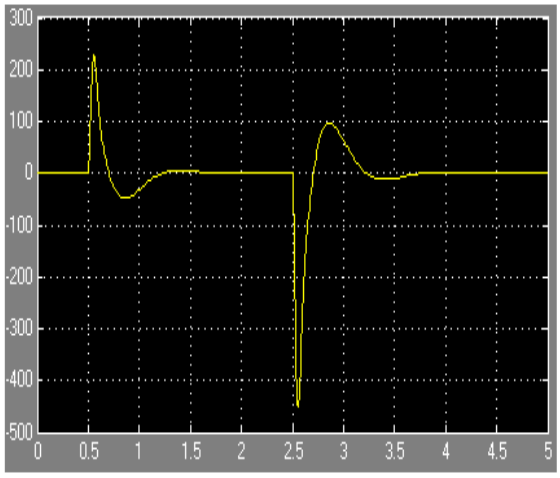


Fig 6.7 (h) Right Side FLC Control Output

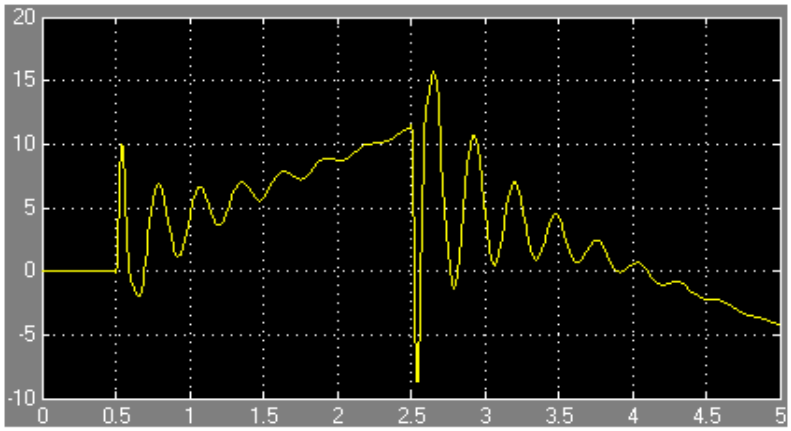


Fig 6.7 (i) Control Output with LQR Method

Figure 6.7 Sprung Mass Displacement & Velocity for FLC vs LQR and Control Outputs for both the system, when displacement is only on the right wheel

Figure 6.7(g) and figure 6.7(h) shows the control signal generated by fuzzy logic controller in Newton with respect to time for left side and right side respectively. Figure 6.7(i) shows the control signal generated by the linear quadratic regulator in Newton with respect to time.

It can be easily seen that linear quadratic regulator method is not as effective as the proposed fuzzy logic controller method when it come to different road disturbances on both sides of the vehicle. The control generated by linear quadratic method is

inadequate to decrease the vibrations quickly. The response of linear quadratic method for no bump on the left wheel clearly shows that the car becomes highly unstable. The car suspension system never settles down which decreases the ride comfort and decreases the handling of the car as well.

When Right wheel has no bump and left wheel has the step variation, the results are as follows:

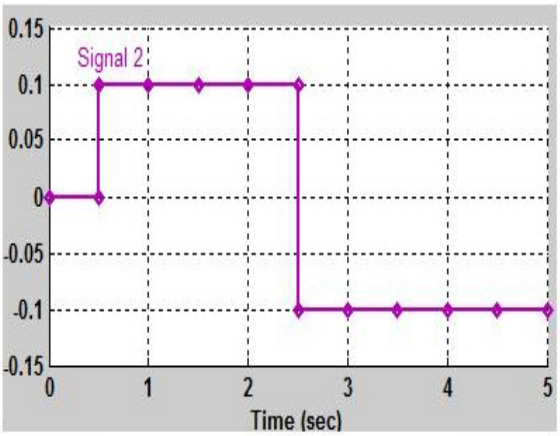


Fig 6.8 (a) Left Wheel Input

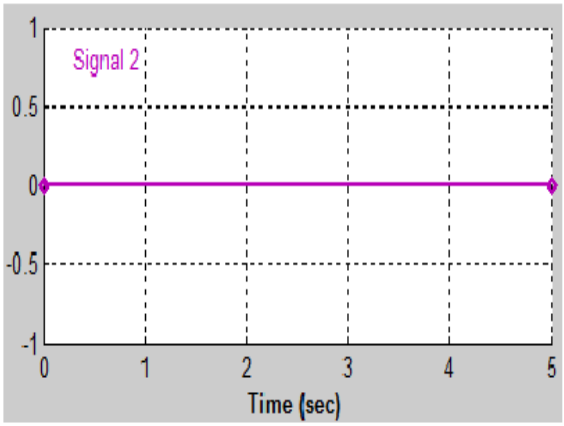


Fig 6.8 (b) Right Wheel Input

Figure 6.8(a) and figure 6.8(b) shows the road disturbances $\dot{z}_{rl}$ and $\dot{z}_{rr}$ which were introduced on both left wheel and right wheel of the front axle suspension system respectively.

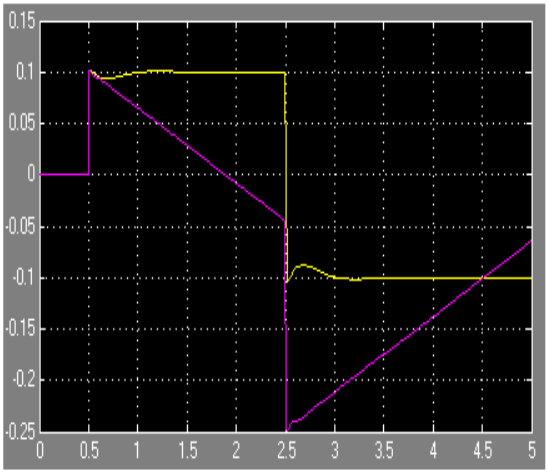


Fig 6.8 (c) Left Side Sprung Mass Displacement

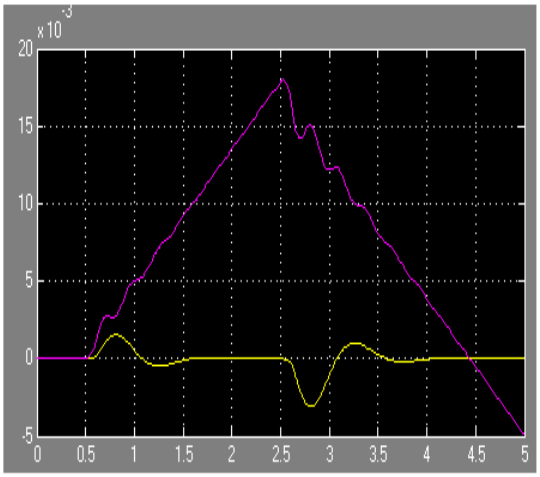


Fig 6.8 (d) Right Side Sprung Mass Displacement

Legends: Yellow: Active Suspension using Fuzzy Logic Controller
Magenta: Active Suspension using LQR

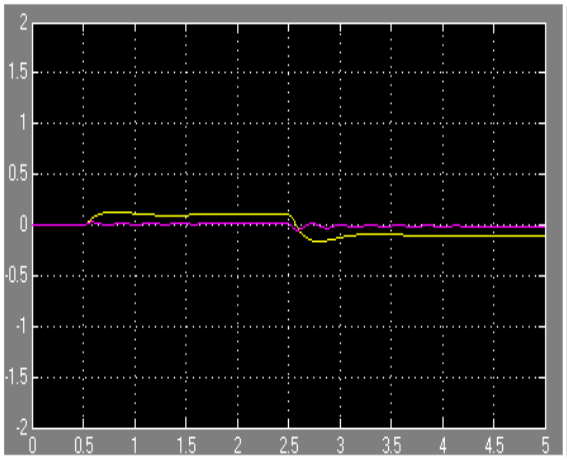


Fig 6.8 (e) Left Side Sprung Mass Velocity

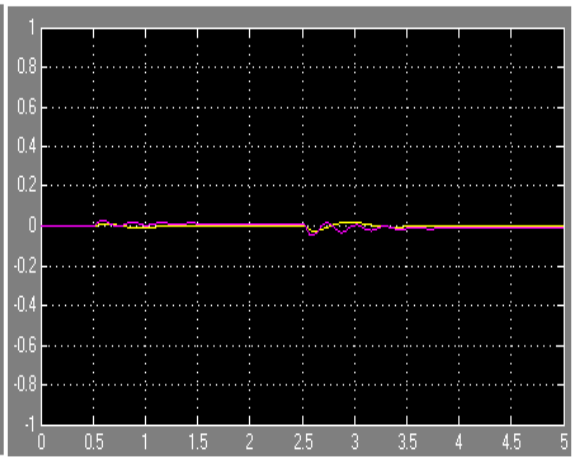


Fig 6.8 (f) Right Side Sprung Mass Velocity

Legends: Yellow: Active Suspension using Fuzzy Logic Controller

Magenta: Active Suspension using LQR

Figure 6.8(c) and figure 6.8(d) shows the relative sprung mass displacement in m with respect time for both left and right side of the front axle respectively for both active suspension system proposed & linear quadratic regulator active suspension system. Figure 6.8(e) and figure 6.8(f) shows the absolute sprung mass velocity in m/s with respect to time for both left and right side of the front axle car suspension system respectively for both active suspension system proposed & linear quadratic regulator active suspension system. The triangular shape for the linear Quadratic method for left and right side sprung mass displacement shows that the linear quadratic method tries to compensate for the raise on the left side of the car to get the car suspension travel to the same level, but is very slow in the process. The linear quadratic method never settles down due to the compensation and decreases the ride comfort as well as provides poor handling for the car.

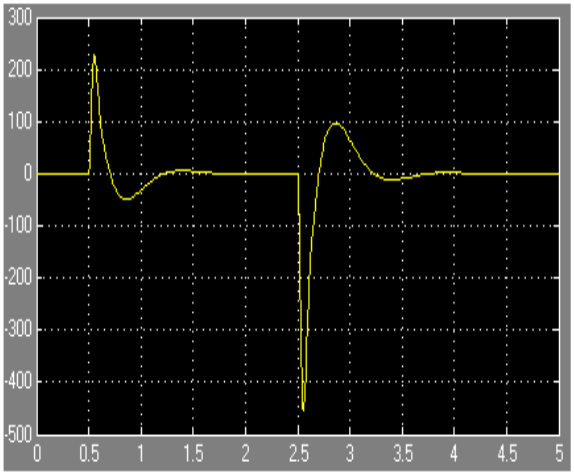


Fig 6.8 (g) Left Side FLC Control Output

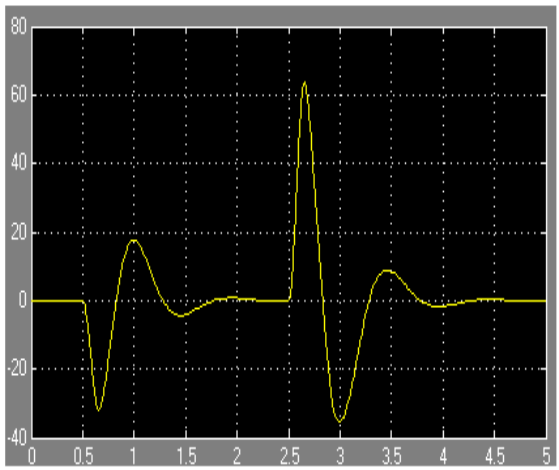


Fig 6.8 (h) Right Side FLC Control Output

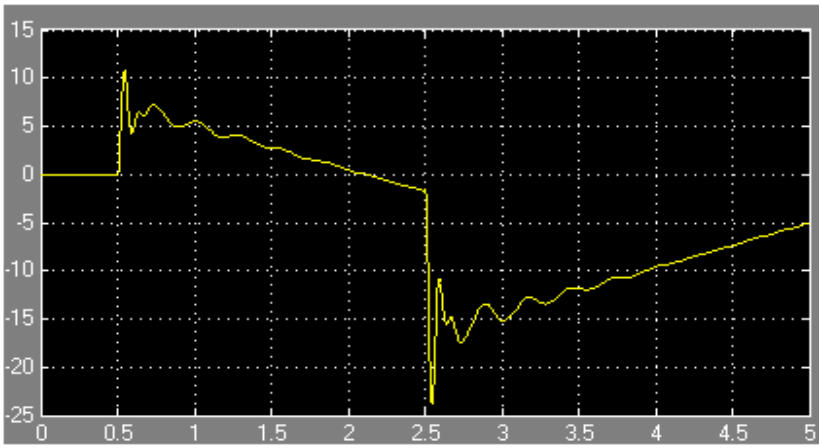


Fig 6.8 (i) Control Ouput with LQR method

Figure 6.8 Sprung Mass Displacement & Velocity for FLC vs LQR and Control Outputs for both the system, when displacement is only on the left wheel

Figure 6.8(g) and figure 6.8(h) shows the control signal generated by fuzzy logic controller in Newton with respect to time for left side and right side respectively. Figure 6.8(i) shows the control signal generated by the linear quadratic regulator in Newton with respect to time.

It clearly shows that Fuzzy logic controller method gives better results than the Linear Quadratic Regulator method. It can be easily seen that linear quadratic regulator method is not as effective as the proposed fuzzy logic controller method when it come to

different road disturbances on both sides of the vehicle. The control generated by linear quadratic method is inadequate to decrease the vibrations quickly. The response of linear quadratic method for no bump on the right wheel clearly shows that the car becomes highly unstable. The car suspension system never settles down which decreases the ride comfort and decreases the handling of the car as well.

All the results in this section help in proving that linear quadratic regulator system is not effective when a vehicle faces different road disturbances on both sides of the vehicle. It makes the vehicle very vulnerable during such different road disturbances on both sides of the vehicle, whereas the proposed system seems to be very effective.

6.4 Simulation III: System with Two Fuzzy Logic Controllers vs. One fuzzy Logic Controller with only Left Wheel Input

In this section two different methodologies with fuzzy logic control are compared. One control method is with two fuzzy logic controllers proposed in the research and the other is with single controller with an input only from the left wheel. The basic reason of the comparison is to prove that the proposed method of using two fuzzy logic controllers for front axle car suspension control is required to achieve better control. The comparison will also help in understanding the performance of proposed in the research even when one of the controller stops performing. Figure 6.9 shows the Matlab-Simulink model used for the front axle suspension control using a single fuzzy logic control with only left side inputs.

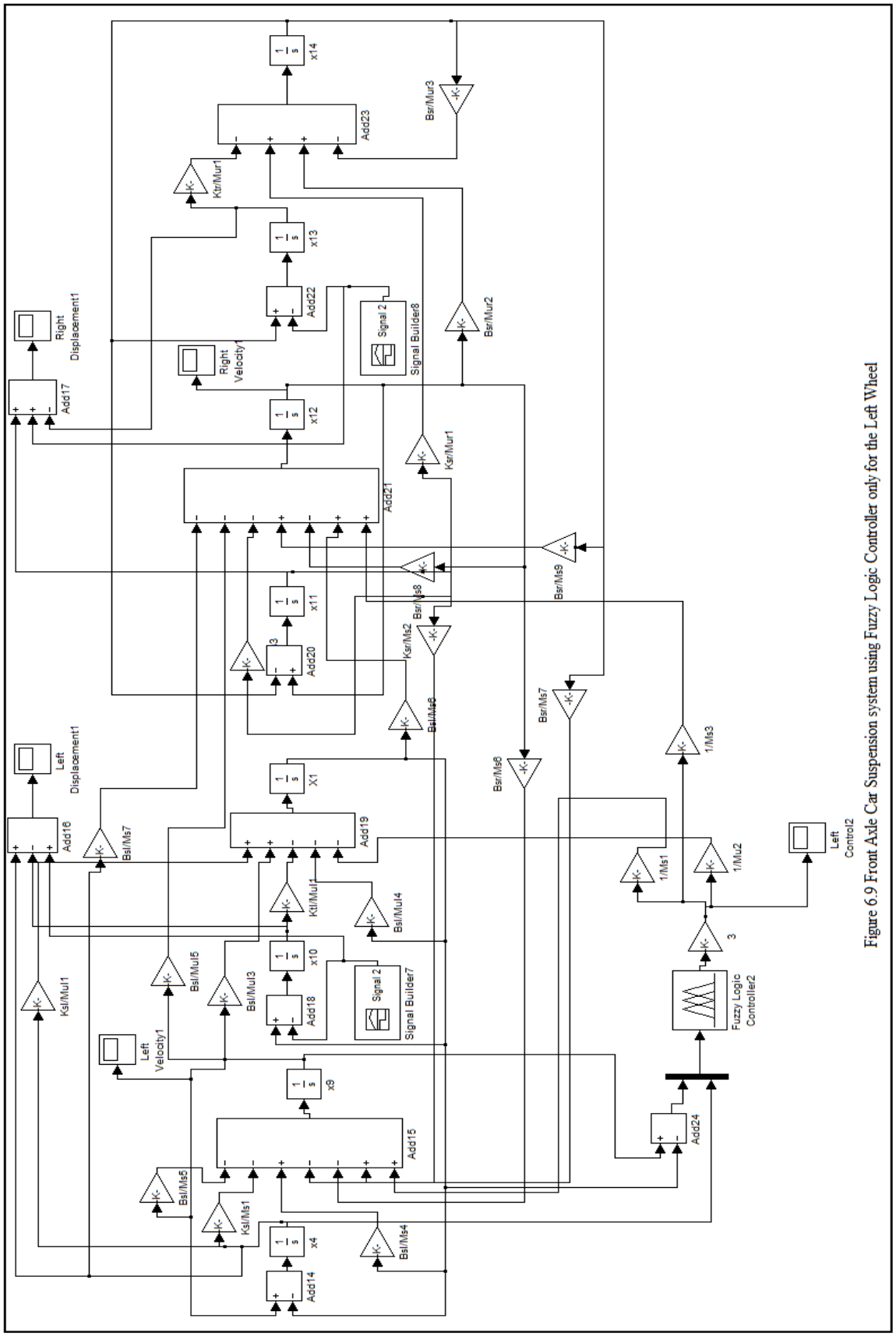


Figure 6.9 Front Axle Car Suspension system using Fuzzy Logic Controller only for the Left Wheel

Responses for both methods with different road irregularities are plotted as shown below:

i. Same bumps on both wheels:

Let both z_{rl} and z_{rr} road disturbances for left and right wheels for front axle car suspension respectively take place in two consequent step input. Figure 6.10(a) and figure 6.10(b) shows the road disturbances which were introduced on both left wheel and right wheel of the front axle suspension system respectively.

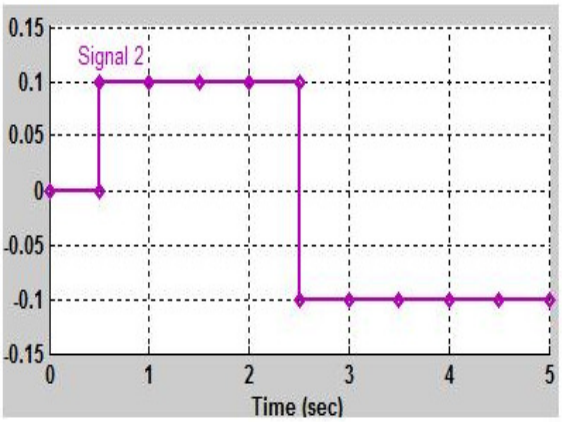


Fig 6.10 (a) Left Wheel Input

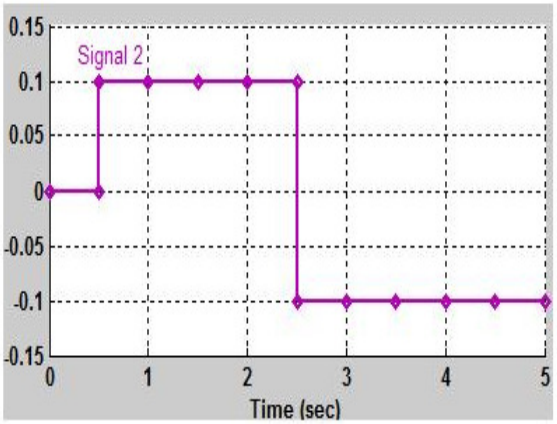


Fig 6.10 (b) Right Wheel Input

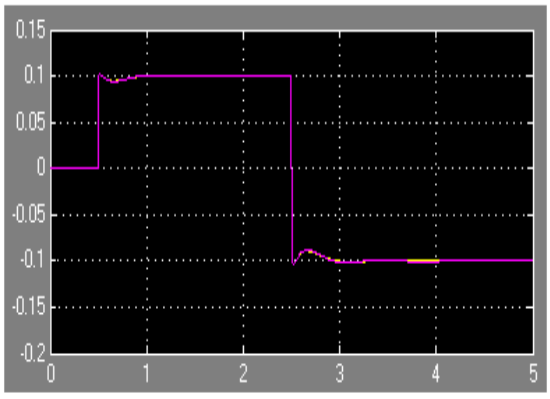


Fig 6.10 (c) Left Side Sprung Mass Displacement

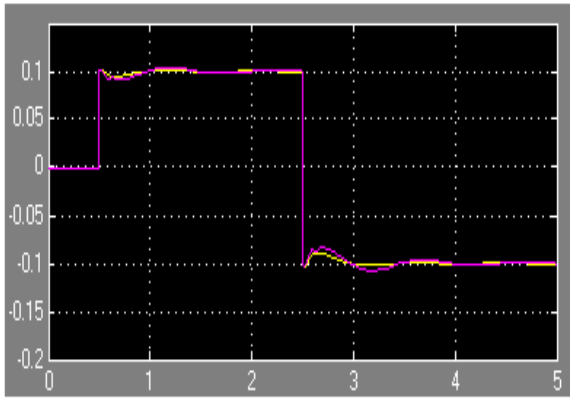


Fig 6.10 (d) Right Side Sprung Mass Displacement

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only Left wheel inputs.

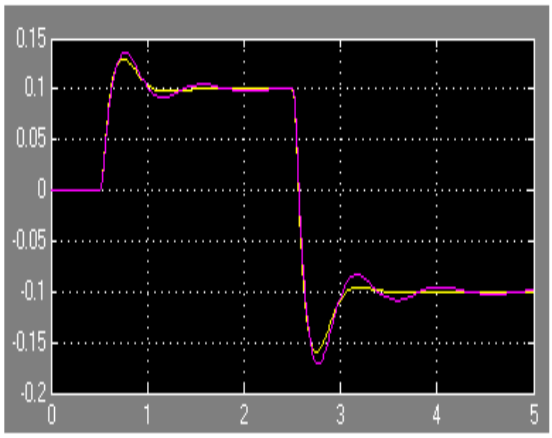


Fig 6.10 (e) Left Side Sprung Mass Velocity

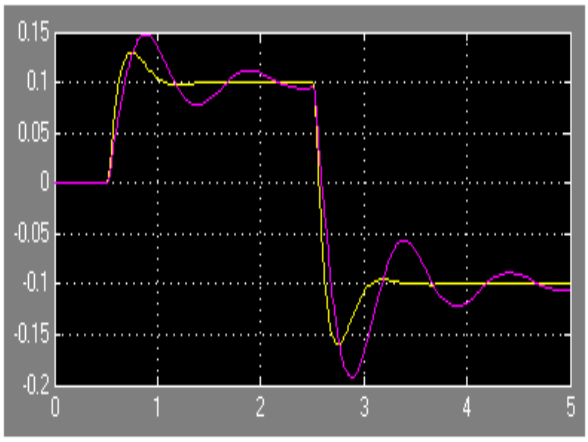


Fig 6.10 (f) Right Side Sprung Mass Velocity

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only Left wheel inputs.

Figure 6.10(c) and figure 6.10(d) shows the relative sprung mass displacement in m with respect to time for both left and right side of the front axle respectively for both active suspension system proposed & active suspension system with single fuzzy logic controller with only left side inputs. Figure 6.10(e) and figure 6.10(f) shows the absolute sprung mass velocity in m/s with respect to time for both left and right side of the front axle car suspension system respectively for both active suspension system proposed & active suspension system with single fuzzy logic controller with only left side inputs.

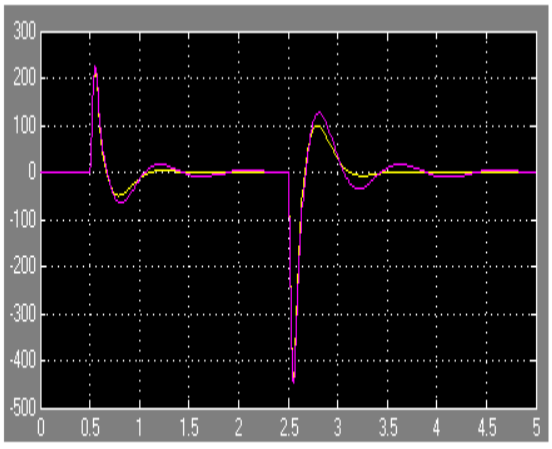


Fig 6.10 (g) Left Control

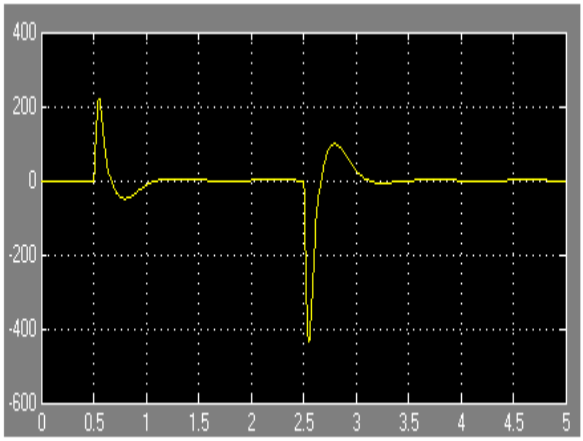


Fig 6.10 (h) Right Control

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only Left wheel inputs.

Figure 6.10 Sprung Mass Displacement & Velocity for two FLC's vs one FLC with only left wheel input and Control Outputs for both the system

Figure 6.10(g) and figure 6.10(h) shows the control signal generated by both the systems in Newton with respect to time for left side and right side respectively.

The response clearly shows the system with two fuzzy logic controllers gets the system to settle system much faster than the system with only one fuzzy logic controller with only left wheel input. It clearly shows that even if one of the controller stops performing it doesn't make the car suspension unstable, it just increases the time taken to suppress the vibrations.

ii. Different bumps on both wheels:

Let both z_{rl} and z_{rr} road disturbances for left and right wheels for front axle car suspension respectively. We consider the case when there's no bump on Left wheel and a step variation is introduced on the Right wheel & vice versa.

When left wheel has no bump and right wheel has the step variation, the results are as follows:

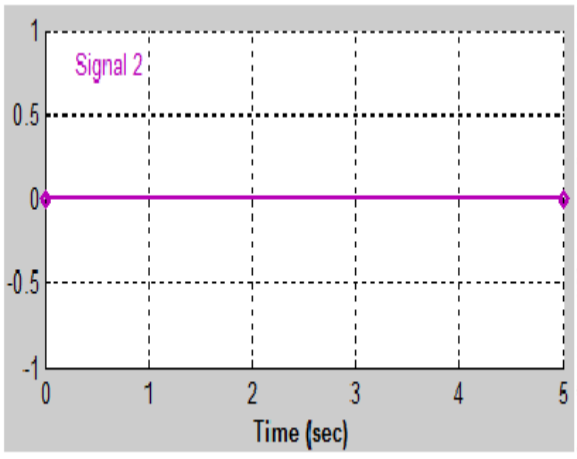


Fig 6.11 (a) Left Wheel Input

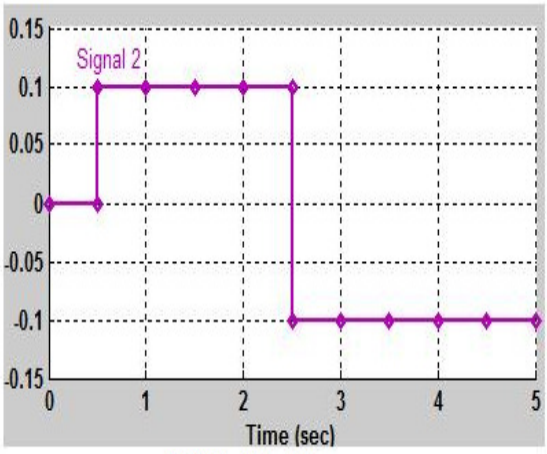


Fig 6.11 (b) Right Wheel Input

Figure 6.11(a) and figure 6.11(b) shows the road disturbances z_{rl} and z_{rr} which were introduced on both left wheel and right wheel of the front axle suspension system respectively.

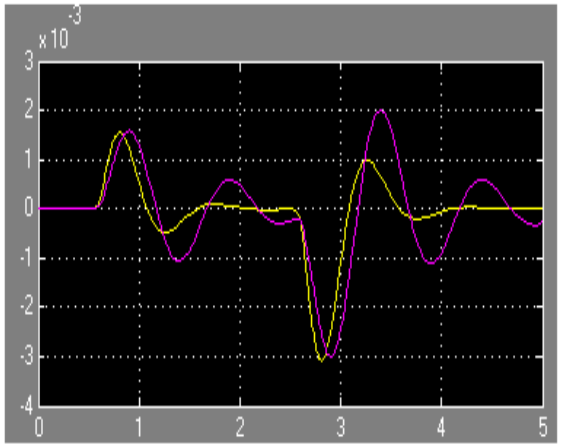


Fig 6.11 (c) Left Side Sprung Mass Displacement

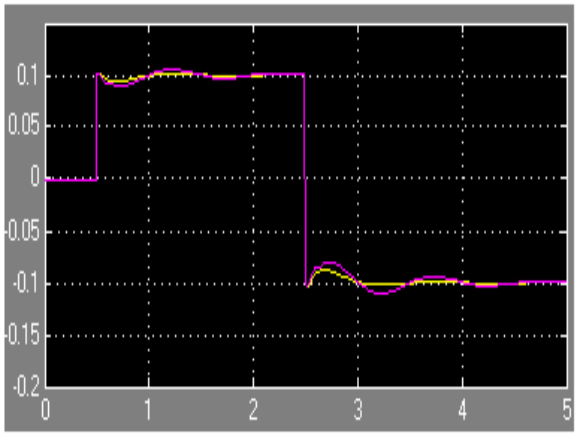


Fig 6.11 (d) Right Side Sprung Mass Displacement

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only Left wheel inputs.

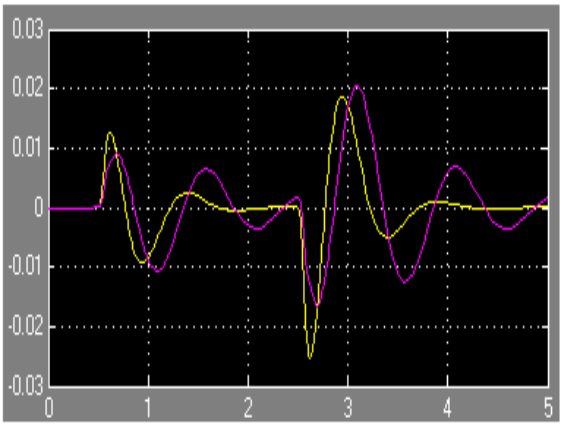


Fig 6.11 (e) Left Side Sprung Mass Velocity

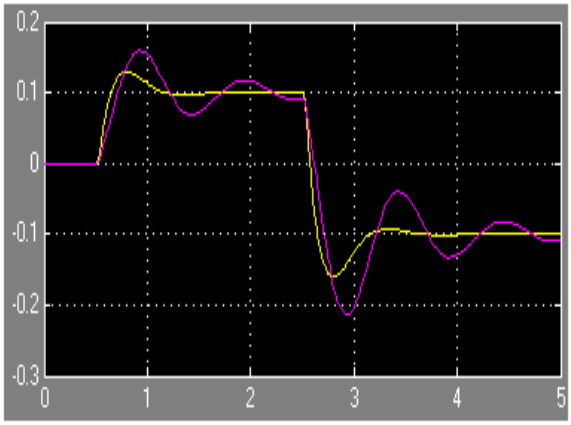


Fig 6.11 (f) Right Side Sprung Mass Velocity

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only Left wheel inputs.

Figure 6.11(c) and figure 6.11(d) shows the relative sprung mass displacement in m with respect to time for both left and right side of the front axle respectively for both active suspension system proposed & active suspension system with single fuzzy logic controller with only left side inputs. Figure 6.11(e) and figure 6.11(f) shows the absolute

sprung mass velocity in m/s with respect to time for both left and right side of the front axle car suspension system respectively for both active suspension system proposed & active suspension system with single fuzzy logic controller with only left side inputs.

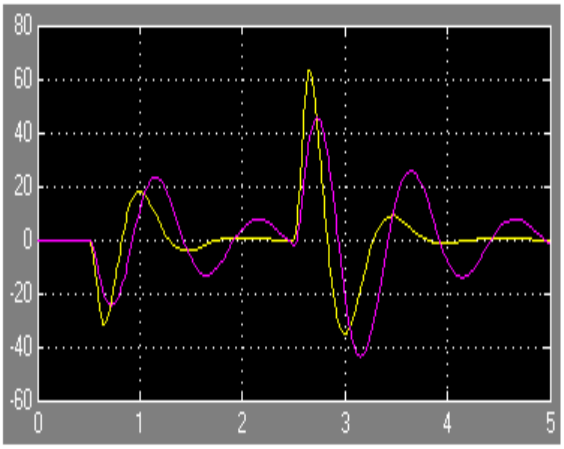


Fig 6.11 (g) Left Side Control

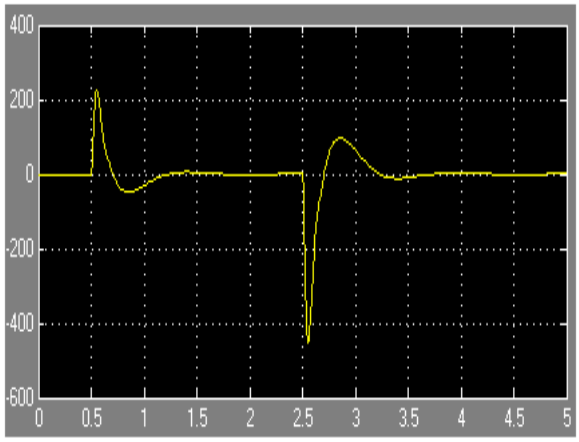


Fig 6.11 (h) Control Right

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only Left wheel inputs.

Figure 6.11 Sprung Mass Displacement & Velocity for two FLC's vs one FLC with only left wheel input and Control Outputs for both the system, with displacement only on the right wheel

Figure 6.11(g) and figure 6.11(h) shows the control signal generated by both the systems in Newton with respect to time for left side and right side respectively.

The response clearly shows the system with two fuzzy logic controllers gets the system to settle system much faster than the system with only one fuzzy logic controller with only left wheel input. It clearly shows that even if one of the controller stops performing it doesn't make the car suspension unstable, it just increases the time taken to suppress the vibrations. Even, when the right side controller is not working and the bump is faced only on the right side the suspension stabilizes but it takes more time to suppress the vibrations.

When Right wheel has no bump and left wheel has the step variation, the results are as follows:

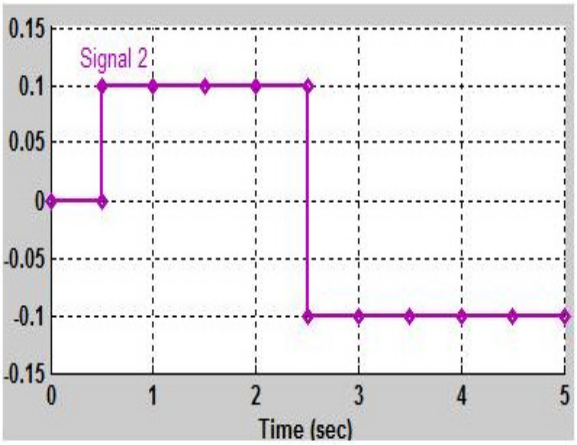


Fig 6.12 (a) Left Wheel Input

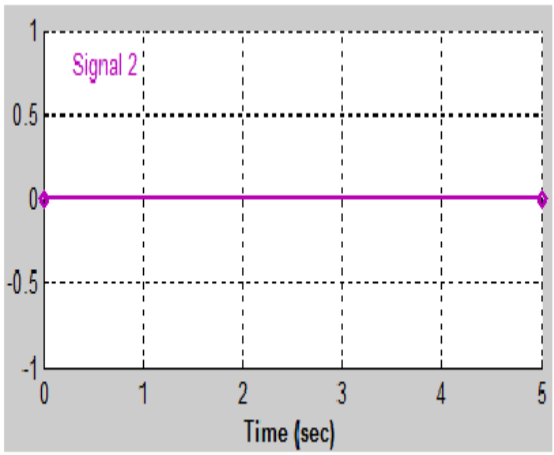


Fig 6.12 (b) Right Wheel Input

Figure 6.12(a) and figure 6.12(b) shows the road disturbances z_{rl} and z_{rr} which were introduced on both left wheel and right wheel of the front axle suspension system respectively.

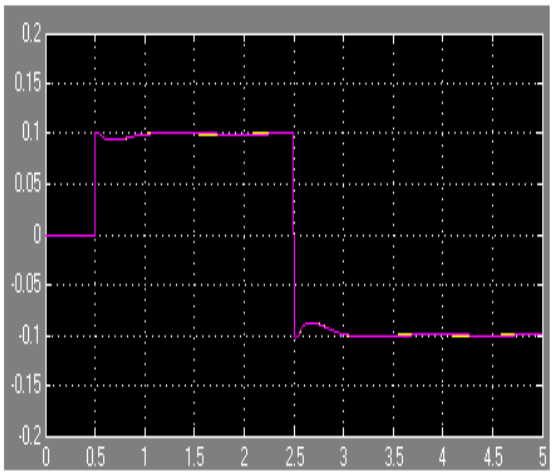


Fig 6.12 (c) Left Side Sprung Mass Displacement

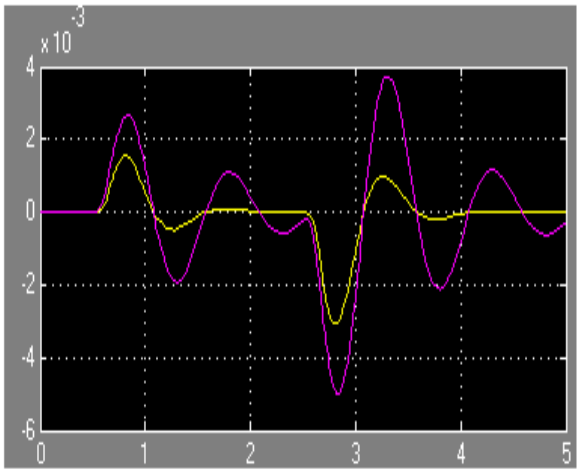


Fig 6.12 (d)Right Side Sprung Mass Displacement

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only Left wheel inputs.

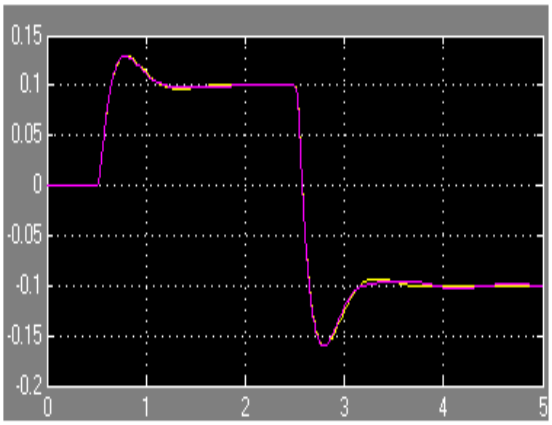


Fig 6.12 (e) Left Side Sprung Mass Velocity

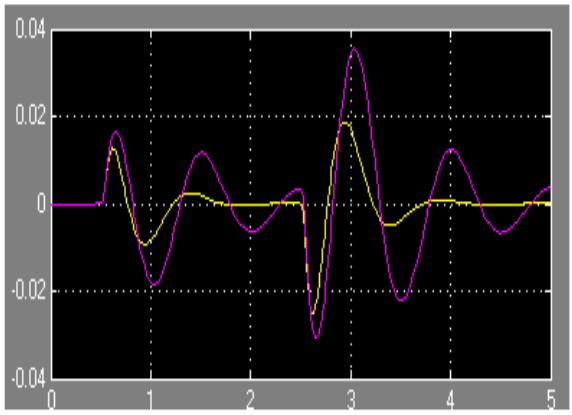


Fig 6.12 (f) Right Side Sprung Mass Velocity

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only Left wheel inputs.

Figure 6.12(c) and figure 6.12(d) shows the relative sprung mass displacement in m with respect to time for both left and right side of the front axle respectively for both active suspension system proposed & active suspension system with single fuzzy logic controller with only left side inputs. Figure 6.12(e) and figure 6.12(f) shows the absolute sprung mass velocity in m/s with respect to time for both left and right side of the front axle car suspension system respectively for both active suspension system proposed & active suspension system with single fuzzy logic controller with only left side inputs.

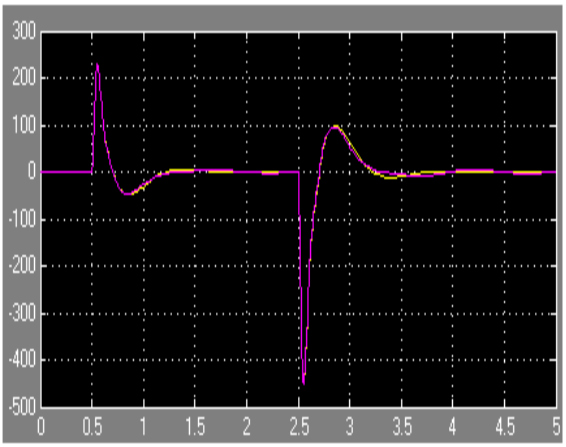


Fig 6.12 (g) Left Side Control

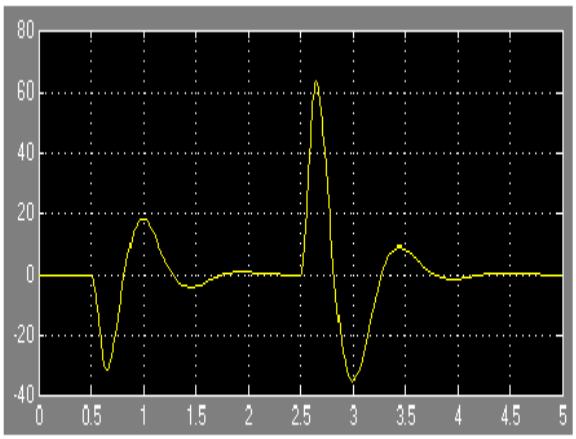


Fig 6.12 (h) Right Side Control

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only Left wheel inputs

Figure 6.12 Sprung Mass Displacement & Velocity for two FLC's vs one FLC with only left wheel input and Control Outputs for both the system, with displacement only on the left wheel

Figure 6.12(g) and figure 6.12(h) shows the control signal generated by both the systems in Newton with respect to time for left side and right side respectively.

The responses achieved in this case are very close. Hardly any difference can be noticed between the responses achieved by both the system.

All the three cases in the section prove that the proposed fuzzy logic controllers system suppresses the vibrations very quickly and performs even when one of the controller stops performing.

6.5 Simulation IV: System with Two Fuzzy Logic Controller vs. One fuzzy Logic Controller with only Right Wheel Input

In this section two different control methodologies are compared. Both the methods are Active Control Suspension using Fuzzy Logic Controllers. One control method is with two fuzzy logic controller proposed in the research and the other is with single controller with an input only from the right wheel. The basic reason of the comparison is to prove that the proposed method of using two fuzzy logic controllers for front axle car suspension control is required to achieve better control. The comparison will also help in understanding the performance of proposed in the research even when one of the controller stops performing. Figure 6.13 shows the Matlab-Simulink model used for the front axle suspension control using a single fuzzy logic control with only right side inputs.

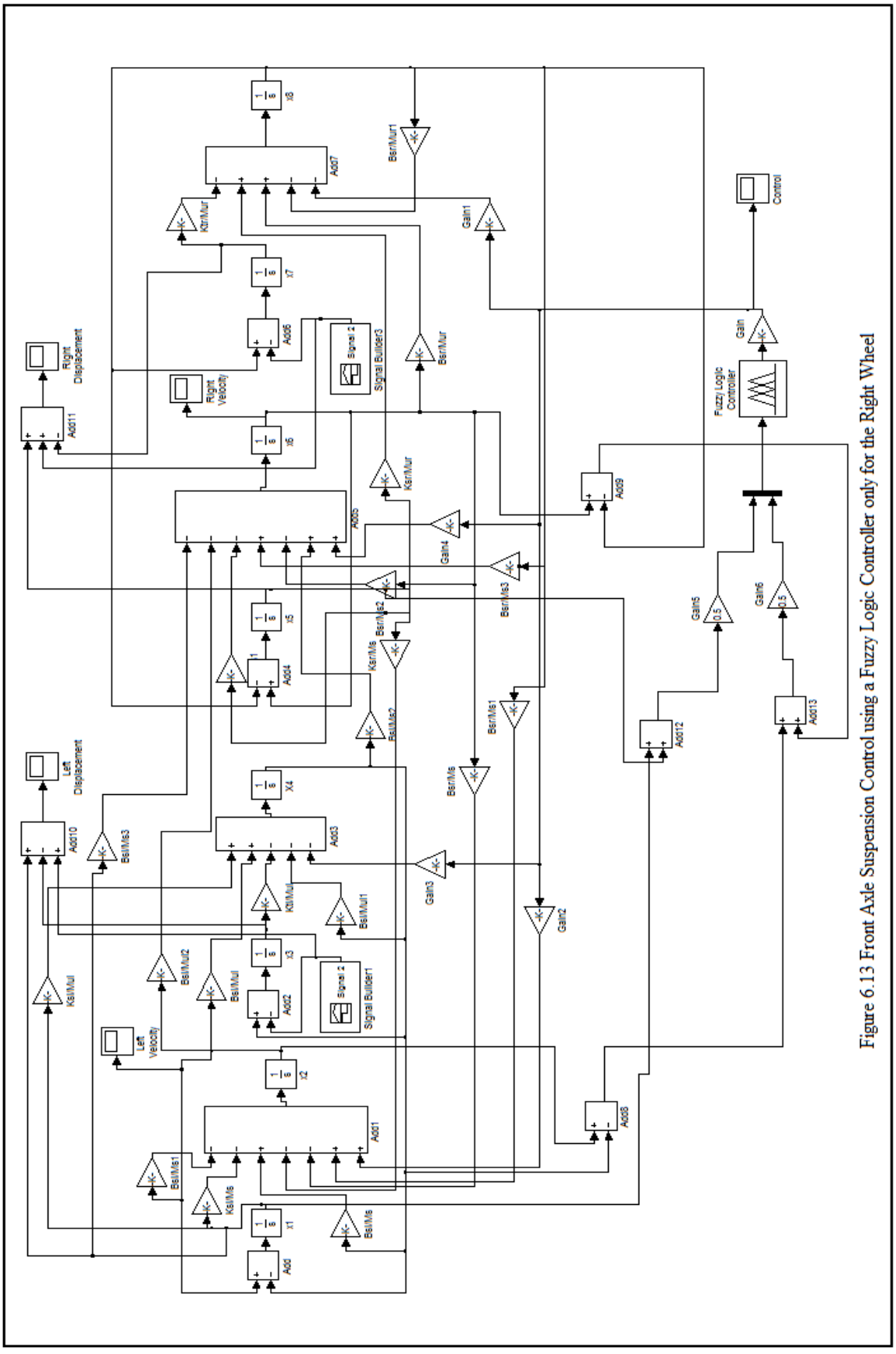


Figure 6.13 Front Axle Suspension Control using a Fuzzy Logic Controller only for the Right Wheel

Responses for both methods with different road irregularities are plotted as shown below:

i. Same bumps on both wheels:

Let both z_{rl} and z_{rr} road disturbances for left and right wheels for front axle car suspension respectively take place in two consequent step input. Figure 6.14(a) and figure 6.14(b) shows the road disturbances which were introduced on both left wheel and right wheel of the front axle suspension system respectively.

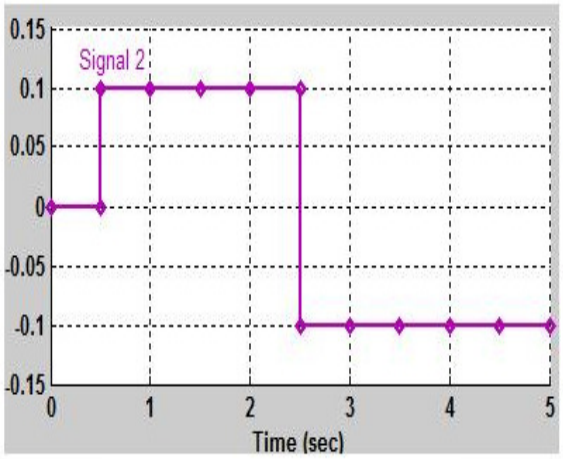


Fig 6.14 (a) Left Wheel Input

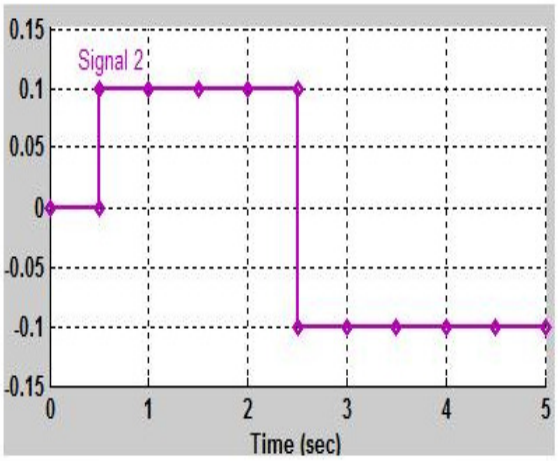


Fig 6.14 (b) Right Wheel Input

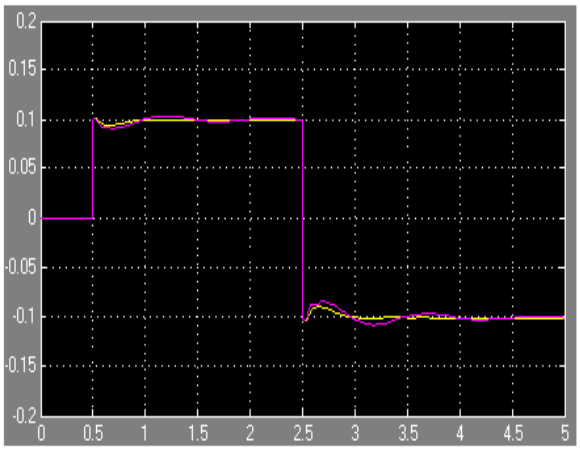


Fig 6.14 (c) Left Side Sprung Mass Displacement

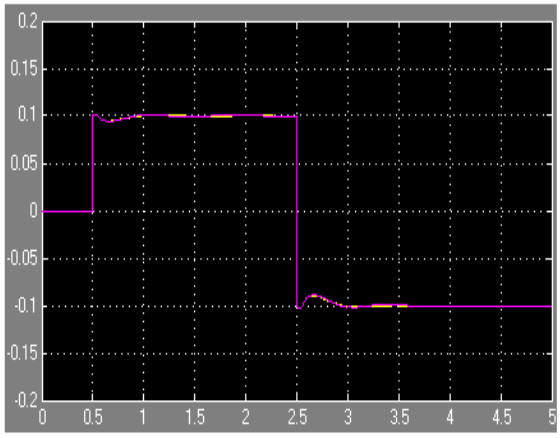


Fig 6.14 (d) Right Side Sprung Mass Displacement

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only right wheel inputs.

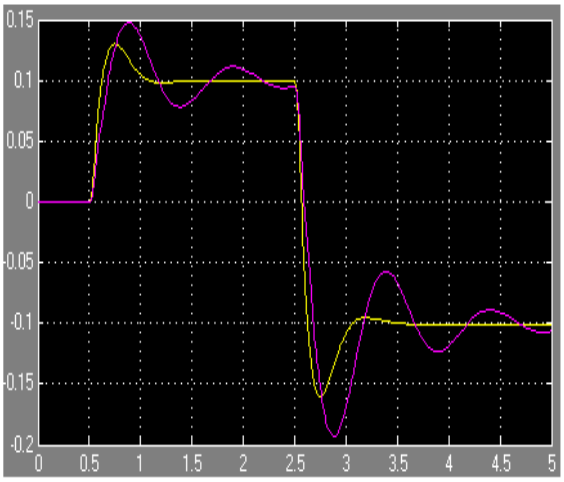


Fig 6.14 (e) Left Side Sprung Mass Velocity

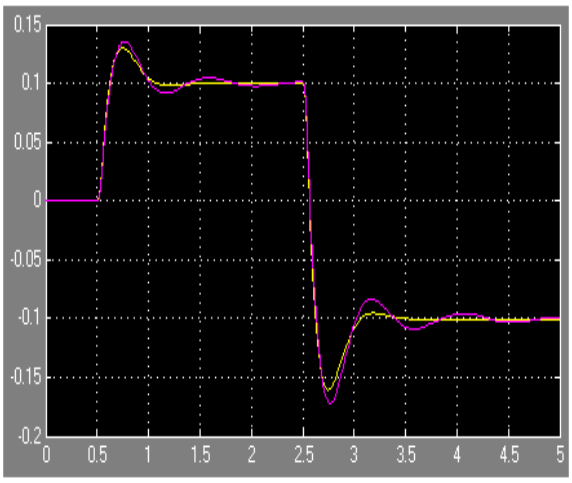


Fig 6.14 (f) Right Side Sprung Mass Velocity

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only right wheel inputs.

Figure 6.14(c) and figure 6.14(d) shows the relative sprung mass displacement in m with respect to time for both left and right side of the front axle respectively for both active suspension system proposed & active suspension system with single fuzzy logic controller with only right side inputs. Figure 6.14(e) and figure 6.14(f) shows the absolute sprung mass velocity in m/s with respect to time for both left and right side of the front axle car suspension system respectively for both active suspension system proposed & active suspension system with single fuzzy logic controller with only right side inputs. Figure 6.14(g) and figure 6.14(h) shows the control signal generated by both the systems in Newton with respect to time for left side and right side respectively.

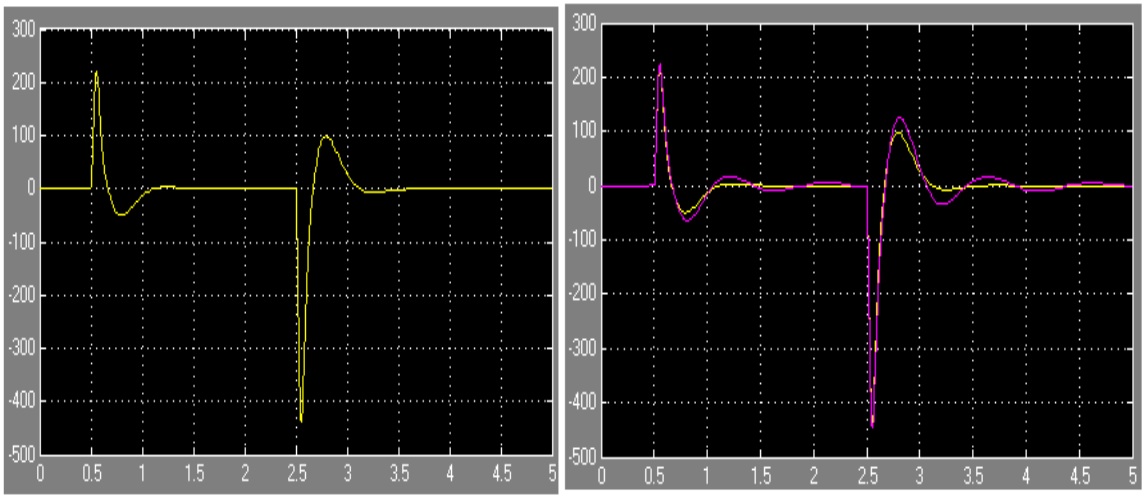


Fig 6.14 (g) Left Side Control

Fig 6.14 (h) Right Side Control

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only right wheel inputs.

Figure 6.14 Sprung Mass Displacement & Velocity for two FLC's vs one FLC with only right wheel input and Control Outputs for both the system

The response clearly shows the system with two fuzzy logic controllers gets the system to settle system much faster than the system with only one fuzzy logic controller with only left wheel input. It clearly shows that even if one of the controller stops performing it doesn't make the car suspension unstable, it just increases the time taken to suppress the vibrations.

ii. Different bumps on both wheels:

Let both z_{rl} and z_{rr} road disturbances for left and right wheels for front axle car suspension respectively. We consider the case when there's no bump on Left wheel and a step variation is introduced on the Right wheel & vice versa.

When left wheel has no bump and right wheel has the step variation, the results are as follows:

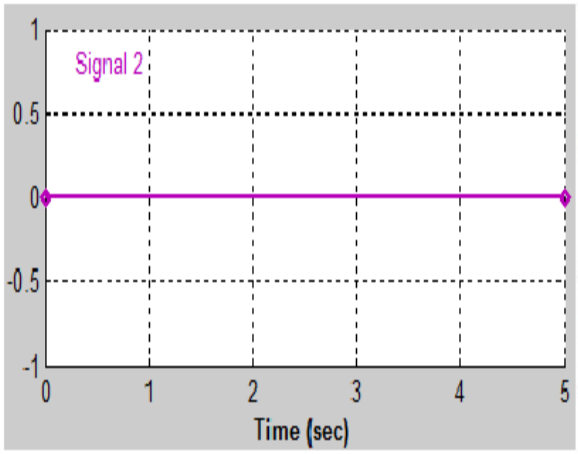


Fig 6.15 (a) Left Wheel Input

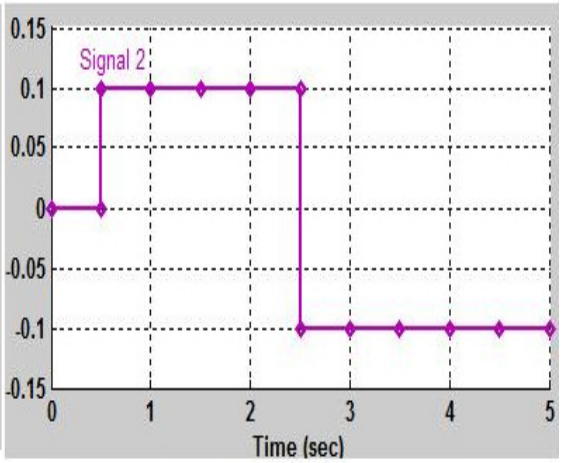


Fig 6.15 (b) Right Wheel Input

Figure 6.15(a) and figure 6.15(b) shows the road disturbances z_{rl} and z_{rr} which were introduced on both left wheel and right wheel of the front axle suspension system respectively.

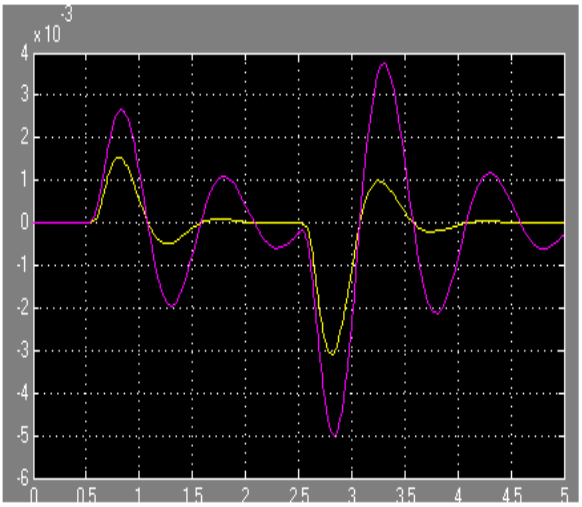


Fig 6.15 (c) Left Side Sprung Mass Dispalcement

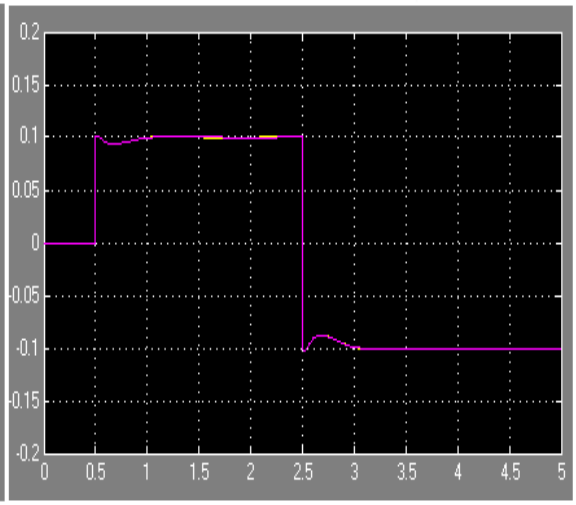


Fig 6.15 (d) Right Side Sprung Mass Displacement

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only Right wheel inputs.

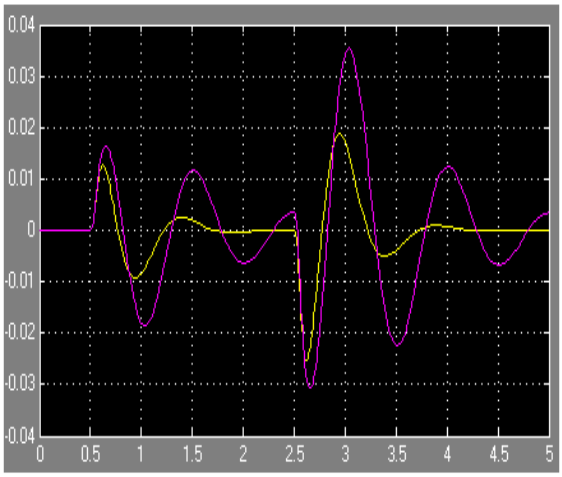


Fig 6.15 (e) Left Side Sprung Mass Velocity

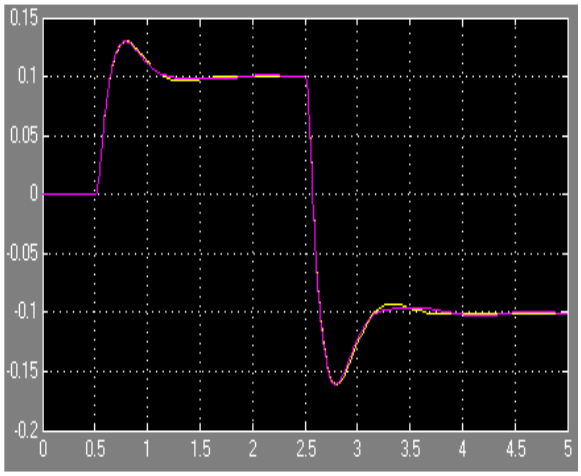


Fig 6.15 (f) Right Side Sprung Mass Velocity

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only Right wheel inputs.

Figure 6.15(c) and figure 6.15(d) shows the relative sprung mass displacement in m with respect to time for both left and right side of the front axle respectively for both active suspension system proposed & active suspension system with single fuzzy logic controller with only right side inputs. Figure 6.15(e) and figure 6.15(f) shows the absolute sprung mass velocity in m/s with respect to time for both left and right side of the front axle car suspension system respectively for both active suspension system proposed & active suspension system with single fuzzy logic controller with only right side inputs.

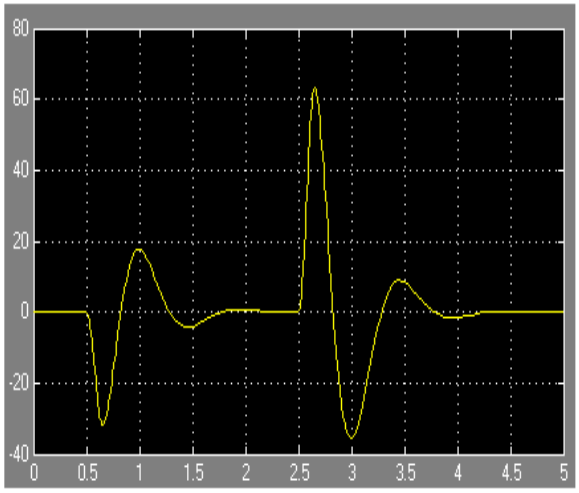


Fig 6.15 (g) Left Side Control

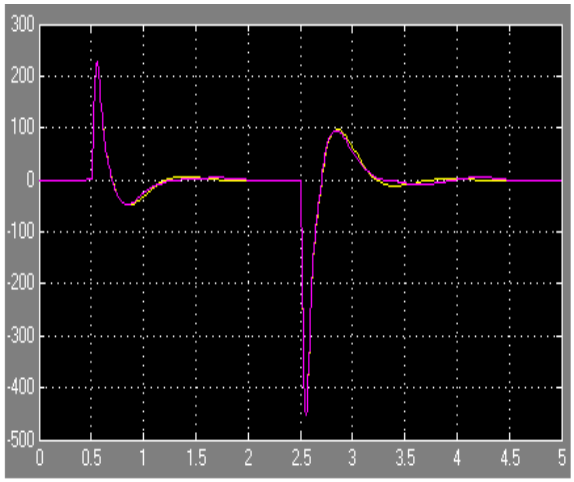


Fig 6.15 (h) Right Side Control

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only Right wheel inputs.
Figure 6.15 Sprung Mass Displacement & Velocity for two FLC's vs one FLC with only right wheel input and Control Outputs for both the system, with displacement only on the right wheel

Figure 6.15(g) and figure 6.15(h) shows the control signal generated by both the systems in Newton with respect to time for left side and right side respectively.

The responses achieved in this case are very close. Hardly any difference can be noticed between the responses achieved by both the system.

When Right wheel has no bump and left wheel has the step variation, the results are as follows:

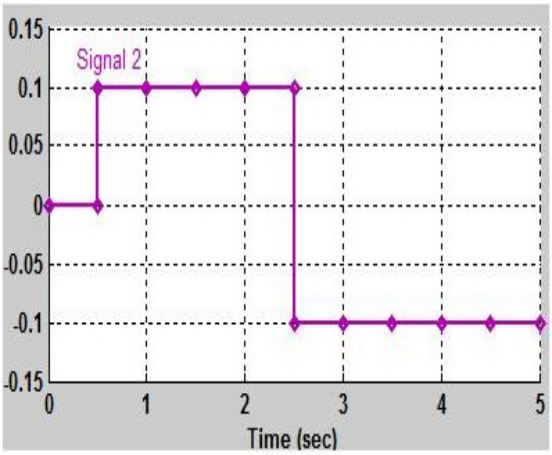


Fig 6.16 (a) Left Wheel Input

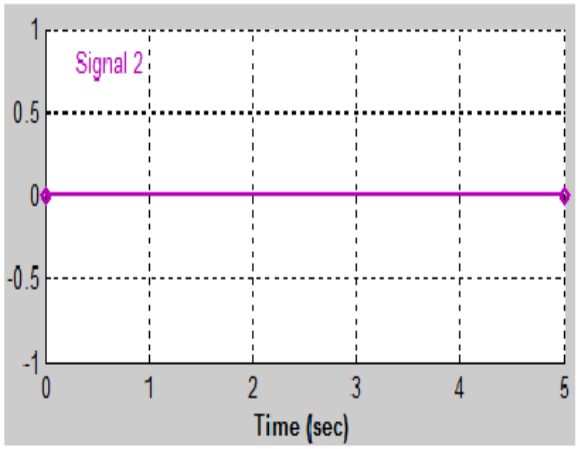


Fig 6.16 (b) Right Wheel Input

Figure 6.16(a) and figure 6.16(b) shows the road disturbances z_{rl} and z_{rr} which were introduced on both left wheel and right wheel of the front axle suspension system respectively.

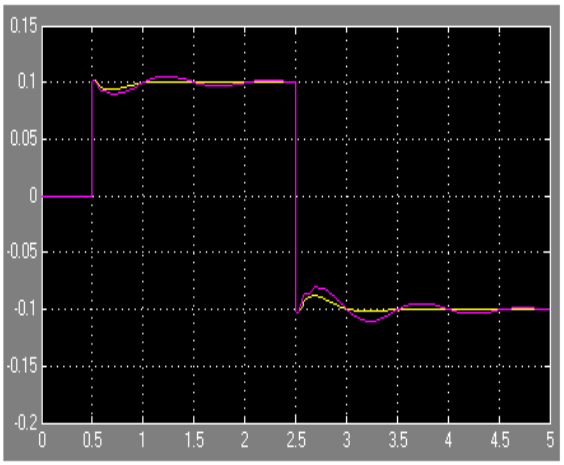


Fig 6.16 (c) Left Side Sprung Mass Displacement

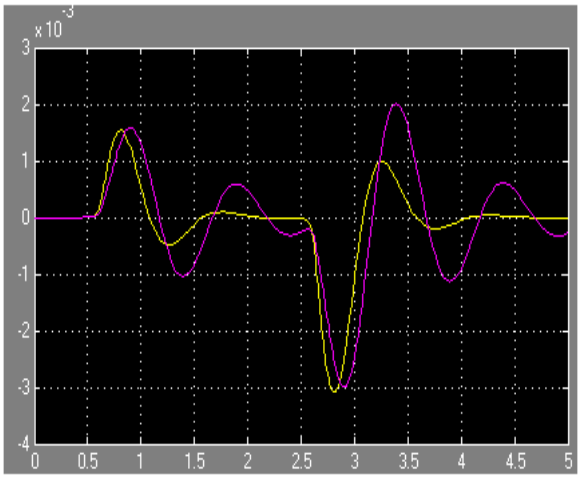


Fig 6.16 (d) Right Side Sprung Mass Displacement

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only Left wheel inputs.

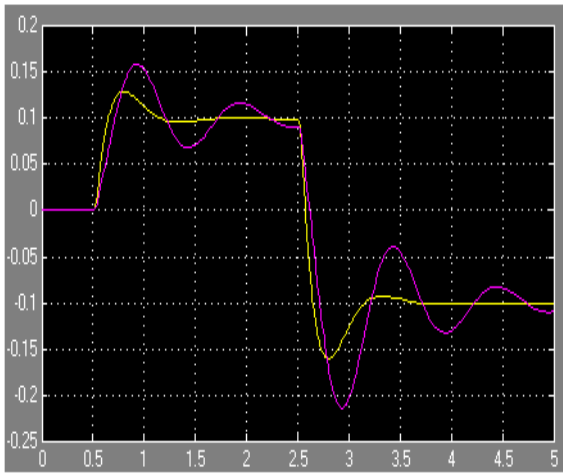


Fig 6.16 (e) Left Side Sprung Mass Velocity

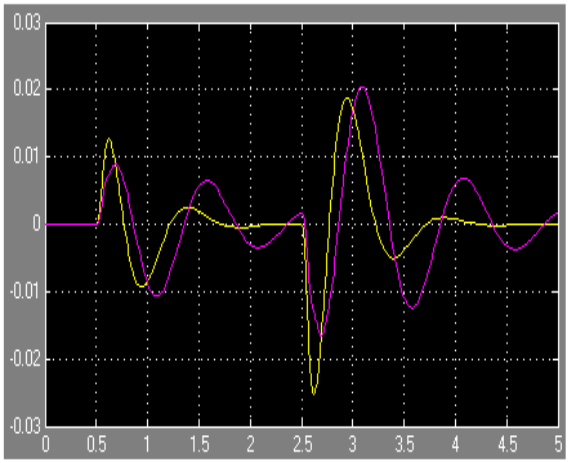


Fig 6.16 (f) Right Side Sprung Mass Velocity

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only Left wheel inputs.

Figure 6.16(c) and figure 6.16(d) shows the relative sprung mass displacement in m with respect to time for both left and right side of the front axle respectively for both active suspension system proposed & active suspension system with single fuzzy logic controller with only right side inputs. Figure 6.16(e) and figure 6.16(f) shows the absolute sprung mass velocity in m/s with respect to time for both left and right side of the front

axle car suspension system respectively for both active suspension system proposed & active suspension system with single fuzzy logic controller with only right side inputs.

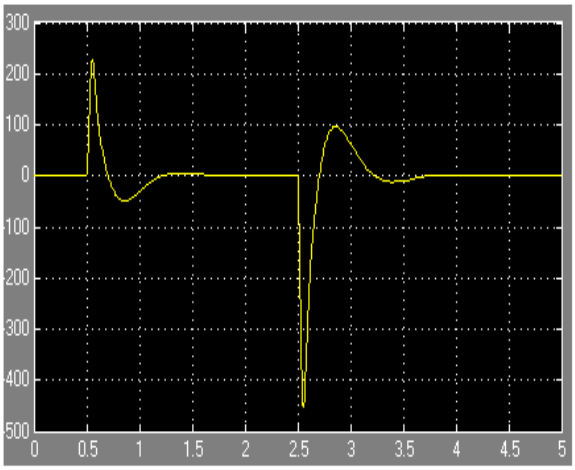


Fig 6.16 (g) Left Side Control

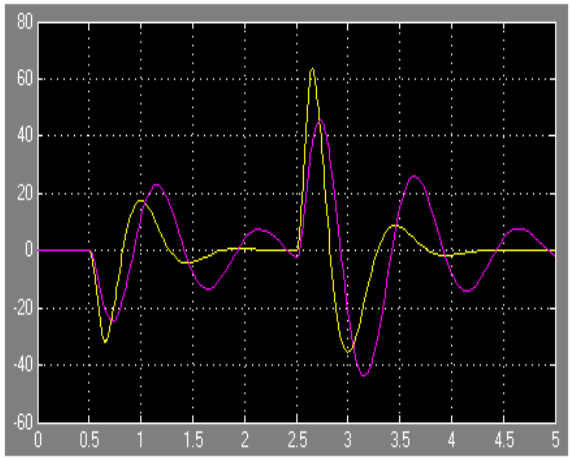


Fig 6.16 (h) Right Side Control

Legends: Yellow: Two Fuzzy Logic Controllers
Magenta: One Fuzzy Logic Controller with only Left wheel inputs.

Figure 6.16 Sprung Mass Displacement & Velocity for two FLC's vs one FLC with only right wheel input and Control Outputs for both the system, with displacement only on the left wheel

Figure 6.16(g) and figure 6.16(h) shows the control signal generated by both the systems in Newton with respect to time for left side and right side respectively.

The response clearly shows the system with two fuzzy logic controllers gets the system to settle system much faster than the system with only one fuzzy logic controller with only left wheel input. It clearly shows that even if one of the controller stops performing it doesn't make the car suspension unstable, it just increases the time taken to suppress the vibrations. It clearly shows that even if one of the controller stops performing it doesn't make the car suspension unstable, it just increases the time taken to suppress the vibrations. Even, when the right side controller is not working and the bump is faced only on the right side the suspension stabilizes but it takes more time to suppress the vibrations.

All the three cases in the section prove that the proposed fuzzy logic controllers system suppresses the vibrations very quickly and performs even when one of the controller stops performing.

Chapter 7: Conclusions & Future Expansion

7.1 Conclusions

In this research, we have developed an approach for fuzzy controller design, of a front axle of car suspension system to enhance ride comfort of passengers and handling. All the results in the last chapter clearly show that the method of control presented in the paper is better than passive suspensions, linear quadratic regulators for the front axle of car suspension system with different road disturbances on both sides of the car as well. Even if one of the controller stops working it doesn't make the car suspension unstable, it just increases the time to suppress the vibrations.

Indeed the output surface of the fuzzy logic controllers can be shaped so as to get this performance, which is inherent property of fuzzy systems. The simulation results clearly show that the designed active suspension system can improve the ride quality and handling by minimizing both the displacement and velocity of vehicle for different road disturbances.

7.2 Future Expansions

The next logical step in the current project is to test the proposed system on the actual experimental test bench to assess the real-world performance of the work in simulation. Another area of good promise for future research is that of proposed system for a full car system.

It will be worth trying out a preview control with fuzzy logic controller, to learn the road disturbance in advance and to compensate for the road variations at exact time. It will help to improve ride quality and handling of the vehicle.

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