Appendix: Schema of Filtering Methodology

Initialize at $\boxed{t=0}$

Pick λ_0, ω_0

Observe u successes

$\boxed{t=u}$

Prior for μ:

$$\mu \sim \text{Exp}(\lambda_0 \delta), \quad \delta \equiv 1/\omega_0$$

Likelihood:

$$\mathcal{L}(\mu; T > u) = \exp\left(\tfrac{-\mu u}{u+1}\right)$$

Posterior (updated) distribution of μ:

$$\mu \sim \text{Exp}(\delta \lambda_u), \quad \lambda_u = \lambda_0 + \frac{u}{u+1},$$
$$\delta \equiv 1/\omega_u, \qquad \omega_u = \omega_0.$$

Observe a failure with n_1 failure modes

$\boxed{t=1}$

Prior for μ_t:

$$\mu \sim \text{Exp}(\lambda_0 \delta), \quad \delta \sim \text{Exp}(\omega_0)$$

Likelihood:

$$\mathcal{L}(\delta; T = 1) = \exp(-\delta n_1 / 1)$$

Posterior (updated) distribution of μ:

$$\mu \sim \text{Exp}(\lambda_1 \delta), \quad \lambda_1 = \lambda_0$$
$$\delta \sim \text{Exp}(\omega_1), \quad \omega_1 = \omega_0 + \frac{n_1}{1}$$

Observe v successes

$\boxed{t=u+v}$

Prior for μ:

$$\mu \sim \text{Exp}(\lambda_0 \delta), \quad \delta \equiv 1/\omega_0$$

Likelihood:

$$\mathcal{L}(\mu; T > u+v) = \exp\left(\tfrac{-\mu(u+v)}{u+v+1}\right)$$

Posterior (updated) distribution of μ:

$$\mu \sim \text{Exp}(\lambda_{u+v} \delta), \; \lambda_{u+v} = \lambda_0 + \tfrac{u+v}{u+v+1}, \; \delta \equiv 1/\omega_0.$$

Observe a failure with n_{u+1} failure modes

$\boxed{t=u+1}$

Prior for μ:

$$\mu \sim \text{Exp}(\lambda_u \delta), \quad \delta \sim \text{Exp}(\omega_u)$$

Likelihood:

$$\mathcal{L}(\delta; T = u+1) = \exp\left(\tfrac{-\delta n_{u+1}}{u+1}\right)$$

Posterior (updated) distribution of μ:

$$\mu \sim \text{Exp}(\lambda_u \delta), \quad \delta \sim \text{Exp}(\omega_{u+1} = \omega_u + \tfrac{n_{u+1}}{u+1})$$

Observe w successes

$\boxed{t=u+1}$

Prior for μ_t:

$$\mu \sim \text{Exp}(\lambda_1 \delta), \quad \delta \equiv 1/\omega_1$$

Likelihood:

$$\mathcal{L}(\mu; T > w) = \exp\left(\tfrac{-\mu w}{w+1}\right)$$

Posterior (updated) distribution of μ:

$$\mu \sim \text{Exp}(\lambda_{w+1} \delta), \; \lambda_{w+1} = \lambda_1 + \tfrac{w}{w+1}, \; \delta \equiv 1/\omega_1.$$

Observe a failure with n_2 failure modes

$\boxed{t=2}$

Prior for μ:

$$\mu \sim \text{Exp}(\lambda_1 \delta), \quad \delta \sim \text{Exp}(\omega_1)$$

Likelihood:

$$\mathcal{L}(\delta; T = 2) = \exp(-\delta n_2 / 2)$$

Posterior (updated) distribution of μ:

$$\mu \sim \text{Exp}(\lambda_2 \delta), \quad \delta \sim \text{Exp}(\omega_2 = \omega_1 + \tfrac{n_2}{2})$$