Improving the Analytical Hierarchy Process Methodology Through Implementation of Consistency Ratio-Based Weights for Multi-Expert Judgement

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Dedication

This body of work is dedicated to…

My wife, Mingi

My children, Luna, Zoe, and Roen

My parents, Mi and Seung, for their lifelong support

And my lifelong friends, Nik, Joe, Kyle, and David

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Abstract of Praxis

Improving the Analytical Hierarchy Process Methodology Through Implementation of Consistency Ratio-Based Weights for Multi-Expert Judgement

The Analytical Hierarchy Process (AHP) is a methodology for multi-criteria decision making where complex decision problems are disaggregated into pairwise comparisons. A critical procedure in AHP is the confirmation of an expert's logical validity through the calculation of a consistency ratio (Saaty, 2008). For analysis where multiple experts are utilized to establish the judgement scale, variability in consistency is not accounted for on an individual basis. Furthermore, inconsistent feedback results in either reduction of the dataset, or reevaluation of the survey by the expert; both conditions confounding the methodology.

Developing a Consistency Ratio-Based Weight (CRW AHP) for multi-expert judgement provides improvements in aggregated consistency. This proposed methodology augments the analysis to increase the impact of judgement from experts with favorable consistency scores, while eliminating the conditions for data loss. The AHP methodology, in any multi-criteria decision problem being supported by feedback from more than one expert, benefits from the proposed improvements because the conditions for data loss or re-surveying are avoided. Industry utilization for the proposed method can range from the selection process of drug therapies to studies in accident precursors; topics addressed as part of the overall research validation strategy.

The CRW AHP methodology requires the definition of a hierarchical structure to set criteria for assessment. Experts are then surveyed to generate a priority matrix

representing assessment of all pairwise comparisons, and a max eigenvalue is calculated to represent an intermediate criteria weight. These results are tested for consistency to ensure an appropriate level of transitivity and reciprocity has been achieved by the expert being surveyed. Based on the consistency ratios of all experts, a weight is then calculated to augment the max eigenvalues prior to aggregation. The intent of the methodology is to increase the aggregated consistency ratio without retraining or normalizing the experts as part of an re-surveying effort.

The results of the research provided four key deductions regarding the analytical functionality of the CRW AHP methodology. First, with a p-value of 0.047 per paired T test, a statistically significant difference between the calculated consistency ratios between CRW AHP and AHP was measured (α=0.05). The consistency ratio from the CRW AHP method was lower, and thus more consistent when compared to AHP. Second, with a p-value of 0.167 per paired T test, no statistical difference between the standard error for the calculated criteria weights between the two methodologies was measured (α=0.05). Third, with a p-value of 0.156 per equivalence testing, the two methodologies resulted in non-equivalent results when calculating criteria weights as part of the AHP assessment. Lastly, with a p-value of 0.014 per two-sample T test, a statistically significant bimodal correlation between consistency ratios and negative versus positive AHP outcomes was measured.

The research efforts have helped define a novel methodology for utilizing an expert's consistency ratio to calculate weights for proportionally adjusted aggregation of max eigenvalues in an AHP analysis. Furthermore, the CRW AHP methodology enables intaking non-normalized or non-reevaluated expert judgement without requiring training

or modulation of pairwise comparisons during the survey process. CRW AHP therefore offers a novel and efficient method of addressing issues with inconsistency without incurring additional costs associated with data loss or re-surveying.

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List of Symbols

$\mathbb{A}$	Pairwise comparison matrix
i, j, k	Alternatives
a_{ij}	A pairwise comparison of alternatives i and j
$\mathbb{E}$	Number of comparisons
$\mathbb{X}$	Normalized standard matrix
$\mathbb{W}$	Weighted analytical vector
λ_{max}	Principal eigenvalue (max eigenvalue)
CI	Consistency index
RI	Random index
CR	Consistency ratio
D	Deviation matrix
$\mathbb{R}$	Fuzzy preference relations matrix
k	Number of iterations for a corrective algorithm
CR_{sum}	Total consistency value
$CR_{norm,i}$	Normalized consistency vector
CR_{sn}	Total normalized consistency value
CW_i	Consistency ratio-based weight
$\lambda_{max\ crw}$	Corrected max eigenvalue
CI_{crw}	Weighted consistency index
$\mathbb{W}_{CR,j}$	Consistency ratio-based weighted vector with j parameters

List of Acronyms

AHP	Analytical Hierarchical Process
CRW AHP	Consistency Ratio-Based Weights for Analytical Hierarchical Process
CR	Consistency ratio
CI	Consistency index
FRP	Fuzzy preference relation

Chapter 1—Introduction

1.1 Background

The analytical hierarchy process (AHP) is a widely adopted methodology for multi-criteria decision making where complex decision problems are disaggregated into pairwise comparisons (R. W. Saaty, 1987). The method derives priority scales for a set of criteria and alternatives based on expert judgement, and structures of consistent feedback matrices are utilized to derive approximate weights from eigenvectors (Forman & Gass, 2001).

Several technical challenges have been identified for the implementation of AHP. Sensitivity in results, lack of control for uncertainty from expert judgement, and lack of consistency within the survey matrix have been studied as common limitations to the methodology (Nukala & Gupta, 2005). The consistency of an expert's judgement, which reflects the degree of logic across the survey matrix, often challenges the validity of the results (T. L. Saaty, 2003). When consistency for any given expert judgement is not achieved, the data is lost from the analysis, or the expert is re-surveyed. These conditions add uncertainty and cost to efforts in decision making.

1.2 Research Motivation

The primary motivation for research into a Consistency Ratio-Based Weight methodology in AHP is to abridge potential failure modes in lack of consistency for any given expert providing judgement, and to provide a mechanism for diluting out low-consistency judgement in a multi-expert setting without incurring data loss. Achieving consistency is a critical premise to the AHP methodology as the pairwise comparison

matrices must satisfy the transitivity property; the arithmetic being characterized in prior work (Alonso & Lamata, 2006). Conditions for data loss or re-surveying and re-training of experts as part of recalibration of the judgement can be restrictive to the utility of the AHP methodology. In order to support future use cases for AHP as a decision-making tool, this research will aim to provide a novel methodology for mitigating the limitations of low-consistency circumstances.

1.4 Problem Statement

The Analytical Hierarchy Process methodology can be inhibited by inconsistent feedback from expert judgement, resulting in up to 52% data loss for a survey (Apostolou & Hassell, 1993).

Inconsistency in an expert's judgement reflects a lack of concurrent logic within the normal survey matrix of pairwise comparisons. As the number of criteria and alternatives increase, it is more likely for an expert's judgement to be hindered by low consistency (Karthikeyan et al., 2017). A majority of AHP-based implementations result in issues with expert consistency, therefore a method for filtering the judgement without incurring data loss will have utility (Kuenz Murphy, 1993). As failed surveys with an AHP assessment can lead an organization to commit to non-optimal decision-making, it is critical that the concept of transitivity and reciprocity in the experts' pairwise judgement is facilitated (Ji & Jiang, 2003).

1.4 Thesis Statement

Implementing a Consistency Ratio-Based Weight (CRW) methodology prior to the aggregation of multi-expert judgement will increase the weight of judgement from experts with favorable transitivity and improve the aggregated consistency score of the survey.

The intent of the research is to evaluate methods of improving the analytical methodology by eliminating the need for secondary surveys or data filtering. Furthermore, experts who provide a higher level of consistency in their pair-wise assessment will be weighed higher during the aggregation of AHP criteria weights. The intended outcome is a more favorable consistency score without a severe impact on the standard error of the analytical output.

1.5 Research Objectives

The objectives of this research are to develop a novel methodology (CRW AHP) to address issues with AHP inconsistency, and to validate the findings per raw data from literature references, where raw AHP analysis is provided across varying industry use cases or research. The use case for the proposed research product is any conditional setting where multi-expert judgement is utilized as part of an AHP decision-making analysis. Surveys in AHP, where a matrix of criteria or alternatives are required, will be examined to assess the impact of CRW AHP on the results relative to Saaty's proposed AHP methodology (R. W. Saaty, 1987). Literature results for AHP implementation across various industries will used to interrogate the utility of the CRW AHP methodology. Details for the use cases are provided in Chapter 2 and Chapter 3.

1.6 Research Questions and Hypotheses

The proposed research questions aim to assess the analytical methodology and utility of the Consistency Ratio-Based Weight AHP approach. Following weighted aggregation of expert judgement, the impact on the resulting consistency and standard error will be interrogated. Furthermore, the research assesses if the proposed methodology augments the results of AHP. Validation of the utility in improving the

consistency ratio will be confirmed by measuring for a correlation between aggregated consistency scores and the outcome or research of a survey from literature references.

RQ1: Will implementing a Consistency-Ratio Based Weight prior to judgement aggregation increase overall analytical consistency?

RQ2: Will including all expert feedback per Consistency-Ratio Based Weights increase the standard error for a comparison matrix?

RQ3: Will implementing a Consistency-Ratio Based Weight prior to judgement aggregation augment the results of comparison with respect to the classical approach?

RQ4: Can a correlation between AHP consistency ratios and reported survey validity from literature case studies be confirmed?

Hypotheses are as follows:

H1: CRW AHP method will achieve at least a 5% improvement in aggregated consistency over the AHP method.

H2: CRW AHP method analysis will result in less than a 10% difference in standard error compared to the AHP method.

H3: CRW AHP method will result in at least a 5% difference in calculated weights compared to the AHP method.

H4: Regression analysis between consistency ratios and qualitative survey success from literature case studies will suggest a statistically significant correlation.

1.7 Scope of Research

With focus on the research questions, the scope of work includes testing of a novel methodology to weigh expert judgement prior to aggregation and analysis. Furthermore, the research aims to evaluate the impact of the proposed method on the standard error and weighted results of the analysis. The research will utilize raw data presented in a set of literature references where multi-expert judgement was collected as part of implementation of the methodology in various industries.

1.8 Research Limitations

The research is limited to the AHP methodology, and other multi-criteria decision analysis where judgement matrices are utilized will not be addressed. Furthermore, the utility of the methodology will be validated as part of literature references involving the selection process of drug therapies, supply chain management, and prediction of accident precursors.

1.9 Organization of Praxis

This praxis is organized into five distinct areas of discussion as part of the main body of the document. In respective order, the five chapters are: introduction, literature review, summary of the research methodology, results, and conclusions derived from the research. In addition to the main body, references and appended material will also be provided to supplement the work.

Chapter 2—Literature Review

2.1 Introduction

An overview of the AHP methodology in both development and implementation since its inception, with a focused review on the significance of the consistency ratio and how eigenvalues are utilized to calculate criteria weights, is necessary to build a case for the research. In addition, a summary of prior efforts in AHP to mitigate issues in consistency will be reviewed to understand how other researchers have approached the problem.

The AHP methodology has been implemented and further characterized by practitioners since its introduction in 1986. In an assessment of the methodical advancement of the tool, Emrouznejad and Marra defines three periods of development for the methodology (Emrouznejad & Marra, 2017). The first period of development (1979 – 1990) acknowledges the inception of AHP by Saaty, to the characterization of inherent limitations for the approach. Two of the critical issues that have been identified for the AHP methodology are the phenomenon of rank reversal, and the requirement for consistency in an expert's judgement (Vargas, 1990). Rank reversal represents the potential of an expert's judgement in a pairwise comparison to reverse when secondary criteria are added to the survey. Extensive work has been dedicated to addressing this issue mathematically. Wang and Elhang summarized efforts in research, where normalization of the eigenvectors derived from the pairwise matrices could be achieved by using value maximums instead of alternative sums (Wang & Elhag, 2006).

The second and third periods (1991 – Current), as defined by Emrouznejad, is highlighted with several mathematical advancements for the methodology, and utilization of AHP in an expanded set of use cases. Arbel and Vargas proposed the utilization of fuzzy numbers to represent interval or pairwise comparisons as a new approach to deriving priorities (Arbel & Vargas, 1993). Furthermore, methods for weight generation using linear programming were proposed to avoid the need for calculating eigenvectors when fitting pairwise indexes (Hosseinian et al., 2012).

These improvements further enabled the use of the AHP methodology in a wide range of industry and academic sectors (Li et al., 2013). In medicine, AHP has been utilized to drive front-line decisions with direct impact to patients; a recent example being a framework for convalescent plasma transfusion in patients suffering from COVID-19 (Mohammed et al., 2021). Additional use cases in standardizing the selection of therapeutic regimens for patients being treated with traditional medicines were described by medical professionals. (Moon et al., 2015). In supply chain, AHP has been implemented to model program risk and assist in selection of critical supplies as part of a manufacturing value chain (Radivojević & Gajović, 2014). Additional examples of the methodology being utilized in supplier management and logistics demonstrated the value of AHP in dynamic systems, where quantitative uncertainty cannot be properly addressed (Babak Daneshvar Rouyendegh, 2012). Adebanjo implemented AHP to supplement lean supply chain management in the healthcare sector to better address inventory risk in the medical field (Adebanjo et al., 2016). Ding utilized the methodology to decide the location of building out industrial landfills based on proximity and impact to the local environment (Ding et al., 2018).

The extensive use of the AHP methodology across a diverse set of industries has potentially demonstrated its value and progressing validity as a tool in multi-criteria decision analysis. However, areas for improvement remain within the mathematical basis of the approach (Li et al., 2013). Inconsistency of expert judgement is an intrinsic constraint that practitioners have to address (Martin et al., 2018). Although several approaches for mitigating issues with low consistency in survey feedback have been proposed, opportunities may remain in augmenting the methodology to provide a basis for analytics without data loss. Section 2.2 provides a review of what the consistency ratio reflects in the context of pairwise comparisons in AHP, and Section 2.3 summarizes key prior efforts in addressing low consistency issues in a multi-expert setting.

2.2 Consistency Scores for Pairwise Comparison Matrices

As defined by Satty's seminal efforts, consistency refers to the measure of the transitivity of preferences in an expert's judgement when pairwise comparisons are utilized (R. W. Saaty, 1987). Lack of consistency, or transitivity in preferences, results in a nullification of the data because of the implications towards issues in the logical structure of the expert's feedback (Brunelli & Fedrizzi, 2015a). Various reasons in survey efforts may result in inconsistent judgement; Cheng proposed three main areas of causality (Cheng & li, 2003):

- *Lack of domain knowledge* – The expert does not possess adequate knowledge or experience in the relevant field, and therefore cannot provide a well-informed assessment of the conditions being interrogated in the pairwise comparisons.

- *Arbitrary judgement* – The expert provides a subjective input to the pairwise comparison due to a lack of opinion in the problem set.
- *Carelessness* – The expert does not provide a thorough assessment of the pairwise comparisons due to a lack of interest or engagement

In addition to the drives for inconsistency proposed by Cheng, an additional condition to be considered may be the quality of the survey design. Saaty advised on the importance of educating involved experts on the methodology and designing surveys with a limited set of criteria and alternative conditions so to minimize the likelihood of misunderstanding (T. L. Saaty, 2003). It is proposed by Stoklasa that a well-designed study, where the criteria are focused on the principal conditions of the problem and appropriate experts are identified and informed on the methodology, will result in a favorable consistency score when implementing AHP (Stoklasa et al., 2013).

In order to calculate the consistency of an expert's judgement in a pairwise comparison, the following rules of reciprocity (Equation 2) and transitivity (Equation 3) must be satisfied (A. Ishizaka & Lusti, 2004).

For a comparison matrix $\mathbb{A}$ (Equation 1), with alternatives $i, j, and\ k$:

$$\mathbb{A} = \begin{bmatrix} 1 & a_{ij} & a_{ik} \\ \frac{1}{a_{ij}} & 1 & a_{jk} \\ \frac{1}{a_{ik}} & \frac{1}{a_{jk}} & 1 \end{bmatrix} \qquad \textit{Equation 1}$$

$$a_{ij} = \frac{1}{a_{ji}} \qquad \textit{Equation 2}$$

$$a_{ij} = a_{ik} \cdot a_{kj} \qquad \textit{Equation 3}$$

With the rule of reciprocity (Equation 1) satisfied, it is satisfactory to confirm transitivity (Equation 2) of one of the two symmetrical components of matrix $\mathbb{A}$. Therefore, a limited number of comparisons, $\mathbb{E}$, must be confirmed per the provided symmetry in the matrix (Equation 4).

$$For\ each\ element\ a_{ij}, \mathbb{E} = j - (i + 1) \qquad \textit{Equation 4}$$

As a supplemental condition to Equation 4, for an $n \times n$ matrix containing n^2 comparisons, $n - 1$ of the matrix components can avoid being tested for transitivity and reciprocity (Pedrycz & Song, 2011).

The practical examples that Ishizaka constructed involve a basic logical comparison of objects (Alessio Ishizaka & Lusti, 2006). Transitivity demonstrates that if *i* is favored twice as much as *j* ($a_{ij} = 2$), and *j* three times as much as *k* ($a_{jk} = 3$), then you must favor *i* six times as much as *k* ($a_{ik} = 6$). Reciprocity demonstrates that if *i* is favored twice as much as *j* ($a_{ij} = 2$), then *j* is favored half as much as *i* ($a_{ji} = \frac{1}{2}$).

Assessment of consistency requires the calculation of the consistency index and consistency ratio (Xu & Xu, 2020). Per Saaty's initial efforts, the AHP methodology requires the normalization of the comparison matrix $\mathbb{A}$ (Equation 1) into a normalized standard matrix, $\mathbb{X}$ (Equation 5):

$$\mathbb{X}_{ij} = \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}} \qquad \textit{Equation 5}$$

The weighted AHP vector, $\mathbb{W}$, is then calculated (Equation 6):

$$\mathbb{W}_{ij} = \frac{\sum_{j=1}^{n} \mathbb{X}_{ij}}{n} \qquad \textit{Equation 6}$$

A critical component to the calculation of the consistency ratio is the principal eigenvalue, λ_{max}, of the weighted AHP vector. The principal eigenvalue is calculated by

summing the products for each elemental eigenvector (weight, $\mathbb{W}$) and the column sums of the reciprocal matrix (Hosseinian et al., 2012). To further elaborate on the logical basis for utilizing the principal eigenvalue in calculation consistency, Lane derived a mathematical reasoning (Lane & Verdini, 1989). With $\mathbb{I}$ as the identity matrix for the problem set:

$$(\mathbb{A} - \lambda \times \mathbb{I})\mathbb{W} = 0 \qquad \textit{Equation 7}$$

Equation 7 implies the following:

$$(\mathbb{A} - \lambda \times \mathbb{I}) = 0 \qquad \textit{Equation 8}$$

Therefore:

$$\begin{vmatrix} 1-\lambda & i & {}^{1}/_{k} \\ {}^{1}/_{i} & 1-\lambda & j \\ k & {}^{1}/_{j} & 1-\lambda \end{vmatrix} = 0 \qquad \textit{Equation 9}$$

Conclusion of the determinant in Equation 9 results in:

$$(1-\lambda)^3 + i \times j \times k + 1/\times j \times k - 3 \times (1-\lambda) = 0 \qquad \textit{Equation 10}$$

Equation 10 reveals that the eigenvalues are equal for all square criteria matrices.

The consistency index, *CI*, is the direct measure of reliability of an expert's pairwise comparison matrix (Brunelli & Fedrizzi, 2015a). In order to calculate *CI*, the principal eigenvalue must be augmented by the size of the square comparison matrix (T. L. Saaty, 2003) :

$$CI = \frac{\lambda_{max} - n}{n - 1} \qquad \textit{Equation 11}$$

The random index, *RI*, represents the consistency index for a matrix of randomly generated pairwise comparisons (Donegan & Dodd, 1991). Inclusion of the random index in the assessment of consistency is required in order to understand the proportion of

inconsistency that is non-random (Donegan & Dodd, 1991). Several statistical methods for calculating *RI* have been proposed, but it is conventional for AHP practitioners to utilize an adaptation of Saaty's average random indices for common matrix sizes (Cheng & li, 2003).

Table 1. Saaty's Average RI Scoring

Size of Matrix	**3**	**4**	**5**	**6**	**7**
Average *RI*	0.58	0.90	1.12	1.24	1.32

Many variants of calculating *RI* have been provided by researchers to address issues of non-relevant estimations of confidence across different use cases for AHP. Tummala described the basic definition for a mean *RI*, *MRI* (Rao Tummala & Wan, 1994).

$$MRI(n) = \frac{\bar{\lambda}_{max} - n}{n - 1} \qquad \textit{Equation 12}$$

A power method has been proposed by Golub to increase the confidence in the computation of the random index, but the efforts did not account for expanded conditions where there exist perturbations within the comparison matrix (Golub et al., 1985). Table 2 provides a referenced summary created by Alonso and Lamata of calculated random indices to demonstrate the level of variability across methodologies (Alonso & Lamata, 2006).

Table 2. Calculation of Random Indices from Various Methods

n	**Oak Ridge**	**Wharton**	**Golden Wang**	**Lane, Verdini**	**Forman**	**Noble**	**Tumala, Wan**	**Aguaron**	**Alonso, Lamata**
3	0.382	0.58	0.5799	0.52	0.5233	0.49	0.500	0.525	0.5245
4	0.946	0.90	0.8921	0.87	0.8860	0.82	0.834	0.882	0.8815
5	1.220	1.12	1.1159	1.10	1.1098	1.03	1.046	1.115	1.1086
6	1.032	1.24	1.2358	1.25	1.2539	1.16	1.178	1.252	1.2479
7	1.468	1.32	1.3322	1.34	1.3451	1.25	1.267	1.341	1.3417
8	1.402	1.41	1.3952	1.40		1.31	1.326	1.404	1.4056
9	1.350	1.45	1.4537	1.45		1.36	1.369	1.452	1.4499
10	1.464	1.49	1.4882	1.49		1.39	1.406	1.484	1.4854
11	1.576	1.51	1.5117			1.42	1.433	1.513	1.5141
12	1.476		1.5356	1.54		1.44	1.456	1.535	1.5365
13	1.564		1.5571			1.46	1.474	1.555	1.5551
14	1.568		1.5714	1.57		1.48	1.491	1.570	1.5713
15	1.586		1.5831			1.49	1.501	1.583	1.5838

For the purposes of this research, Saaty's predefined *RI* values is utilized as part of the proposed methodology, as the mathematical basis for improving consistency is relative to the consistency index. As the denominator to calculating the consistency ratio, it is critical for practitioners of AHP to understand the implications when selecting a methodology for calculating the random index (Cheng & li, 2003). Furthermore, since the estimated value of the random index increases in uncertainty as the comparison matrix increases in size, limiting the scope of the survey should be considered as part of the overall design (Martin et al., 2018).

The final step for assessing consistency is the calculation of the consistency ratio, *CR*. The consistency ratio is a proportional assessment between the confidence index and then random index, per Equation 13 (R. W. Saaty, 1987).

$$CR = \frac{CI}{RI} \qquad \textit{Equation 13}$$

Saaty proposed that an expert's judgement is consistent if $CR \leq 0.10$. This, by definition, claims that less than or equal to 10% of the lack of transitivity in an expert's feedback is non-random. The higher the *CR* for an expert, the more it implies that a lack of logic exists within the pairwise comparison (Tung, 1998).

The AHP methodology requires the assessment of the consistency ratio per expert before completion of the analytical process (T. L. Saaty, 2003). If consistency is not achieved, the expert's feedback is either removed from the assessment prior to multi-expert aggregation, or the individual can be revisited with an additional opportunity to execute the pairwise comparisons. Successfully intaking consistent feedback from a survey is critical to the validity of the assessment, as the AHP is inherently built from a qualitative basis (Brunelli & Fedrizzi, 2015b). Various efforts in literature have been presented in addressing issues with inconsistency. The methodologies thus far have been focused on survey design, expert re-assessment, and augmentation of how consistency is calculated based on the construction of the pairwise comparison matrix. In cases where mathematical methods are utilized to correct issues with consistency, the risk of impacting information fidelity from the original pairwise comparison must be considered (Alessio Ishizaka & Lusti, 2006). Furthermore, the point in which corrections are implemented in the serial components of the AHP methodology must be assessed to ensure improvements in consistency are not artificial (Dong et al., 2010).

2.3 Prior Efforts in Addressing Inconsistency

Table 3 provides an overview of methodologies previously proposed to address concerns around inconsistency when collecting expert judgement for AHP. Additional background information and evaluation of each methodology is subsequently provided.

Table 3. Overview of Inconsistency Improvement Methodologies

Methodology & Background	Author(s)	Date Introduced
1. Use of expert modules Implementation of an expert module to detect transgressions from objectives of analysis during survey process	Ishizaka, Lusti	2004
2. Use of an evolutionary algorithm and random search Adjustment of surveys based on random screening and categorization of consistency scores	Han	2014
3. Modification of the random index Definition of desired confidence, $\boldsymbol{\alpha}$, and modification of the value as part of the random index based on the use case	Alonso, Lamata	2006
4. A non-matrix-based approach for estimating CR For 3 tuple matrices, a linear equation is utilized to gauge and adjust consistency	Xu, Xu	2020

Table 2 Continued…

Methodology & Background	Author(s)	Date Introduced
5. Interactive consistency Implementation of algorithm for assisting experts during judgement of pairwise comparisons	Ishizaka, Siraj	2020
6. Expert confidence-based assessment of consistency Experts provide confidence measures that can be used to calculate consistency across criterion	Yu	2017
7. Discretization of revisited surveys Algorithm is provided to experts being re-surveyed to focus on logical gaps	Jin, Wu, Xiao	2019
8. Use of fuzzy preference relations Transformation of comparison matrix into an fuzzy preference matrix to recalculate consistency	Hamka, Harjono	2020
9. Exponential smoothing technique Use of a recalculated deviation matrix to continuously improve maximum consistency	Cao	2012

1. *Use of expert modules* (A. Ishizaka & Lusti, 2004)

   The proposed methodology requires direct interactions with the expert being surveyed. Based on a predefined threshold for confidence, the expert is notified of potential errors in consistency during the survey process. In order to accomplish this, the rule of transitivity must be supplemented with a term representing error:

$$a_{i,j} = a_{i,j+1} \cdot a_{i+1,i+2} \cdot \ldots \cdot a_{j-1,j+l} \pm \frac{e \cdot h}{100} \qquad \textit{Equation 14}$$

   where e represents the tolerated error, and h represents the heigh value of the comparison scale. If the supplemented transitivity rule is not satisfied, the expert is informed to modify the entered pairwise comparison value. Although this methodology provides a proactive approach in correcting inconsistency, it requires additional training of experts to respond to the correcting algorithm. Furthermore, the active approach to correction may introduce unintentional biasing of feedback. The proposed CRW AHP methodology intends to bypass the need for directly influencing the expert during the survey process by implementing adjustments during data aggregation, and not during collection.

2. *Use of an evolutionary algorithm and random search* (Han, 2014)

   The proposed methodology utilizes a mutation function to gradually augment the comparison matrix when an expert's feedback is inconsistent. A random search and filtering across the comparison matrix is implemented to avoid potential polarization of biases. The methodology filters out comparisons per individual, and then mutates the index at a rate of 5%. If the aggregated consistency is still greater than 0.10, the screening process is re-performed. Although this method requires low computational cost and is mathematically feasible, its adjustments do

not have a relative correlation to the logic of the initial feedback from the expert. The proposed CRW AHP methodology limits modifications to the aggregated criteria weights solely based on initial pairwise comparisons from the experts and does not randomly adjust the survey data.

3. *Modification of the random index* (Alonso & Lamata, 2006)

   The proposed methodology stems from the acknowledgement that experts cannot provide perfect judgements; this is especially true as the size of the comparison matrix increases. An equation for generating a R*I* curve is proposed in the research based on the following condition:

$$\lim_{n\to\infty}[RI(n+1) - RI(n)] = 0 \qquad \textit{Equation 15}$$

$$\therefore \quad RI(n) = -0.0021n^2 + 0.1183n - 0.001 \qquad \textit{Equation 16}$$

   In addition to providing an algorithm for calculating RI, an average error value is calculated:

$$Avg\ Err(n) = \bar{\lambda}_{max}(n) - n \qquad \textit{Equation 17}$$

$$\therefore \quad \frac{Err(matrix)}{Avg\ Err(matrix)} \leq \alpha \qquad \textit{Equation 18}$$

   Based on the predefined confidence, if Equation 18 is satisfied, the expert feedback is considered consistent. This methodology provides a powerful basis for correcting issues with inconsistency because it accounts for the size of the pairwise comparison matrix. However, the approach does not guarantee that the standard method of calculating *RI* will work to improve consistency for all scenarios, a limitation not intrinsic to the proposed CRW AHP methodology.

4. *A non-matrix-based approach for estimating CR* (Xu & Xu, 2020)

   The proposed methodology utilizes a direct linear approach for testing consistency by utilizing the ratio between a subset of comparison values. For a square comparison matrix with three criteria (3×3), the following evaluations are proposed based on the calculated consistency score:

$$CR < 0.05 \Leftrightarrow 0.505 < \frac{a_{ik}}{a_{ij}a_{jk}} < 1.979 \qquad \textit{Equation 19}$$

$$CR < 0.10 \Leftrightarrow 0.382 < \frac{a_{ik}}{a_{ij}a_{jk}} < 2.620 \qquad \textit{Equation 20}$$

   This methodology does not provide mathematical proof for comparison matrices of all sizes. Practitioners will be limited to calculating ratio values fit for the size of their survey, and therefore the analysis would lack flexibility if changes to the criteria or alternatives are required. CRW AHP proposed a methodology in which matrix size for pairwise comparisons is not a constraint in the analysis.

5. *Interactive consistency* (Alessio Ishizaka & Siraj, 2020)

   This methodology proposes the introduction of a tolerance factor into the transitivity assessment.

$$a_{ij} \pm a_{ij} \cdot t = a_{ix} \cdot a_x \cdot \ldots \cdot a_z a_{zj} \qquad \textit{Equation 21}$$

   Where *t* represents the predefined tolerance, as a percentage. When Equation 21 is not satisfied as part of the construction of the comparison matrix, the expert is informed of the violation for transitivity. The expert is then given the option to modify either the aggregated comparison value, or the independent comparison values of the matrix. This methodology provides a consistent mathematical basis

for continually revising the pairwise comparisons until consistency in improved, but the actions are not directly tied to the initial intended evaluation of the pairwise comparisons of the expert. Furthermore, this approach may bias the re-grading efforts of the expert, as a continuous notification of inconsistency can motivate illogical conclusions during the survey. For the CRW AHP methodology, no means of intervention during the survey process is proposed since all modifications to the survey results follow eigenvalue calculations.

6. *Expert confidence-based assessment of consistency* (Yu, 2017)

   The proposed methodology accounts for the expert's confidence in understanding the relationships between each pairwise comparison criteria. To quantify the lack of consistency, a state of total consistency is defined, where *i, j, k*, and *m* represent all possible pairwise comparisons:

$$a_{ij} = a_{mj}\left(\frac{a_{ik}}{a_{mk}}\right) \qquad \textit{Equation 22}$$

   The independent pairwise comparisons driving the highest level of inconsistency are iteratively silenced as a proportion of the total consistency of the matrix. This methodology requires multiple iterations, and each stage of correction drives deviation of comparisons from the expert's initial survey feedback. With this approach, data is incrementally lost until a favorable consistency ratio is achieved per expert. No means of data filtering of the pairwise comparisons is required for the CRW AHP methodology since the weights derived from the initial consistency ratios are utilized to modify the eigenvalue of each expert.

7. *Discretization of revisited surveys* (Jin et al., 2019)

   In order to diminish expert feedback with low consistency, an iterative process of decreasing the weight of low-consistency feedback is proposed. A deviation matrix, ***D***, is defined per Equation 23 and Equation 24:

$$D = \left(d_{ij}\right)_{n\times n} \qquad \textit{Equation 23}$$

$$d_{ij} = a_{ij} \times \frac{w_j}{w_i} - 1 \qquad \textit{Equation 24}$$

   The variables *a* and *w* represent values from the comparison and priority matrices, respectively. As part of the screening process, $max\{d_{ij}\}$ is identified to be modified, as it is implied that majority of the inconsistency is from its pairwise comparison value. The expert is then advised to reevaluate the pairwise comparisons identified in the deviation matrix. This approach influences the feedback of the expert during the survey, and therefore poses the risk of biasing the results towards favorable consistency versus the feedback the expert intended to provide. The proposed CRW AHP methodology does not require retraining or re-surveying of the expert based on the consistency.

8. *Use of fuzzy preference relations* (Hamka & Harjono, 2020)

   The proposed methodology utilizes the transformation of the comparison matrix using the following steps:

   a. Transform comparison matrix $\mathbb{A} = a_{ij}$ into a fuzzy preference relations (FRP) matrix $\mathbb{R} = r_{ij}$

$$r_{ij} = g\left(a_{ij}\right) = \frac{1}{2}(1 + \ log_9 a_{ij}) \qquad \textit{Equation 25}$$

b. Construct a new FRP matrix that satisfies the desired level of consistency

$$g_{ik} = \left(\frac{1}{n} \sum_{j=1}^{n} (r_{ij} + r_{jk}) - \frac{1}{2} \right) \qquad \textit{Equation 26}$$

c. Construct a new consistent pairwise matrix based on results of b

$$a_{ij} = 9^{2(g_{ik} - 0.5)} \qquad \textit{Equation 27}$$

If the new pairwise matrix is still inconsistent, the results are transformed again using the steps described. Although this methodology ensures that inconsistency can be correct for all scenarios, the steps for correcting the lack of transitivity in the expert's judgement incrementally deviates away from the initial feedback.

9. *Exponential smoothing technique* (Cao et al., 2008)

The methodology hinges on the identification of a deviation matrix, ***D***, a reciprocal matrix to the priority matrix, ***A***. The deviation matrix represents the set of pairwise comparisons with transitivity issues within the priority matrix, and zero deviations from inconsistency is represented per Equation 28.

$$d_{ij} = 1 \qquad \textit{Equation 28}$$

An iterative process of modifying ***A*** with *k* iterations until a desired level of consistency is proposed per Equation 29.

$$A^{(k+1)} = \left[w_i^k \Big/ w_j^k \right] \times D'^{(k)} \qquad \textit{Equation 29}$$

The method defined by Cao successively changes the priority matrix, and therefore changes the initial pairwise comparison provided by the expert. The deviation from the initial feedback would be increase as inconsistency increases.

2.4 Summary and Conclusion

The developed methodologies described in the literature provide several approaches for addressing inconsistency in pairwise comparison matrices. The efforts effectively utilize one of the following four high-level strategies:

- Active influencing of the expert during the survey, based on assessment of single entries in the pairwise comparison
- Passive influencing of the expert as part of revisiting the survey, based on assessment of the aggregated consistency matrix
- Algorithmic transformation or bypassing of the comparison matrix until transitivity is achieved
- Augmentation of the random index or desired confidence to diminish impact of the principal eigenvector on *CR*

The effectiveness of the proposed methodologies has been demonstrated in practice across various use cases, however, additional discussion is required around the impact of the approaches on the initial feedback of the experts. The rules of reciprocity and transitivity demand consistency, but it is noted that perfect consistency should not be expected from expert judgement (Lane & Verdini, 1989). Furthermore, aggregated inconsistency does not imply that all independent pairwise comparisons must be revised, and therefore the preservation of the cumulative judgement should be prioritized in order to account for the fidelity of the expert's feedback. This notion is even more critical when multi-expert decision analysis is required as part of the AHP methodology, as asymmetry in the correction of pairwise judgement for a subset of experts in a survey may need to be considered. Transitivity is a surrogate measure of the expert's proficiency of the criteria at hand, however, methods of augmenting an expert's feedback prior to calculation of

criteria weights may impact the intended judgement. Furthermore, eliminating an expert's pairwise comparison results or influencing the survey process until consistency is achieved may impact the fidelity of the experts' judgement in aggregation.

The proposed CRW AHP methodology, in the context of information fidelity, potentially provides three key benefits compared to prior efforts in defining methods of improving the consistency index in a multi-expert setting:

- The experts are not subjected to active or passive suggestions around changing their pairwise comparisons in order to achieve consistency
  - *Potential risk being mitigated by CRW AHP:* The expert is not influenced during the survey, thus the intended feedback in totality is not manipulated
- The experts' pairwise comparison matrices are not transformed in order to achieve consistency
  - *Potential risk being mitigated by CRW AHP:* The experts' intended feedback is not manipulated prior to calculation of eigenvalues, thus the intended feedback in totality is not manipulated
- As part of calculating the consistency ratio, the random index value is not adjusted in order to dilute out issues with transitivity
  - *Potential risk being mitigated by CRW AHP:* Consistency ratio scores are not adjusted independently from eigenvalues, thus ensuring there is significance in changing the results

In addition to the proposed benefits that differentiate CRW AHP from prior efforts, a novel method for utilizing weights generated from consistency ratios is proposed. Chapter 3 provides details for how the novel methodology augments the eigenvalues of the comparison matrices prior to aggregation.

Chapter 3—Methodology

3.1 Introduction

The primary data utilized in this research was derived from literature sources, where the AHP methodology was implemented as part of a multi-criteria decision analysis. The identified sources provided raw data for pairwise comparisons from a minimum of three experts, and an array of criteria for analysis. A wide range of topics associated with demonstrating novel research in AHP were collected. The objective of the data collection was to capture a diverse set of independent pair-wise comparisons. Furthermore, a research tool was developed to aid AHP and CRW AHP analysis once data was prepared. The following steps were pursued in preparation of the dataset utilized in this research:

1. Identification of literature source
    a. Prior published results from implementation of the AHP methodology across research topics were identified. All literature sources stem from efforts in improving AHP or implementing the methodology in an industry setting.
2. Data collection from literature
    a. Literature containing raw data from the experts' pairwise comparisons were selected for use. Varying levels of the number of experts and criteria for comparison were accounted for in the data sources.

3. Data preparation for analysis
    a. The raw data from literature was transformed and transferred into a standardized template for analysis. A verification process of testing calculated eigenvalues and criteria weights was performed to ensure data was properly transcribed.
4. Application of AHP methodology
    a. Using Microsoft Excel (v16.51), a custom AHP tool was developed to process pairwise comparison matrices, and to calculate criteria weights and consistency scores.
5. Cross-verification of consistency calculations between research tool and data source
    a. The results from the custom AHP tool developed for this research were cross verified against the reported consistency calculated directly from the literature source. This ensured that the reported results for the methods of this research were aligned with Saaty's AHP algorithms (Cheng & li, 2003).
6. Application of CRW AHP methodology
    a. The same tool developed for AHP in this research was modified to enable the CRW AHP data processing methodologies. The details of the methodology are provided in Section 3.4.

3.2 Data Source and Design of Experiment

As an analytical tool was developed to support this research, all results were cross verified against the data source. Once results were confirmed to align, the data source was further analyzed per CRW AHP methodology. Table 4 provides a summary of the literature sources used for raw data input.

Table 4. Summary of Literature Raw Data Sources

Title: Norm-dist Monte-Carlo integrative method for the improvement of fuzzy analytic hierarchy process **Author:** (Wicaksono et al., 2020) **Number of Experts in Analysis:** 10 **Number of Criteria in Analysis:** 5 **Data Used For:** RQ1, RQ2, RQ3, RQ4 **Subject Summary**: A novel methodology for aggregating pairwise comparisons from multiple experts is described, where Monte-Carlo simulations are utilized to represent fuzzy numbers from an array of survey data. For the AHP analysis, the paper implements the methodology for comparisons regarding non-destructive testing technology.
Title: Time Complexity and Consistency Issues in Using the AHP for Making Group Decisions **Author:** (Tung, 1998) **Number of Experts in Analysis:** 3 **Number of Criteria in Analysis:** 3 **Data Used For:** RQ1, RQ2, RQ3

Subject Summary: The paper explores impact on computational processing time requirements as a function of when multi-expert data is aggregated. The first option poses aggregation of eigenvalues from pairwise comparisons prior to completion of AHP analysis, and the second option allows for aggregation of final results from AHP per expert. For the second method, averages are calculated per weights derived from the analysis as a basic form of aggregation.

Title: Analytic Hierarchy Process (AHP) in Group Decision Making and its Optimization With an Allocation of Information Granularity

Author: (Pedrycz & Song, 2011)

Number of Experts in Analysis: 5

Number of Criteria in Analysis: 5

Data Used For: RQ1, RQ2, RQ3

Subject Summary: The paper explores implementation of varying levels of granularity during pairwise comparisons. The object of the research is to provide experts more flexibility in surveys to ensure the provided judgement is not abstracted by the standard structure of the methodology. Particle swarm optimization is utilized t drive the research forward in terms of modifications.

Title: The Expert Perspective in Treatment of Functional Gastrointestinal Conditions: A Multi-Criteria Decision Analysis Using AHP and BWS

Author: (Mühlbacher & Kaczynski, 2016)

Number of Experts in Analysis: 6

Number of Criteria in Analysis: 7

Data Used For: RQ4 **Subject Summary**: The paper implemented AHP to survey patients in therapeutic decision making to address impact to gastrointestinal conditions. The overall results of the exercise were reported to yield favorable outcomes.
Title: A Consistency Test for AHP Decision Makers **Author:** (Lane & Verdini, 1989) **Number of Experts in Analysis:** 3 **Number of Criteria in Analysis:** 3 **Data Used For:** RQ4 **Subject Summary**: The paper explores a statistical basis for a decision rule and utilizes the AHP methodology in a three-criteria setting to assess impact of the resulting consistency score on the outcome of the decision-making process.
Title: Analyzing Factors that Influence the Efficiency of Airport Passenger Terminal by Using AHP (Analytic Hierarchy Process) Method (Case Study of Juanda Airport – Surabaya, Indonesia) **Author:** (Park & Lee, 2020) **Number of Experts in Analysis:** 10 **Number of Criteria in Analysis:** 9 **Data Used For:** RQ4 **Subject Summary**: AHP was utilized to assess factors that impact efficiency scores at airport terminals. It is reported in the paper that the results of the survey did not fully align with the measured traffic per terminal.

As the raw data from the literature sources were presented in varying form, the data was prepared to align with the research tool. The key factor was arranging the data in a formal pairwise comparison matrix to standardize the AHP analysis. The research tool for calculating criteria weights and consistency scores was developed in Microsoft Excel (16.47.1). As stated previously, results derived from the research tool were cross verified against the source for AHP criteria weight calculation, thus validating all steps required to derive the solution from the raw pairwise comparisons per expert. When analyzed results were provided in the literature sources for criteria weights, max eigenvalues, or consistency ratios, the values were directly compared to the output of the research tool. Not all cross-comparable data were presented in the literature, however, Figure 1 through Figure 3 provide the results for verification.

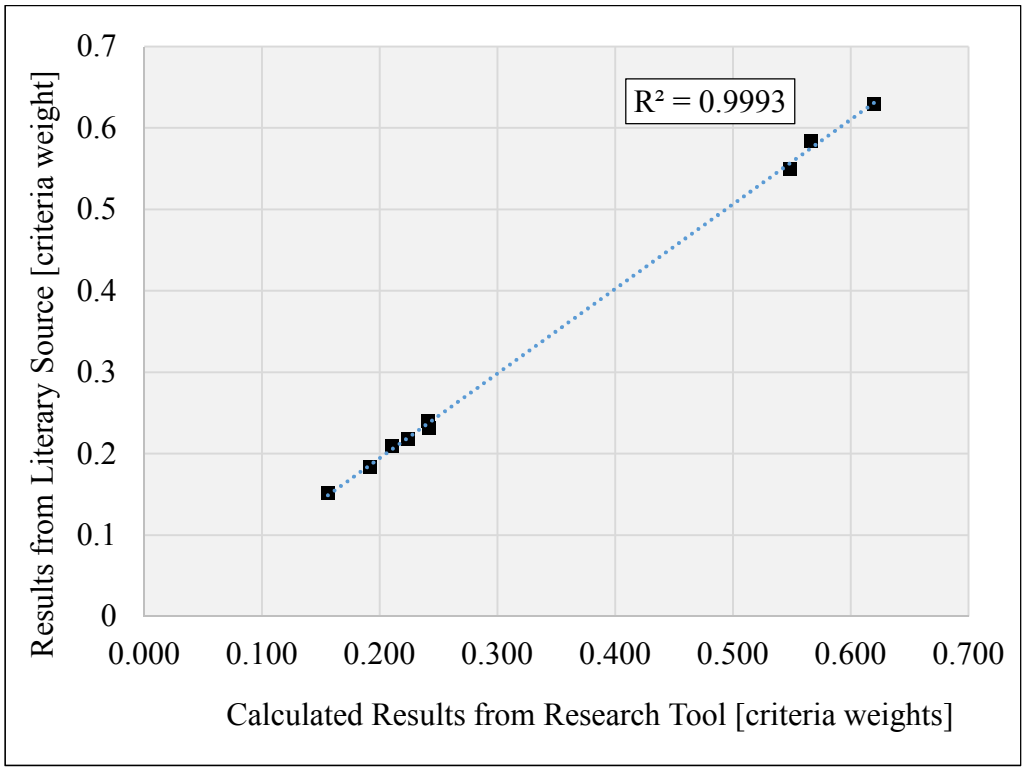

Figure 1. Comparison of Final AHP Criteria Weights (N=2 Literature Source)

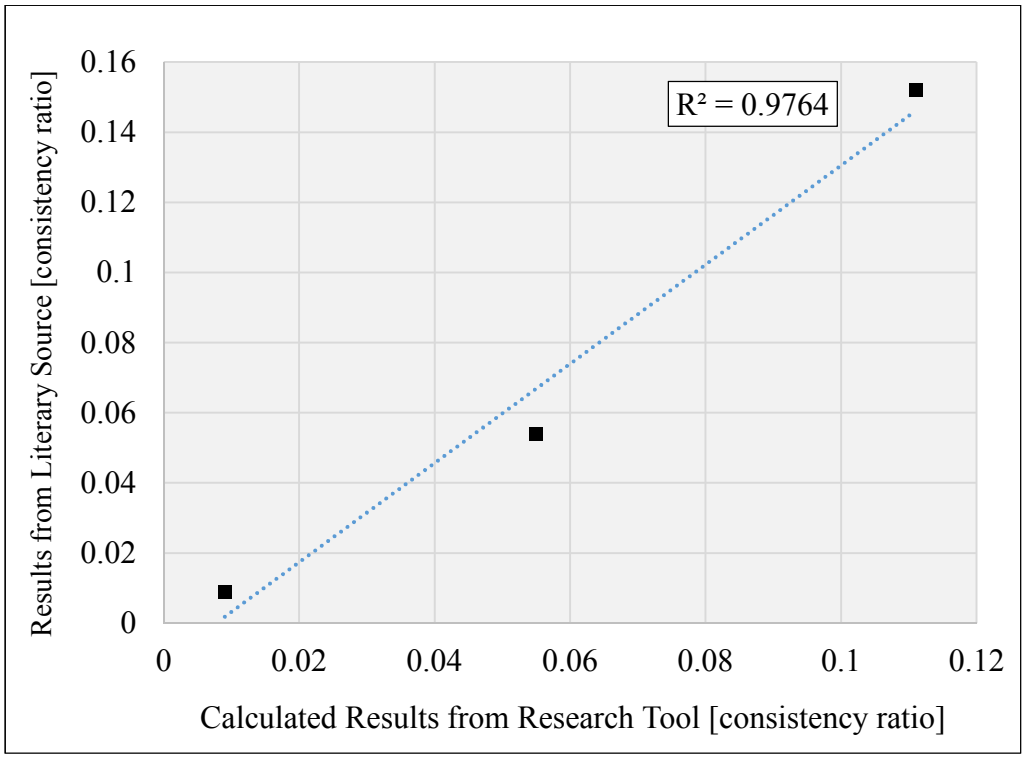

Figure 2. Comparison of Consistency Ratios (N=1 Literature Source)

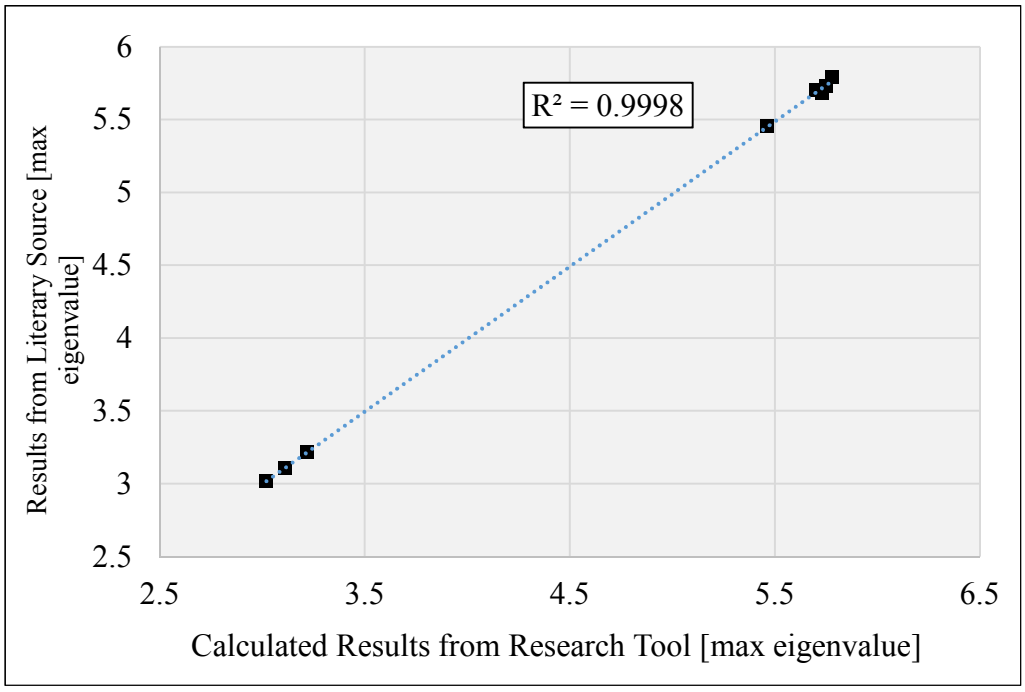

Figure 3. Comparison of Max Eigenvalues (N=2 Literature Source)

In addition to the test for linear correlation for the research tool and literature sources, a Paired T test for the calculated AHP criteria weights was performed, and a p-value of 1.0 was calculated to demonstrate no statistically significant difference. The results of the test are provided in Figure 4.

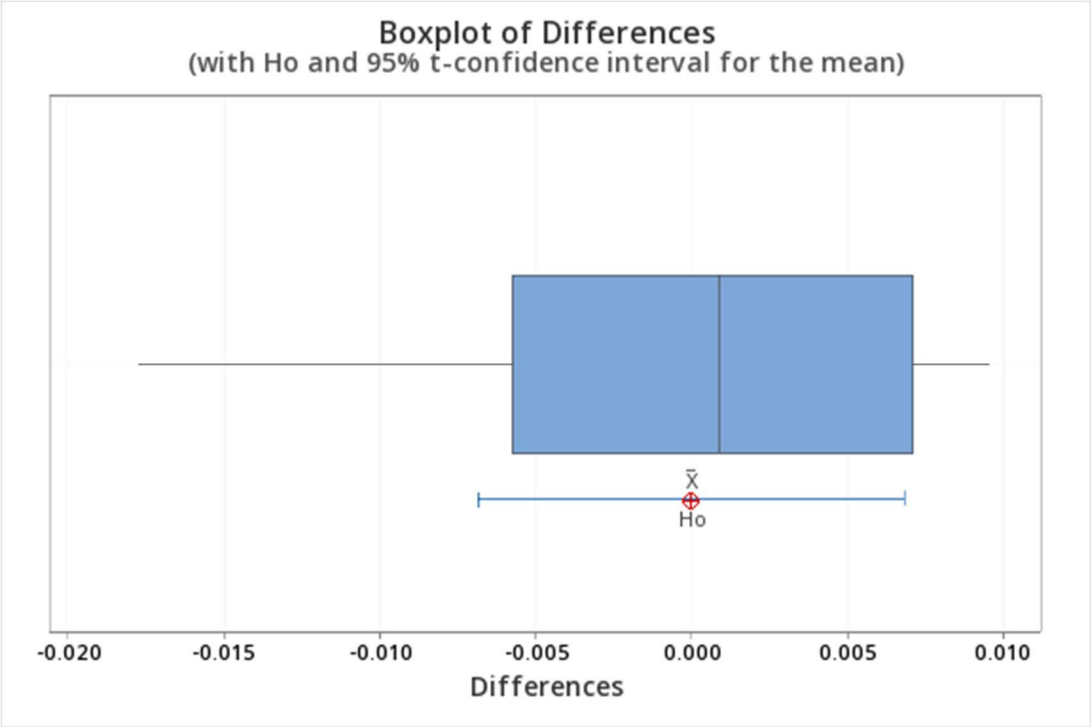

Figure 4. Summary of Differences from Paired T test of Criteria Weights

The overall design of experiment summarizing how data was collected and analyzed is presented in the Appendix, as part of Figure 29.

3.4 Application of AHP and CRW AHP Methodologies

For comparison of research methodologies between a control and experimental analysis, Saaty's original definition of the AHP method was utilized for control data generation (R. W. Saaty, 1987). In Saaty's seminal work, the following process for analysis was proposed for multi-criteria decision analysis (Cheng & li, 2003):

1. Define a problem set for multi-criteria decision, with criteria and sub-criteria defined
2. Represent the problem set as a standard hierarchical analysis
3. Deploy a pairwise comparison per paired criteria
4. Calculate consistency of pairwise comparison to assess transitivity and reciprocity
5. Calculate the criteria weights

For the standard hierarchical analysis, Saaty proposed a "Fundamental Scale" for aligning on pairwise comparisons (R. W. Saaty, 1987). Table 5 provides a summary of the Fundamental Scale for the AHP methodology.

Table 5. The Fundamental Scale for AHP Pairwise Comparisons

Scalar	**Definition**	**Explanation**
1	Equal importance	Two activities contribute equally to the objective or analysis
3	Moderate importance of one over the other	Experience and judgement strongly favor one activity over another
5	Essential or strong importance	Experience and judgement strongly favor one activity over another
7	Very strong importance	An activity is strongly favored, and its dominance demonstrated in practice
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation

Figure 5 provides a process flow diagram of Saaty's AHP method (R. W. Saaty, 1987). This methodology was defined as a the classical AHP approach and utilized as the control analysis in this research for AHP implementation.

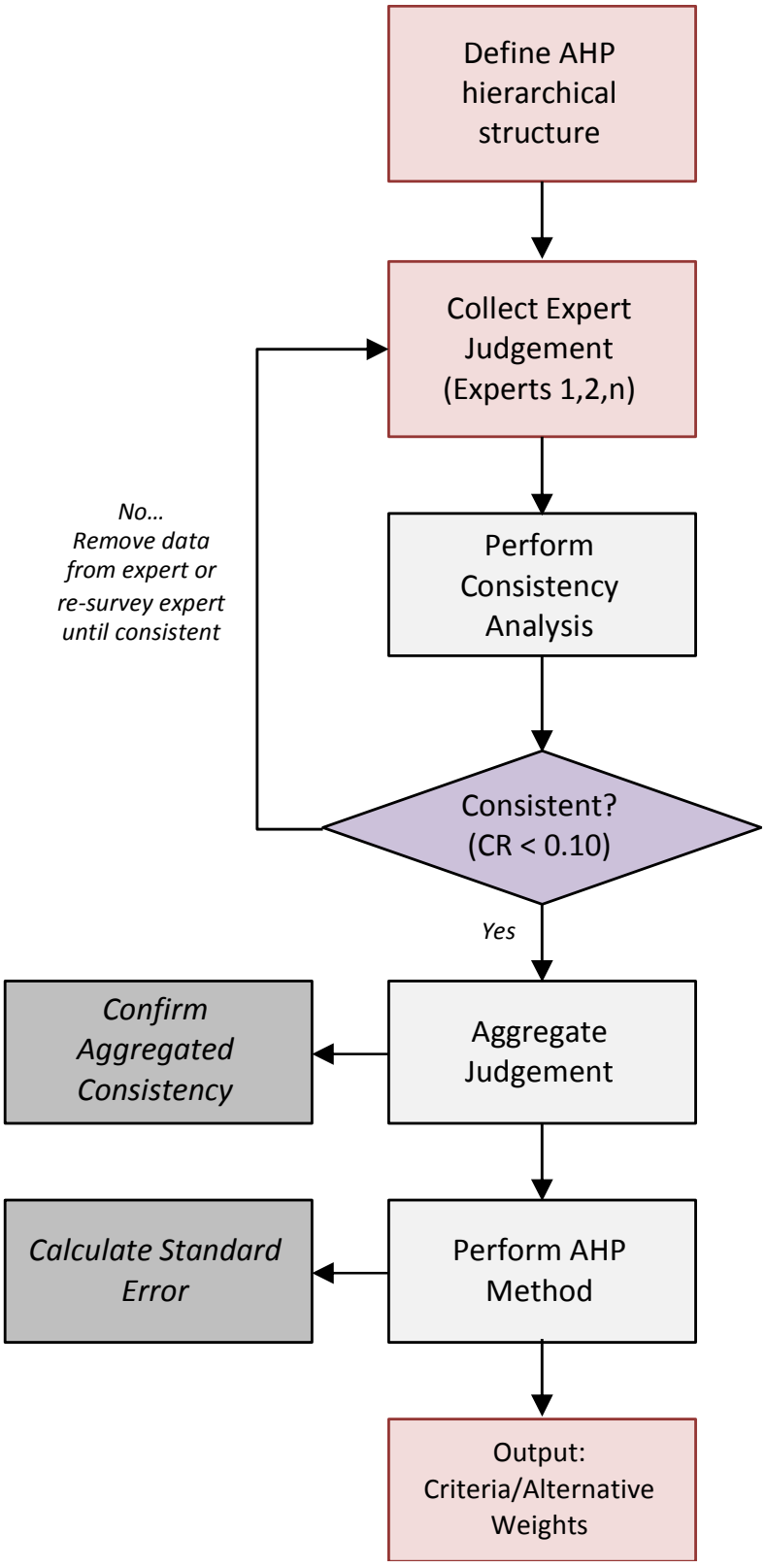

Figure 5. Process Flow for Saaty's AHP Methodology (Control)

To provide an example of AHP implementation in an industry use case, we use Wicaksono's work in selection of non-destructive testing techniques in the petroleum industry using the AHP methodology (Wicaksono et al., 2020). In this example, five criteria for pairwise comparisons are defined:

- *Criteria 1:* Reliability and precision
- *Criteria 2:* Capital and operational cost
- *Criteria 3:* Detection coverage
- *Criteria 4:* Training and development costs
- *Criteria 5:* Maturity and market availability.

Saaty's grade of importance of the comparisons was utilized for the survey (R. W. Saaty, 1987). A team of ten experts were surveys per pairwise comparisons to assess the criteria. In the results, five out of the ten experts provided feedback that resulted in a consistency ratio above 0.10. This implies that the overall survey contained a high level of issues with transitivity; however, the survey results were still utilized for decision making. Per Saaty's method design, it would have been favorable to retrain and re-survey the experts to attempt to capture more consistent feedback. For the five criteria in the example survey, weights of 0.399, 0.297, 0.152, 0.102, and 0.041, were calculated, respectively (Wicaksono et al., 2020).

Figure 6 provides an overview of the proposed CRW AHP methodology. The proposed method was the primary focus of this research and is detailed as the experimental analysis for AHP implementation.

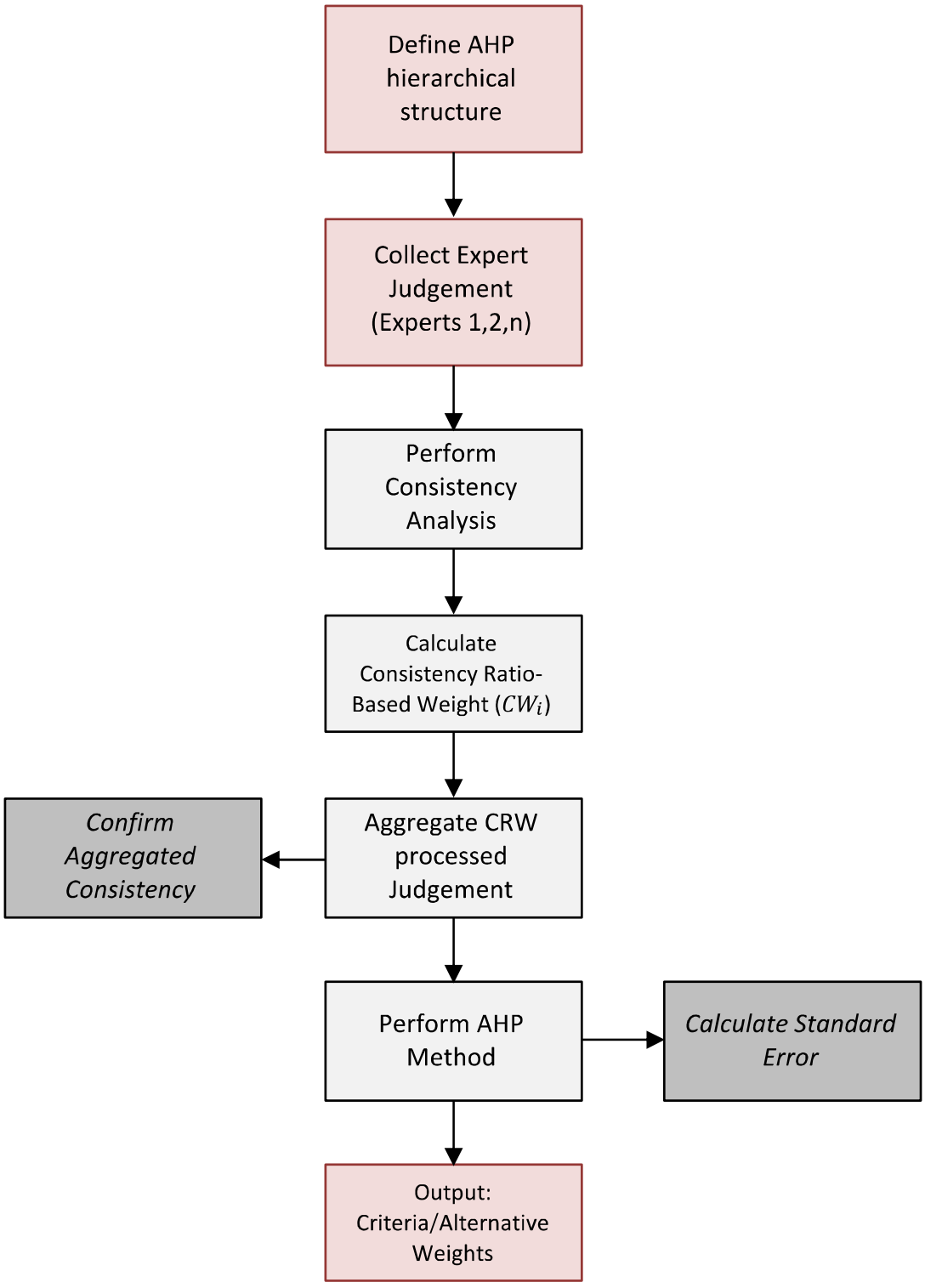

Figure 6. Process Flow for CRW AHP Methodology (Experimental)

The following details provide a comprehensive overview of the proposed CRW AHP method as the key product of this research:

1. *Step 1: Define the AHP hierarchical structure*
    a. With alignment with the control methodology, the AHP hierarchical structure for CRW AHP represents a framework for defining a series of criteria, sub-criteria, and alternatives from a multi-criteria decision problem. The elements for analysis for each criteria or alternative represent constraints or methods of measuring favorability.
2. *Step 2: Collect expert judgement per pairwise comparison*
    a. Expert judgement per pairwise comparison is obtained using Saaty's Fundamental scale for comparisons. This exercise is performed for all pairwise combinations of criteria and alternatives at each level of the hierarchical structure. Between criteria and alternatives, the analysis requires a standard set of elements for judgement. Just as the control AHP methodology, the pairwise comparison for CRW AHP is the fundamental exercise for deriving expert feedback as is the raw data analysis. No novel methodologies for survey implementation or pre-processing of expert feedback is proposed for the CRW AHP method. Furthermore, expert judgement is independently stored without further aggregation at this step.

3. *Step 3: Perform consistency analysis*

    a. Consistency analysis for each expert's pairwise comparison matrix is performed prior to aggregation. With full alignment to Saaty's AHP methodology, a maximum eigenvalue, λ_{max}, is calculated as the principal variable for calculating the Consistency Index, *CI*. The number of components, or the matrix size, is represented by *n*.

$$CI = \frac{\lambda_{max} - n}{n - 1} \qquad \textit{Equation 30}$$

In order to calculate the Consistency Ratio, the Consistency Index for each expert is normalized by the Random Index. The Random Index is an estimated value calculated to represent a standard level of inconsistency allowed as a function of the order of the pairwise matrix (Rao Tummala & Wan, 1994). It is critical to calculate consistency prior to any form of aggregation for the experts' judgement. The purpose of this constraint is to ensure the max eigenvalue for each expert can be derived prior to any loss or modification of information. No additional processing for the pairwise comparisons is required due to this approach. The fundamental axioms for the AHP methodology, proposed by Saaty, are fully relevant for the CRW AHP methodology (R. W. Saaty, 1987).

4. *Step 4: Calculate consistency ratio-based weights (CW_i)*

    a. In order to calculate weights based on each expert's consistency ratio, a total consistency value, CR_{sum}, is calculated for the survey.

$$CR_{sum} = \sum_{i=1}^{n} CR_i \qquad \textit{Equation 31}$$

Once the total consistency value is calculated, the consistency ratios for each expert is normalized, $CR_{norm,i}$.

$$CR_{norm,i} = \frac{CR_i}{CR_{sum}} \qquad \textit{Equation 32}$$

The next step is to calculate the total normalized consistency value, CR_{sn}.

$$CR_{sn} = \sum_{i=1}^{n} CR_{norm,i} \qquad \textit{Equation 33}$$

The final step in calculating the consistency ratio-based weight is to derive the fundamental weight based on the normalized consistency ratios.

$$CW_i = \frac{{}^{1}/_{CR_{norm,i}}}{CR_{sn}} \qquad \textit{Equation 34}$$

5. *Step 5: Aggregate consistency ratio-based weighted judgement*

    a. With the consistency ratio-based weights calculated per expert, the next step is to calculate the aggregated max eigenvalue per CRW AHP method.

$$\lambda_{max\ crw} = \sum_{i=1}^{n} \lambda_{max,i} \times CW_i \qquad \textit{Equation 35}$$

From this point, the aggregated and weighted consistency index can be calculated.

$$CI_{crw} = \frac{\lambda_{\max crw} - n}{n - 1} \qquad \textit{Equation 36}$$

6. *Step 6: Perform AHP methodology per Standard Matrix*
    a. To calculate the consistency ratio-based weighted vector with j parameters, $\mathbb{W}_{CR,j}$, also referred to as the vector of preferences in common AHP use (Pedrycz & Song, 2011), the CRW AHP methodology requires the sum-product of the criteria per expert feedback.

$$\mathbb{W}_{CR,j} = \sum_{i=1}^{n} \left(CW_i \times \mathbb{W}_{i,j} \right) \qquad \textit{Equation 37}$$

The CRW AHP weighted vector is the endpoint of analysis if requirements for consistency have been met.

7. *Step 7: Review output for analyzed criteria weights*
    a. The final step in the CRW AHP methodology is to review the results of the expert judgement. Just as Saaty's AHP method initially proposed, the CRW AHP method will provide weights to represent preference for all criteria, sub-criteria, and alternatives. The total number of experts involved, or the total number of criteria are not limited with the proposed methodology. However, survey complexity and computational requirements should be considered.

Using the same literature example provided in the AHP methodology overview, the expert's feedback, where five of the ten pairwise comparisons were deemed inconsistent, can be modified prior to aggregation using the CRW AHP methodology. Instead of filtering out the inconsistent feedback or having to re-survey the experts, the calculated eigenvalues can be modified using the consistency ratio-based weights and then aggregated. The resulting aggregated consistency ratio for the example is 0.0787.

Another qualitative example is in reference to Lee's implementation of AHP for decision-making for homeowners in a plumbing failure-prone area (Lee et al., 2013). It is reported in the paper that 250 household participants were surveyed as part of the AHP analysis, however, only 133 were used due to inconsistency issues. If CRW AHP was implemented in this use case, all data captured as part of the survey would have been part of the analysis. Even though some experts may provide inconsistent feedback, it does not imply that all of their judgement in pairwise comparisons has issues with transitivity (Rao Tummala & Wan, 1994). Furthermore, the CRW AHP methodology would correlatively decrease the weight of an expert's feedback as relative inconsistency increases. The value of enable use of all data in a pairwise comparison survey without modifying the data prior to aggregation of eigenvalues is a novel benefit of the CRW AHP methodology.

Chapter 4—Results

4.1 Introduction

The results of the research involve the presentation of the AHP and CRW AHP analysis of the raw data identified in key literature sources. Furthermore, results from the tests of the research questions are provided for assessment relative to the research hypothesis. Figure 7 provides a recapitulation of the process flow for generating the results for the research.

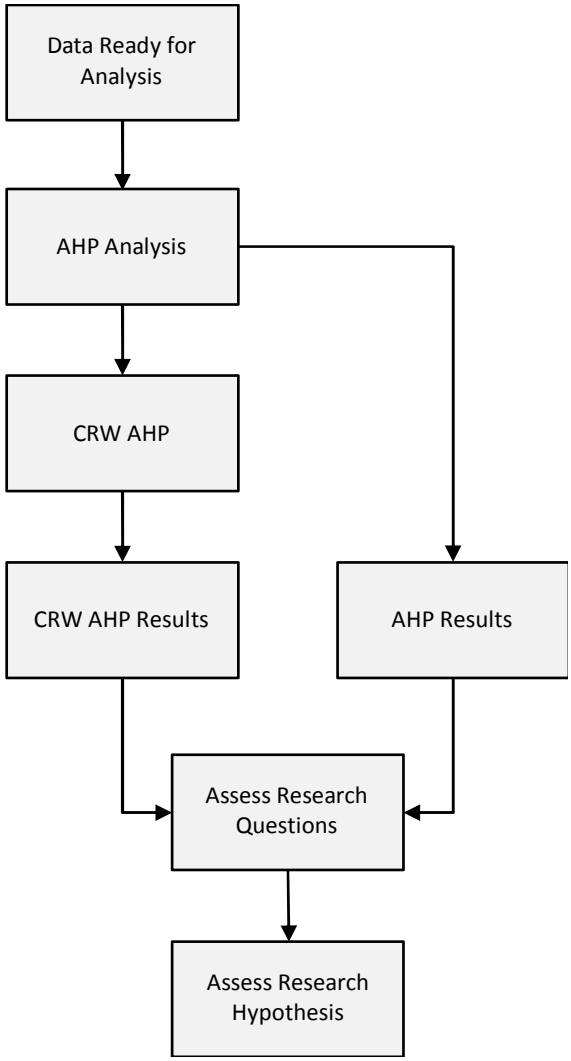

Figure 7. Results Generation Process Flow

4.2 AHP Analysis of Raw Data

Using the pairwise comparison as raw data from the literature sources, the following criteria were calculated using the Saaty's AHP methodology (R. W. Saaty, 1987). For the three sources identified to support this research, the results are presented for the AHP analysis in the following order:

1. Calculated criteria weights from the pairwise comparison raw data
    a. Tables 6 – 9
2. Calculated max eigenvalue and consistency ratio per expert
    a. Tables 10 – 13
3. Aggregated criteria weights per geometric mean
    a. Tables 14

All results were calculated using the AHP tool developed for this research. Raw pairwise comparison data is provided in the appendix.

Additional analysis required for the CRW AHP methodology is performed on the output of the AHP analysis. Namely, as defined in methodologies, the consistency ratios for each expert is utilized to calculate a weight that augments the input from each expert based on the level of consistency in their pairwise comparison. Furthermore, once the consistency ratio-based weights have been calculated, the values of the AHP criteria weights are recalculated as part of testing the research product.

Table 6. AHP Criteria Weights Calculated from Wicaksono, 2020 – Dataset 1

	Criteria Weight 1	*Criteria Weight 2*	*Criteria Weight 3*	*Criteria Weight 4*
Expert 1	0.229	0.375	0.267	0.128
Expert 2	0.335	0.335	0.245	0.084
Expert 3	0.304	0.344	0.272	0.080
Expert 4	0.472	0.199	0.241	0.088
Expert 5	0.291	0.241	0.341	0.128
Expert 6	0.474	0.274	0.195	0.057
Expert 7	0.265	0.299	0.307	0.130
Expert 8	0.499	0.262	0.143	0.096
Expert 9	0.310	0.314	0.243	0.134
Expert 10	0.416	0.241	0.277	0.065

Table 7. AHP Criteria Weights Calculated from Wicaksono, 2020 – Dataset 2

	Criteria Weight 1	*Criteria Weight 2*	*Criteria Weight 3*	*Criteria Weight 4*
Expert 1	0.460	0.271	0.119	0.106
Expert 2	0.379	0.323	0.154	0.105
Expert 3	0.364	0.315	0.183	0.101
Expert 4	0.338	0.322	0.191	0.111
Expert 5	0.352	0.334	0.156	0.117
Expert 6	0.455	0.266	0.117	0.121

Table 7 continued…

Expert 7	0.382	0.288	0.182	0.106
Expert 8	0.483	0.243	0.141	0.097
Expert 9	0.463	0.263	0.148	0.085
Expert 10	0.353	0.372	0.147	0.077

Table 8. AHP Criteria Weights Calculated from Tung, 1998

	Criteria Weight 1	*Criteria Weight 2*	*Criteria Weight 3*
Expert 1	0.224	0.620	0.156
Expert 2	0.548	0.241	0.211
Expert 3	0.242	0.192	0.566

Table 9. AHP Criteria Weights Calculated from Pedrycz, 2011

	Criteria Weight 1	*Criteria Weight 2*	*Criteria Weight 3*	*Criteria Weight 4*	*Criteria Weight 5*
Expert 1	0.060	0.085	0.184	0.289	0.381
Expert 2	0.074	0.153	0.300	0.288	0.184
Expert 3	0.043	0.060	0.113	0.261	0.524
Expert 4	0.071	0.104	0.181	0.346	0.297
Expert 5	0.349	0.246	0.269	0.099	0.036

The following tables provide a summary of the calculated eigenvalues for expert judgement from the literature sources. The calculations were performed per Saaty's AHP methodology (R. W. Saaty, 1987). Furthermore, the resulting consistency ratios are also provided.

Table 10. Summary of Consistency Calculations from Wicaksono, 2020 – Dataset 1

	λ_{max}	*Consistency Index*	*Consistency Ratio*
Expert 1	4.51	0.170	0.193
Expert 2	4.67	0.223	0.252
Expert 3	4.61	0.204	0.231
Expert 4	4.27	0.090	0.102
Expert 5	4.27	0.090	0.102
Expert 6	4.44	0.146	0.166
Expert 7	6.75	0.917	1.040
Expert 8	4.51	0.169	0.192
Expert 9	5.43	0.476	0.539
Expert 10	4.82	0.274	0.310

Table 11. Summary of Consistency Calculations from Wicaksono, 2020 – Dataset 2

	λ_{max}	*Consistency Index*	*Consistency Ratio*
Expert 1	5.15	0.039	0.034
Expert 2	5.44	0.110	0.098
Expert 3	5.58	0.144	0.129
Expert 4	5.55	0.137	0.122

Table 11 continued…

Expert 5	5.37	0.093	0.083
Expert 6	5.23	0.058	0.052
Expert 7	5.84	0.210	0.188
Expert 8	5.46	0.115	0.103
Expert 9	5.22	0.056	0.050
Expert 10	5.83	0.207	0.185

Table 12. Summary of Consistency Calculations from Tung, 1998

	λ_{max}	*Consistency Index*	*Consistency Ratio*
Expert 1	3.11	0.055	0.104
Expert 2	3.02	0.009	0.017
Expert 3	3.22	0.111	0.211

Table 13. Summary of Consistency Calculations from Pedrycz, 2011

	λ_{max}	*Consistency Index*	*Consistency Ratio*
Expert 1	5.73	0.183	0.163
Expert 2	5.78	0.194	0.174
Expert 3	5.46	0.116	0.104
Expert 4	5.75	0.188	0.168
Expert 5	5.70	0.176	0.157

The following results in Table 14 provide a summary of the aggregation process of the multi-expert judgement, per Saaty's AHP methodology (R. W. Saaty, 1987). Geometric means of the calculated weights were used for the aggregation process.

Table 14. Summary of Aggregated Criteria Weights per Geometric Mean

	Wicaksono 1	*Wicaksono 2*	*Tung*	*Pedrycz*
Criteria 1	0.357	0.403	0.352	0.102
Criteria 2	0.291	0.300	0.348	0.136
Criteria 3	0.254	0.153	0.301	0.234
Criteria 4	0.098	0.103	N/A	0.280
Criteria 5	N/A	0.041	N/A	0.247

4.3 CRW AHP Analysis of Raw Data

As part of the first step in the CRW AHP methodology, a normalized consistency ratio-based weight for each expert contributing to a pairwise comparison is calculated. The resulting weights are then inverted and re-normalized based on the sum of the consistency ratio-based weights. As defined in the methodology, the inversion of the weight is to reflect the fact that a lower consistency ratio is favorable, and therefore needs to result in a higher consistency ratio-based weight. The results of the analysis for each literature source are provided in Tables 15 to 18.

Table 15. Summary of CRW Calculations for Wicaksono, 2020 - Dataset 1

	Normalized CR	*Inversed Norm. CR*	*CRW*
Expert 1	0.062	16.23	0.223
Expert 2	0.081	12.40	0.170
Expert 3	0.074	13.53	0.186

Table 15 Continued….

Expert 4	0.032	30.78	0.422
Expert 5	0.033	30.53	0.194
Expert 6	0.053	18.84	0.120
Expert 7	0.332	3.01	0.019
Expert 8	0.061	16.28	0.103
Expert 9	0.172	5.80	0.037
Expert 10	0.099	10.08	0.064

Table 16. Summary of CRW Calculations for Wicaksono, 2020 - Dataset 2

	Normalized CR	*Inversed Norm. CR*	*CRW*
Expert 1	0.033	30.25	0.228
Expert 2	0.094	10.67	0.080
Expert 3	0.123	8.10	0.061
Expert 4	0.117	8.54	0.064
Expert 5	0.080	12.52	0.094
Expert 6	0.049	20.22	0.153
Expert 7	0.180	5.56	0.042
Expert 8	0.099	10.14	0.077
Expert 9	0.048	20.92	0.158
Expert 10	0.177	5.65	0.043

Table 17. Summary of CRW Calculations for Tung, 1998

	Normalized CR	*Inversed Norm. CR*	*CRW*
Expert 1	0.313	3.19	0.134
Expert 2	0.052	19.05	0.800
Expert 3	0.634	1.58	0.066

Table 18. Summary of CRW Calculations for Pedrycz, 2011

	Normalized CR	*Inversed Norm. CR*	*CRW*
Expert 1	0.213	4.69	0.181
Expert 2	0.227	4.41	0.170
Expert 3	0.135	7.39	0.285
Expert 4	0.220	4.55	0.176
Expert 5	0.205	4.88	0.188

With consistency ratio-based weights calculated for each expert, the augmented criteria weights for the pairwise comparisons can be reevaluated. Table 19 provides the results of the calculated CRW AHP weights, and Table 20 provides the augmented max eigenvalues and consistency ratios utilized for calculating the criteria weights.

Table 19. Summary of CRW AHP Calculated Criteria Weights

	Wicaksono 1	*Wicaksono 2*	*Tung*	*Pedrycz*
Criteria 1	0.376	0.423	0.485	0.377
Criteria 2	0.273	0.288	0.288	0.313
Criteria 3	0.254	0.144	0.227	0.163
Criteria 4	0.097	0.104	N/A	0.107
Criteria 5	N/A	0.041	N/A	0.040

Table 20. CRW AHP Max Eigenvalues and Consistency Ratios

	λ_{max}	*Consistency Index*	*Consistency Ratio*
Wicaksono, 2020 - 1	4.53	0.18	0.20
Wicaksono, 2020 - 2	5.35	0.09	0.08
Tung, 1998	3.04	0.02	0.04
Pedrycz, 2011	5.43	0.11	0.10

4.4 Research Question 1 – Analytical Results

Research Question 1

Will implementing a Consistency-Ratio Based Weight prior to judgement aggregation increase overall analytical consistency?

The first research question assesses whether the CRW AHP methodology improves the aggregated consistency relative to Saaty's proposed AHP method, in a multi-expert setting. In order to perform a paired T-test on the dataset (sample group 1: AHP, sample group 2: CRW AHP), key properties of the data were assessed.

The test for normality of the difference between the sample groups, the Shapiro-Wilk test was selected to account for the small sample size (Bee Wah & Mohd Razali, 2011); however, statistical power is still limited due to the N=4 sample size.

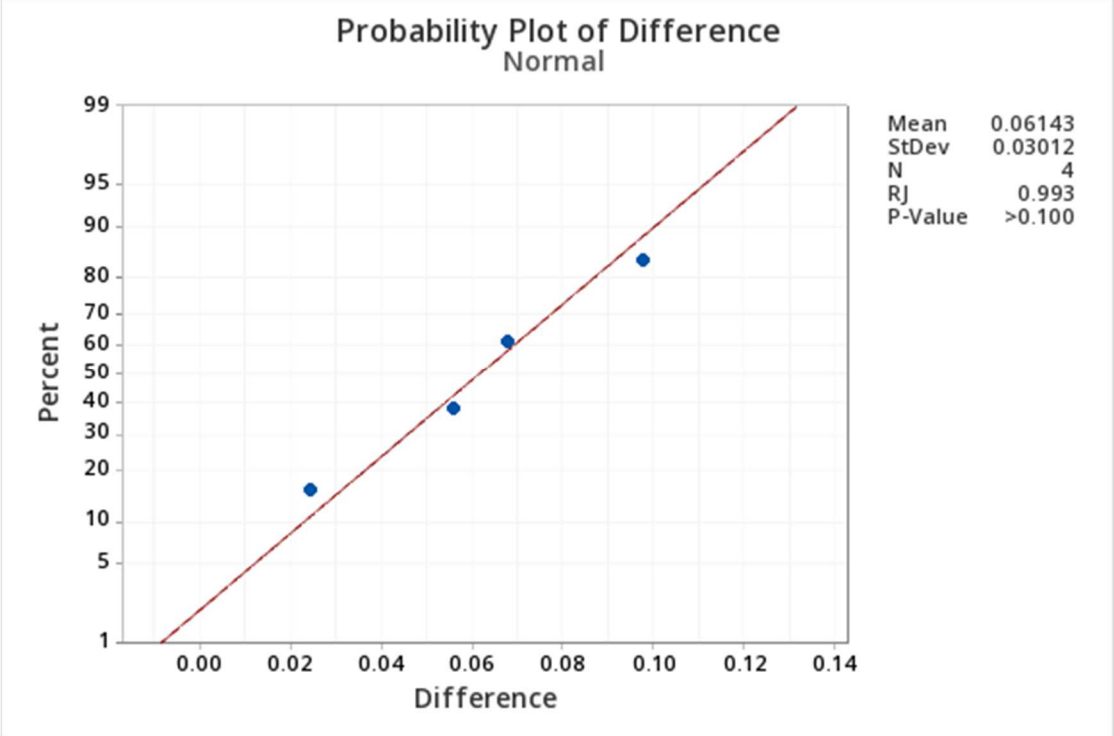

Figure 8. Results of Normality Test for CR Difference Between Methodologies

With a p-value of greater-than 0.100 ($\alpha = 0.05$), the results of the test in Figure 8 suggests the data is normally distributed.

Outlier analysis was also performed on the differences between the two datasets. Grubb's test was utilized to detect outliers with $\alpha = 0.05$.

Table 21. Results of Grubb's Test for Difference between AHP and CRW AHP

Variable	*N*	*Mean*	*StDev*	*Min*	*Max*	*G*	*P*
Difference	4	0.0614	0.0301	0.0246	0.0974	1.22	0.740

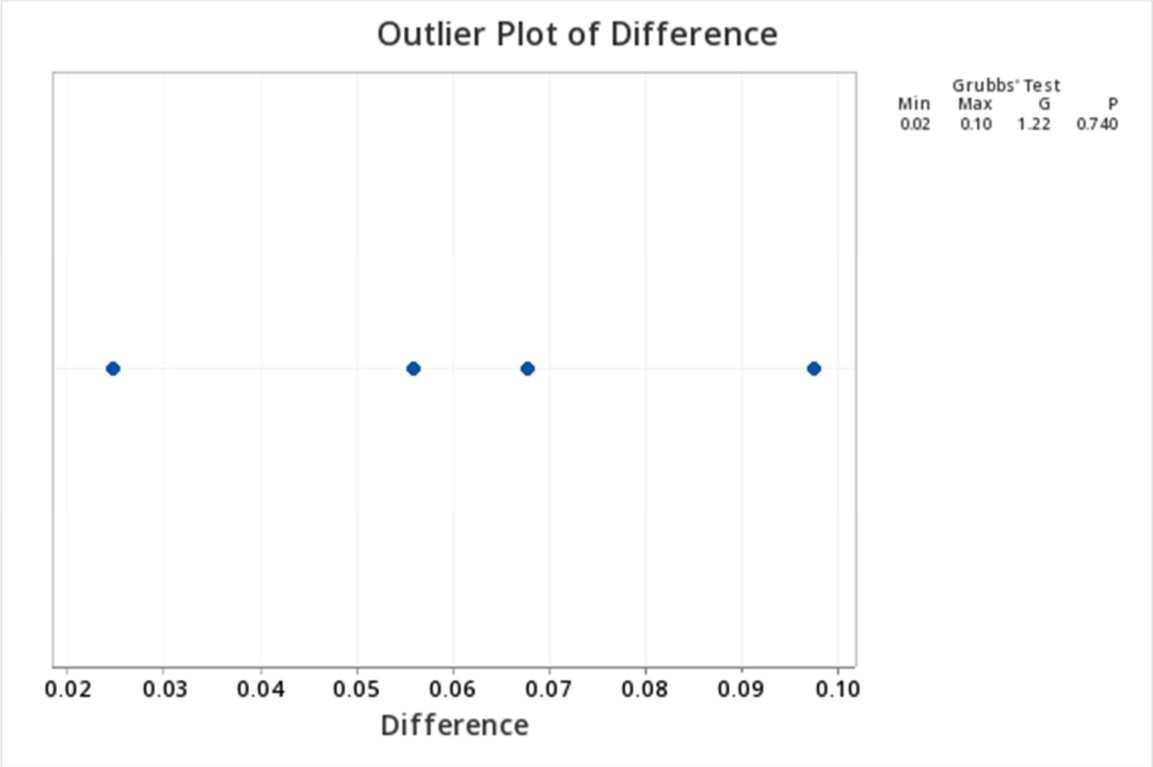

Figure 9. Graphical Summary of Grubb's Test for Difference Between AHP and CRW AHP

With a p-value of 0.740, Figure 9 suggests no statistically significant outliers were identified in the dataset.

In addition to normality and outlier analysis, another key assumption of the dataset is that the results being utilized for the research represent a random sampling of potential consistency ratios.

Once key statistical assumptions have been satisfied, Paired t-test was performed to compare the differences in consistency between AHP and CRW AHP.

Table 22. Summary Statistics for Comparison of Consistency

Sample	**N**	**Mean**	**StDev**	**SE Mean**
AHP	4	0.1654	0.0898	0.0449
CRW AHP	4	0.1040	0.0671	0.0335

Table 23. Estimation for Paired Difference for Consistency

Mean	**StDev**	**SE Mean**	**95% CI for μ_difference**
0.0614	0.0301	0.0151	(0.0135, 0.1094)

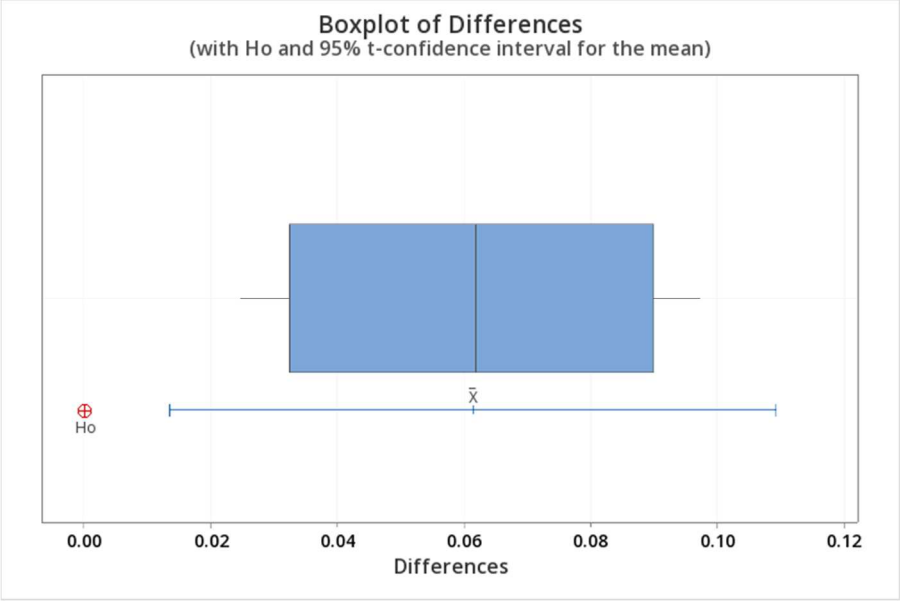

Figure 10. Boxplot of Differences for Consistency Between AHP and CRW AHP

P-value = 0.047

Figure 10 shows the results of the paired analysis as a boxplot of the differences between the AHP and CRW AHP criteria weights. With the results not aligned with the null hypothesis of no differences, a difference in means between the methodologies is suggested. The results in Table 22 suggests that the consistency ratio resulting from the CRW AHP methodology is significantly lower than the CRW method.

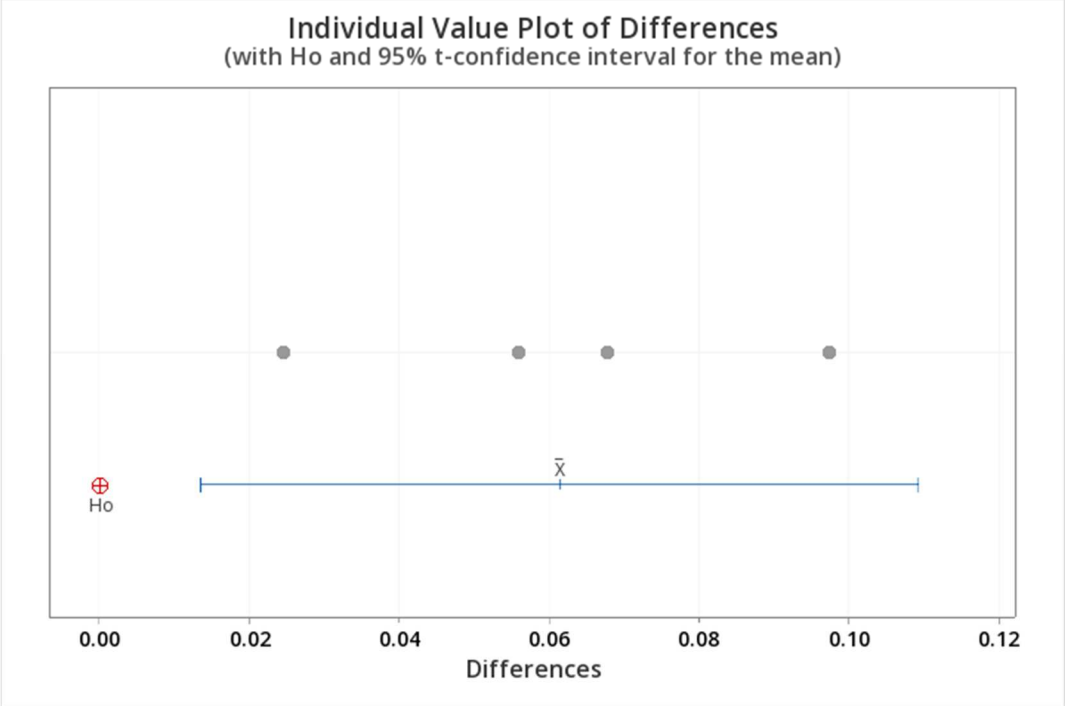

Figure 11. Individual Value Plots of Differences for Consistency Between AHP and CRW AHP

The individual value plot presented in Figure 11 reiterates that the paired test results in a statistically significant difference between the calculated criteria weights. Furthermore, the results imply that all results from the CRW AHP methodology resulted in a lower consistency ratio than the CRW methodology.

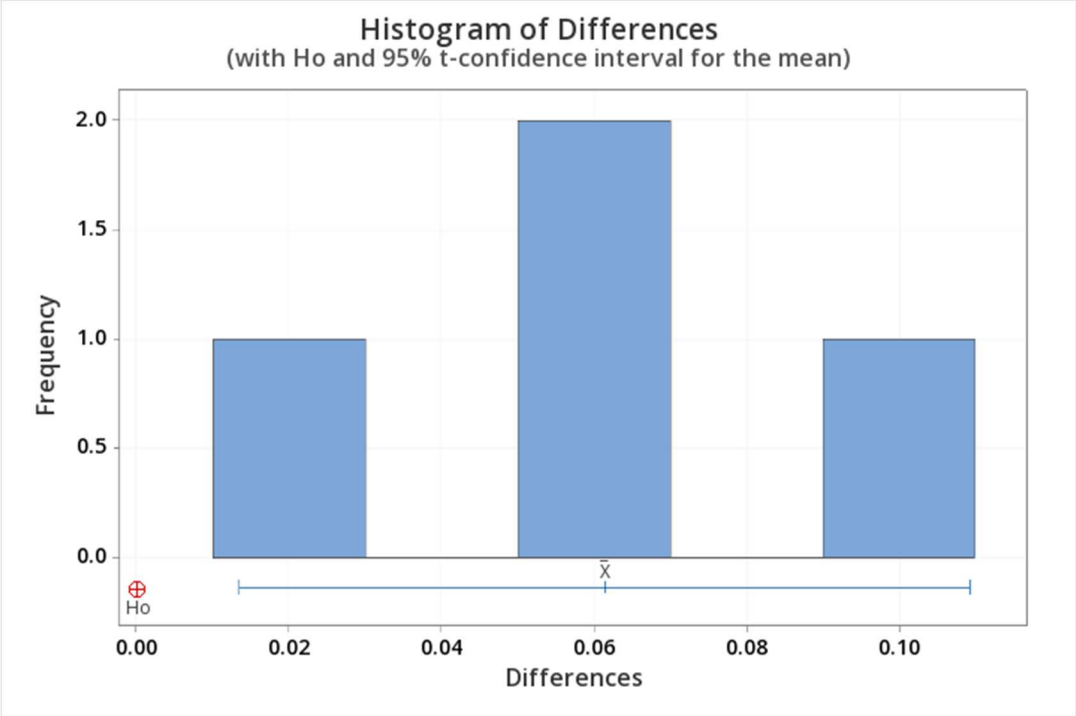

Figure 12. Histogram of Differences for Consistency Between AHP and CRW AHP

Based on the results presented in Figure 12, the difference calculated per paired T test is normally distributed around 0.06. With a null hypothesis of no difference between the paired sample groups, the results suggest that there is no significant difference.

With a p-value of 0.047 resulting from the paired T test, at an alpha of 0.05, the null hypothesis is rejected and a statistically significant difference between the consistency ratios between the AHP and CRW AHP methodologies is noted. Furthermore, the results show that the difference is driven by a lower consistency ratio resulting from CRW AHP compared to AHP; this implies a higher level of consistency for the prior methodology.

4.5 Research Question 2 – Analytical Results

Research Question 2

Will including all expert feedback per Consistency-Ratio Based Weights increase the standard error for a comparison matrix?

The second research question assesses whether the CRW AHP methodology increases the standard error of the criteria weights relative to Saaty's proposed AHP method, in a multi-expert setting. In order to perform a paired T-test on the dataset (sample group 1: AHP, sample group 2: CRW AHP), key properties of the data were assessed.

To assess the normality of the differences in standard error between the methodologies, Shapiro-Wilk test was selected to account for the low sample size.

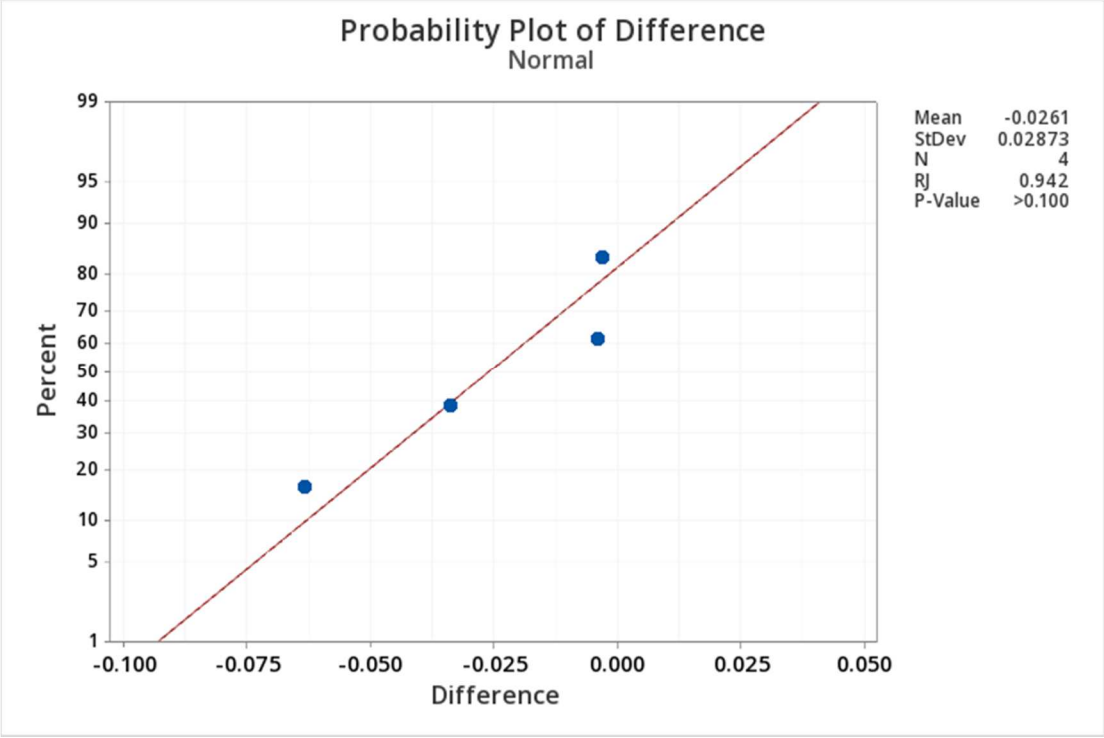

Figure 13. Normality Test for Differences in Standard Error Between AHP and CRW AHP

With a p-value greater-than 0.100, Figure 13 shows no significance of non-normality. The mean difference was measured to be -0.0261.

Outlier analysis was also performed on the differences between the two datasets. Grubb's test was utilized to detect outliers with $\alpha = 0.05$. The summary results are presented in Table 24.

Table 24. Results of Grubb's Test Comparing Standard Error Between AHP and CRW AHP

Variable	*N*	*Mean*	*StDev*	*Min*	*Max*	*G*	*P*
Difference	4	-0.0261	0.0287	-0.0634	-0.0030	1.30	0.538

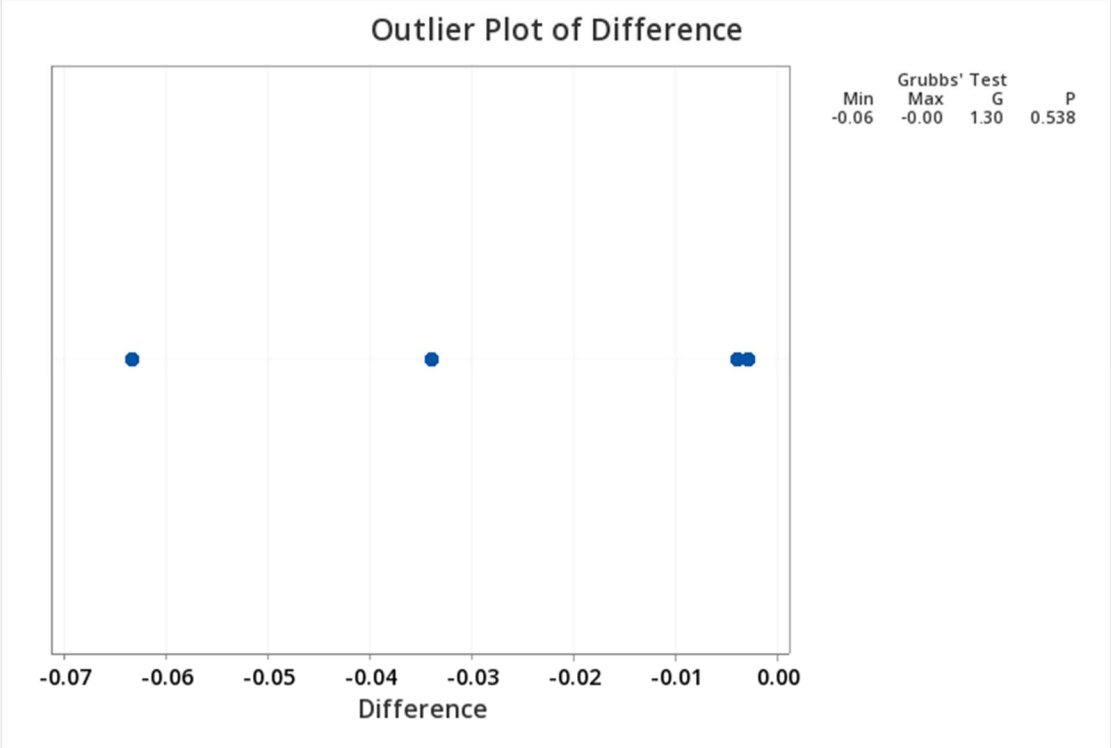

Figure 14. Graphical Summary of Grubb's Test for Differences in Standard Error Between AHP and CRW AHP

With a p-value of 0.538, Figure 14 shows that no statistically significant outliers were identified in the dataset. The G-value resulting from the test also suggests that the results are below the critical value for the sample size at an alpha of 0.05.

With key statistical assumptions satisfied for the dataset, a paired T-test was performed to assess potential difference in standard error between AHP and CRW AHP methodologies. Summary results are presented in Table 25 and Table 26.

Table 25. Summary Statistics of Paired T-test for Differences in Standard Error Between AHP and CRW AHP

Sample	**N**	**Mean**	**StDev**	**SE Mean**
AHP	4	0.0408	0.0235	0.0117
CRW AHP	4	0.0669	0.0085	0.0042

Table 26. Estimation for Paired Difference for Standard Error

Mean	**StDev**	**SE Mean**	**95% CI for μ_difference**
-0.0261	0.0287	0.0144	(-0.0718, 0.0196)

The AHP methodology resulted in a mean standard error of 0.0408, while the CRW AHP methodology results in a mean standard error of 0.0669. The mean difference between the paired sample groups was measured to be -0.0261.

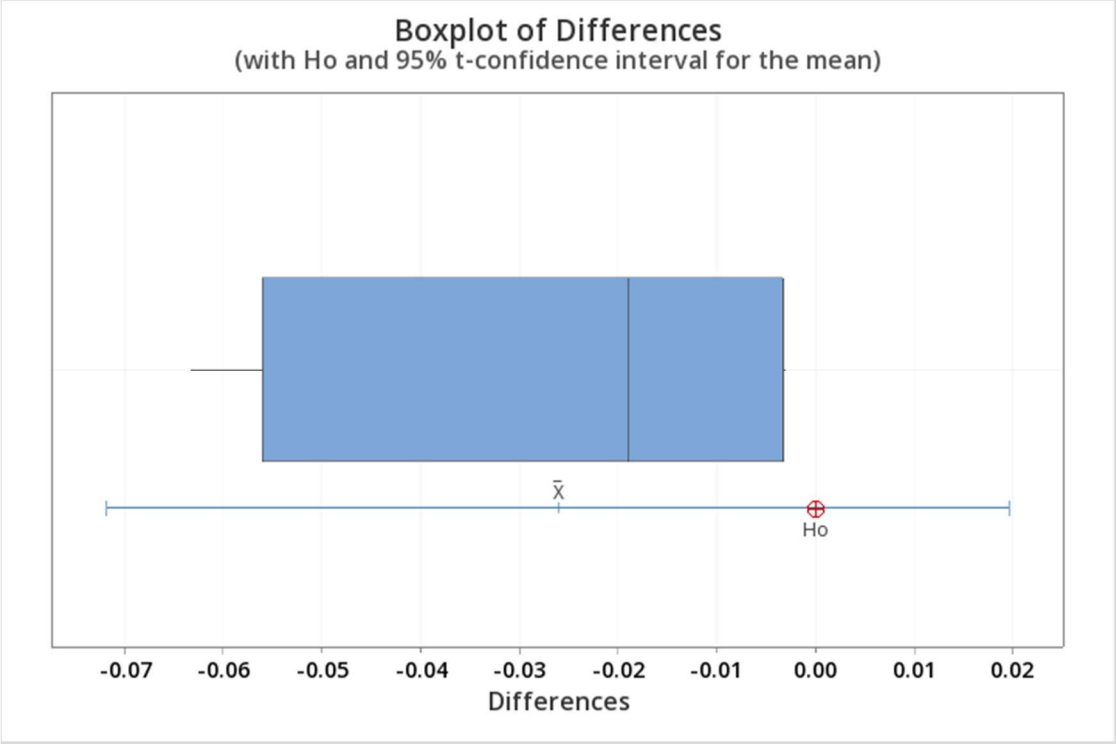

Figure 15. Boxplot of Differences of Standard Error Between AHP and CRW AHP

P-value = 0.167

Figure 15 presents the boxplot of differences between the paired sample groups. The null hypothesis of no difference is marked at 0, while the median difference is measured at -0.02. The overall range of differences in standard error between the two methodologies is from -0.063 to -0.003. An overall p-value for the paired T test is reported as 0.167.

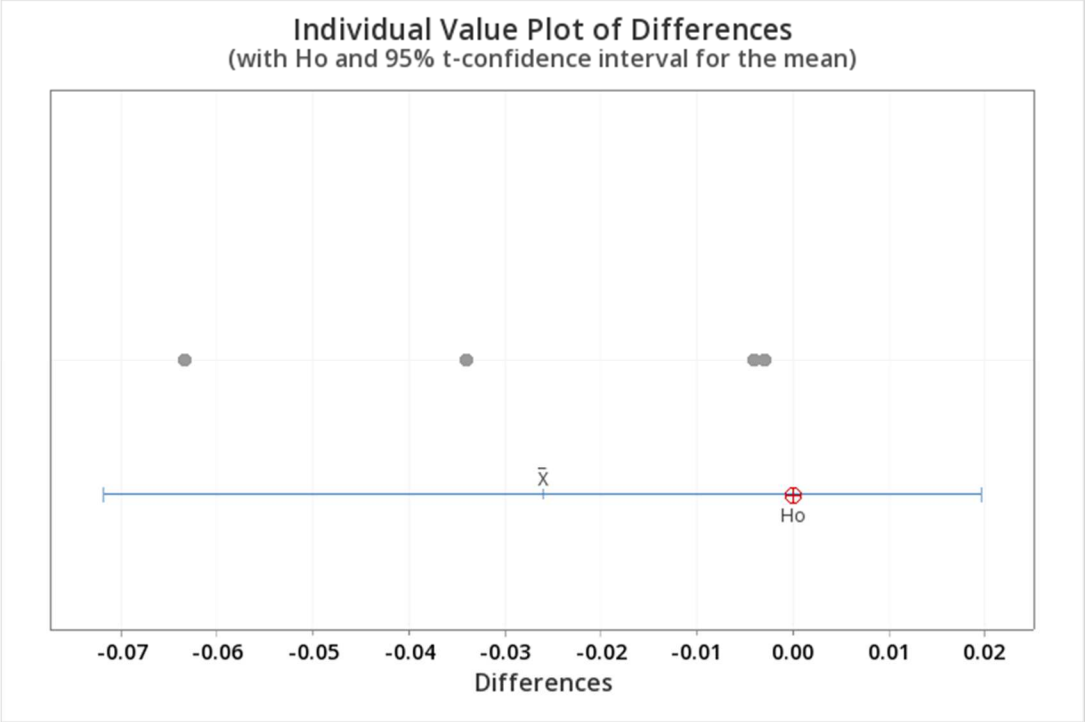

Figure 16. Individual Values of Differences in Standard Error Between AHP and CRW AHP

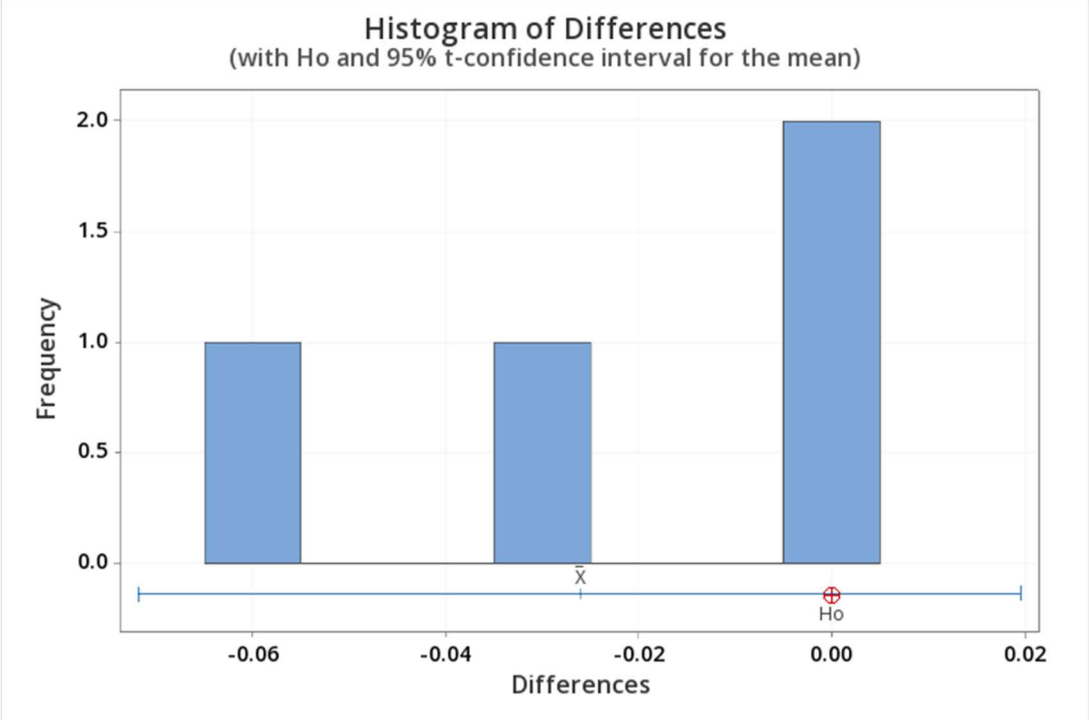

Figure 17. Histogram of Differences of Standard Error Between AHP and CRW AHP

Figure 16 and Figure 17 shows the distribution of calculated differences relative to the null hypothesis of no difference. Majority of differences in standard error between the methodologies were measures a distribution between 0.00 and -0.06.

With a measured p-value of 0.167 for the paired T test, at an alpha of 0.05, the null hypothesis cannot be rejected and no statistically significant difference in standard error between the AHP and CRW AHP methodologies can be claimed.

4.6 Research Question 3 – Analytical Results

Research Question 3

Will implementing a Consistency-Ratio Based Weight prior to judgement aggregation augment the results of comparison with respect to the classical approach?

The third research question assesses whether the CRW AHP methodology changes the calculated criteria weights relative to Saaty's proposed AHP method, in a multi-expert setting. In order to perform a paired equivalence test on the dataset (sample group 1: AHP, sample group 2: CRW AHP), key properties of the data were assessed. Equivalence testing, or two one-sided t-Test, with a null hypothesis that there would be a difference greater than 5% between the methodologies.

For paired equivalence testing, normality of the input data was assessed. Anderson-Darling tests were utilized for the analysis.

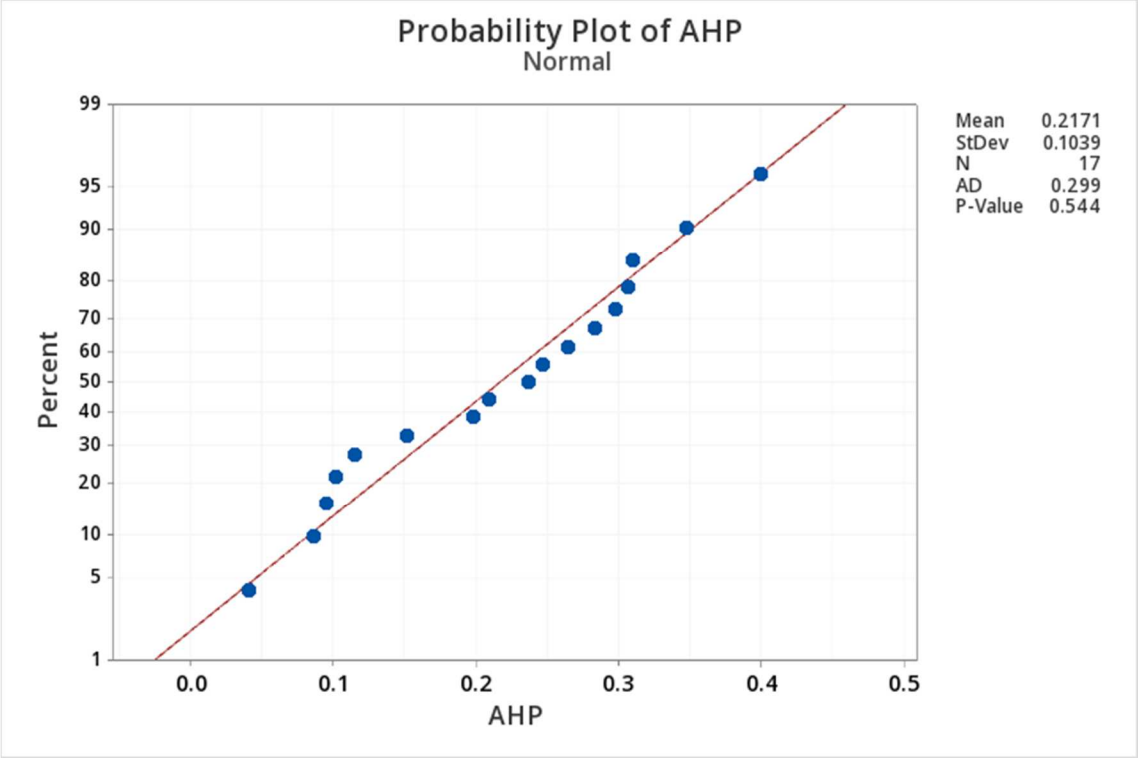

Figure 18. Normality Test for AHP Criteria Weights

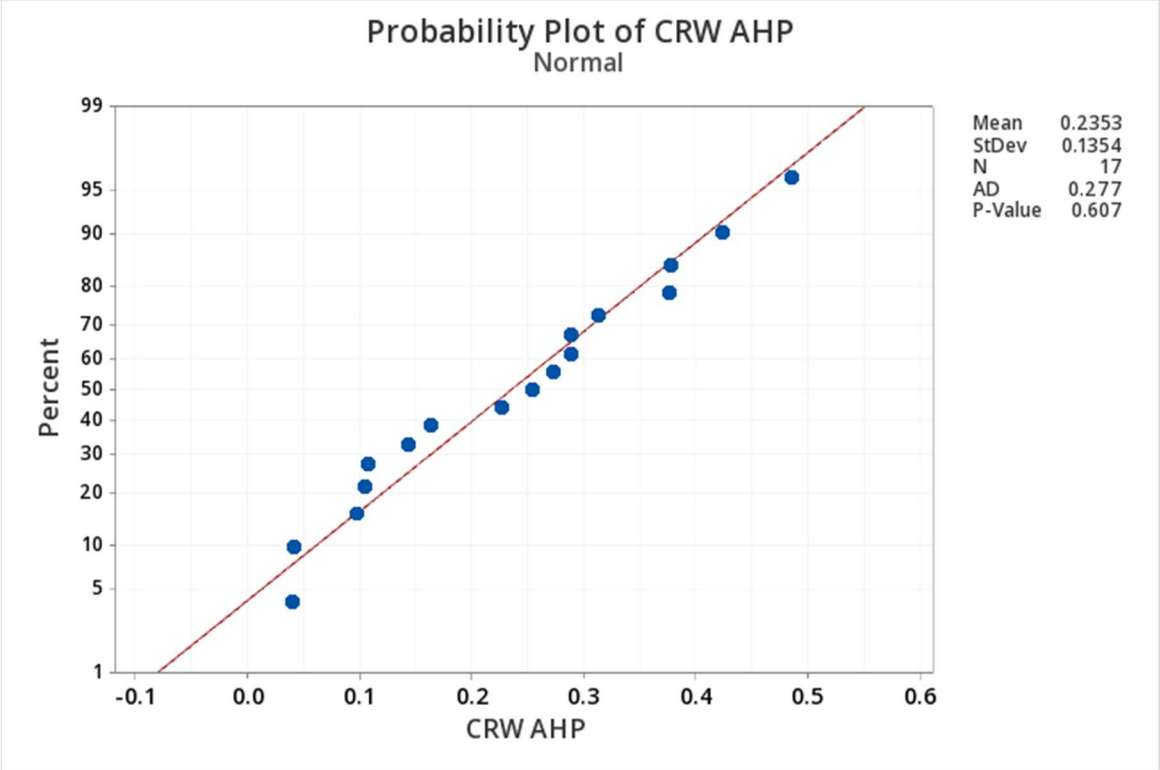

Figure 19. Normality Test for CRW AHP Criteria Weights

Both datasets presented in Figure 18 and Figure 29, no significant evidence of non-normality is reported. A p-value of 0.544 and 0.607 for the AHP and CRW AHP datasets were reported, respectively.

Grubb's test for outliers was also performed on the input data for AHP and CRW AHP.

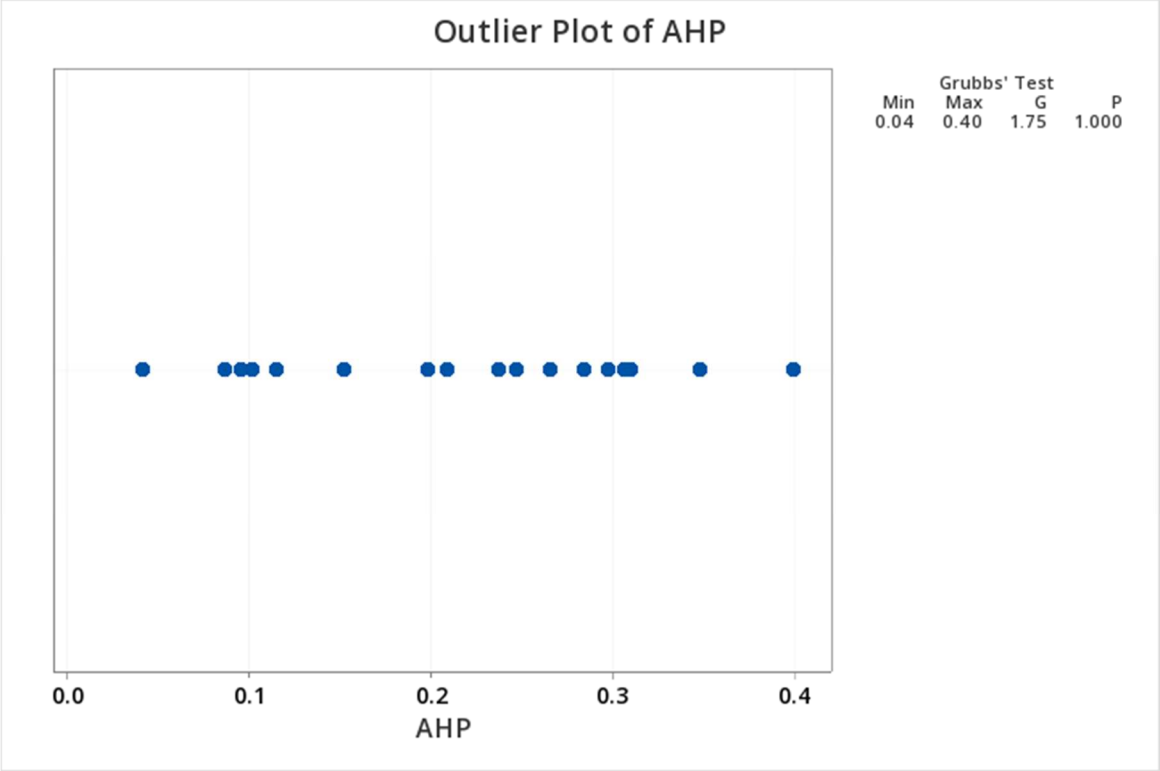

Figure 20. Results of Grubb's test for AHP Criteria Weights

Figure 20 presents the results of the Grubbs outlier test for the AHP dataset. At a p-value of 1.0, no significant outliers were measures in the criteria weight dataset being utilized for the research.

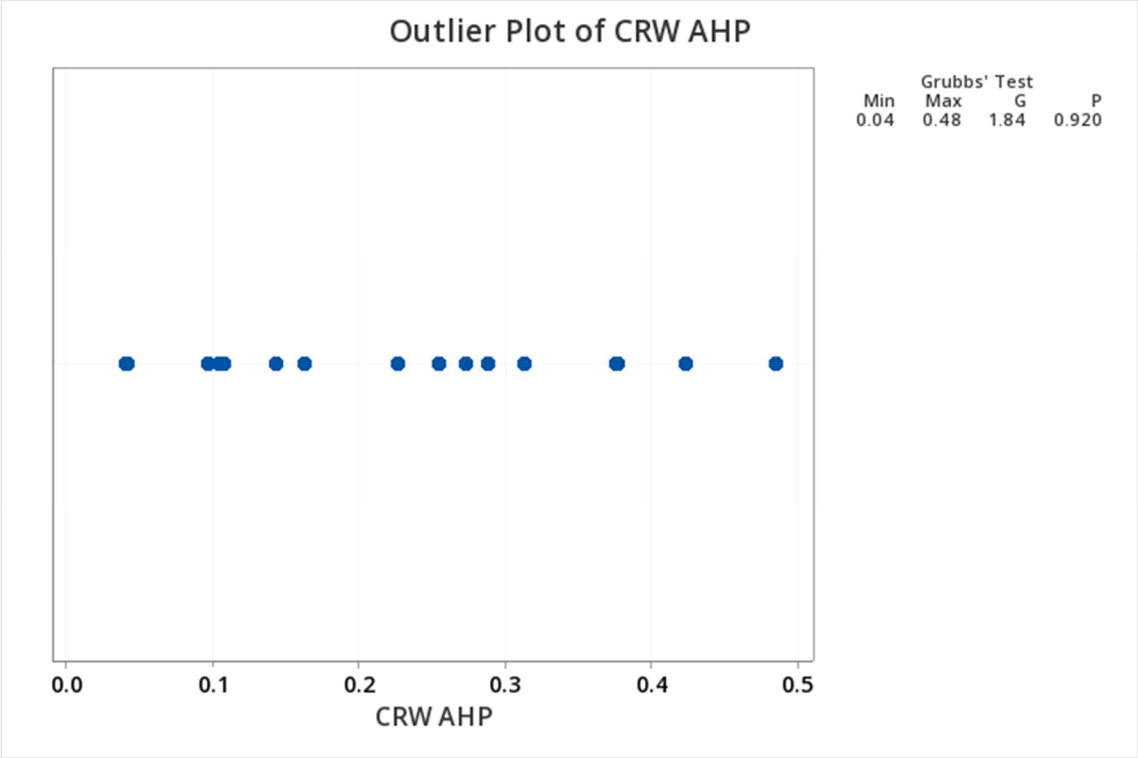

Figure 21. Results of Grubb's test for CRW AHP Criteria Weights

Figure 21 presents the results of the Grubbs outlier test for the CRW AHP dataset. At a p-value of 0.92, no significant outliers were measures in the criteria weight dataset being utilized for the research.

With key statistical assumptions of the dataset satisfied, equivalence testing was utilized to analyze potential differences in the criteria weights between AHP and CRW AHP.

To perform the equivalence test, an equivalence range from -0.01 to 0.01 was identified assess potential differences. This range represents a greater-than 5% potential difference, thus satisfying the respective research hypothesis. Table 27 and Table 28 provide a summary of the two sample groups being compared in the equivalence test.

Table 27. Summary Statistics for Criteria Weights for AHP and CRW AHP

Variable	N	Mean	StDev	SE Mean	Variable
CRW AHP	17	0.23529	0.13538	0.032835	CRW AHP
AHP	17	0.21708	0.10394	0.025208	AHP

Table 28. Summary of Differences in Criteria Weights Between AHP and CRW AHP

Difference	StDev	SE	90% CI for Equivalence	Equivalence Interval
0.018217	0.111452	0.027031	(-0.0179173, 0.0543508)	(-0.01, 0.01)

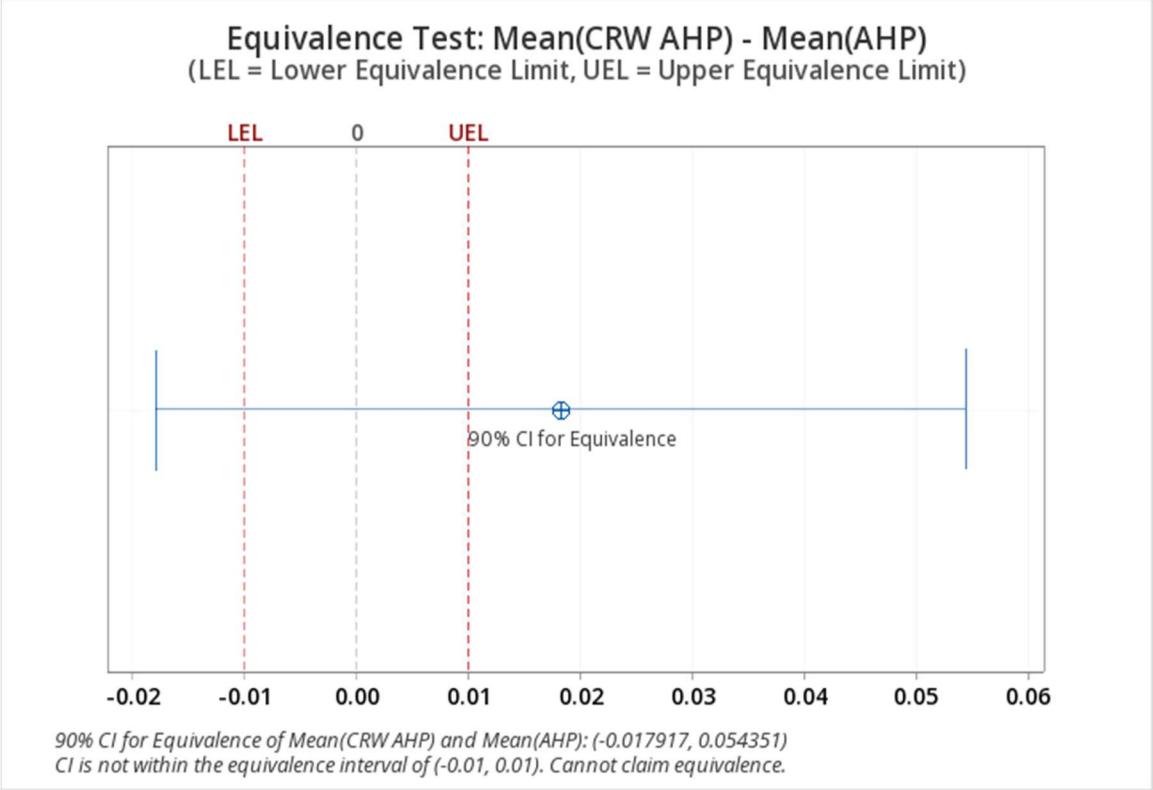

Figure 22. Equivalence Test - Limits Graphed Relative to Differences in Criteria Weights

Figure 22 provides a graphical summary of the equivalence test with the relative limits for equivalence, and the target mean for equivalence at a 90% confidence interval. The general range of differences in means is presented to overlap the defined lower and upper equivalence limits.

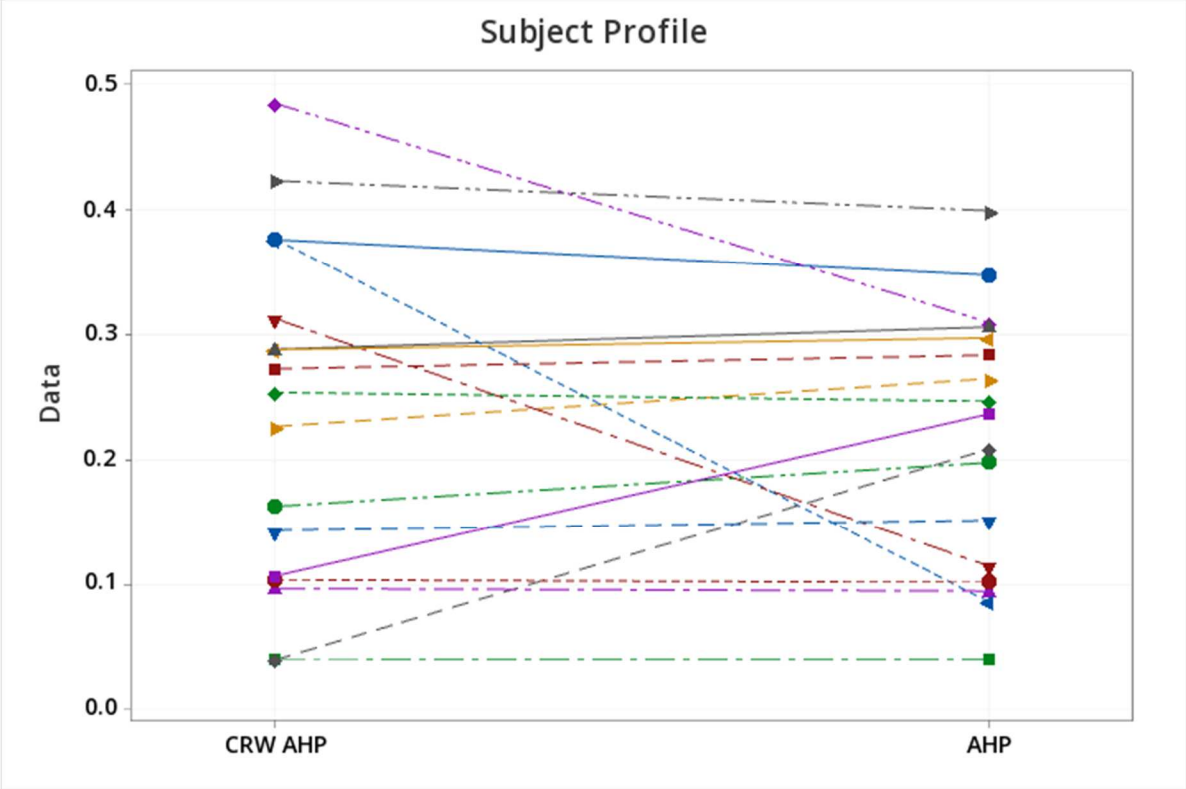

Figure 23. Paired Subject Profiles for Differences in Criteria Weights

Figure 23 presents the direct comparison of criteria weights calculated per CRW AHP and AHP. A range of both positive and negative differences are presented in the dataset. Figure 24 presents the histogram of direct differences between the two datasets. A majority of differences is presented at 0, with a general range from -0.2 to 0.3. A total of N=17 datapoints per sample group were part of the equivalence testing.

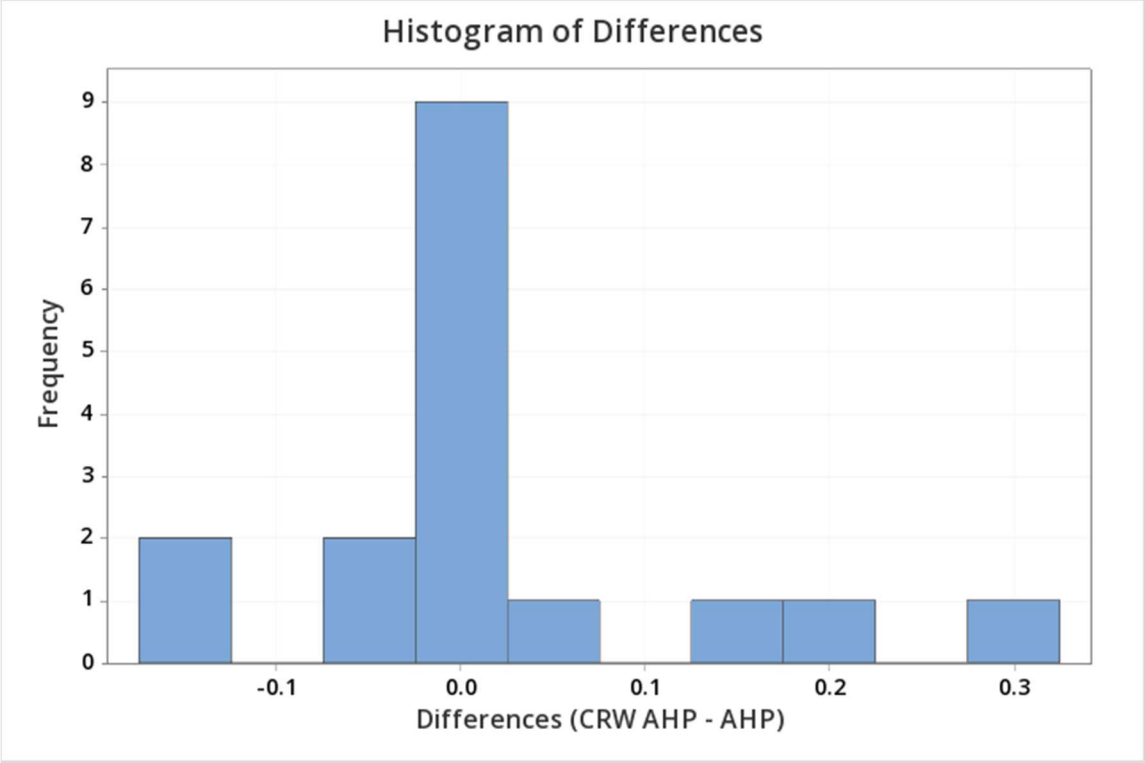

Figure 24. Histogram of Differences in Criteria Weights

Table 29. Summary Results of the Equivalence Test Between AHP and CRW AHP Criteria Weights

Null Hypothesis	**DF**	**T-Value**	**P-Value**
Difference ≤ -0.01	16	1.0439	0.156
Difference ≥ 0.01	16	0.30397	0.617

Table 29 provides a summary of the paired equivalence test to compare criteria weights calculated per AHP and CRW AHP. As a recapitulation, the null hypothesis for equivalence testing proposes that there is a difference between sample groups. With lower and upper p-values of 0.156 and 0.617, respectively, the null hypothesis cannot be rejects, and a statistically significant difference in the sample groups must be implied. The CRW AHP results in different criteria weights when compared to the AHP method.

4.7 Research Question 4 – Analytical Results

Research Question 4

Can a correlation between AHP consistency ratios and reported survey success from literature case studies be confirmed?

To confirm the criticality of a survey's consistency ratio based on the results of the analysis, a two sample T-test was utilized, with the consistency ratio as the continuous independent variable, and a binary categorical assessment as the dependent variable. For the outcome, as stated in the methodologies, a negative outcome from the assessment reflected either unsuccessful results following implementation or reported issues during completion of the analysis that resulted in reprocessing of the pairwise comparison.

Before conducting a two sample T-test, key statistical assumptions were verified on the dataset.

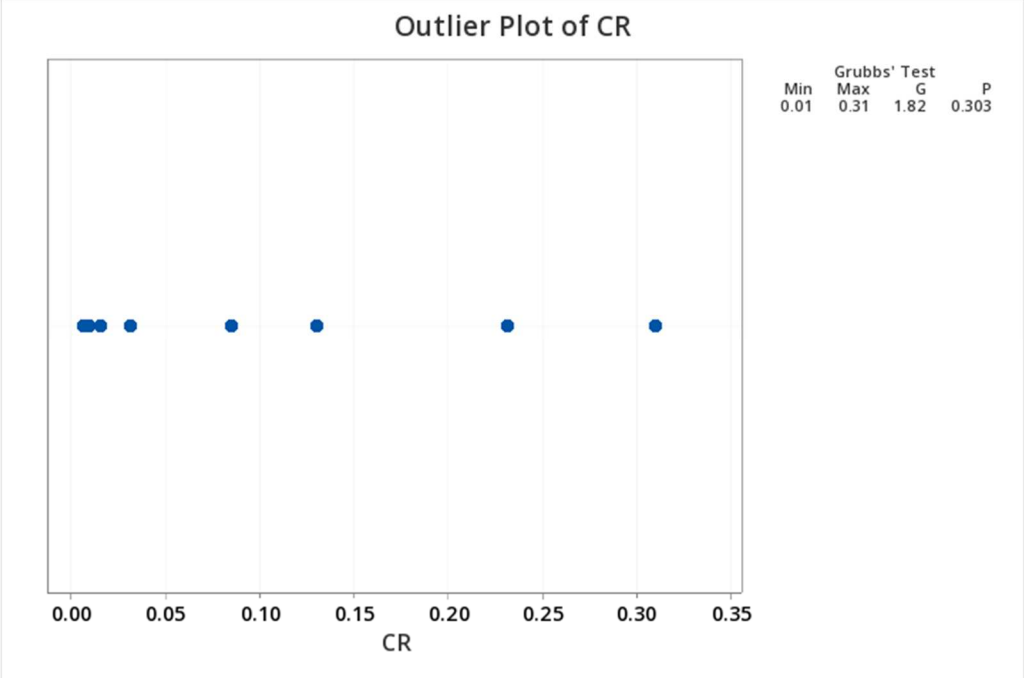

Figure 25. Grubb's Outlier Test for Consistency Ratios

Based on the results of the Grubb's test presented in Figure 26, a p-value of 0.303 is reported and thus no statistically significant outliers were identified in the dataset. In Addition, Figure 27 presents the results of a normality test. Based on the results of the Shapiro-Wilk test, a p-value of 0.098 is reported and thus no statistically significant evidence of non-normality is suggested.

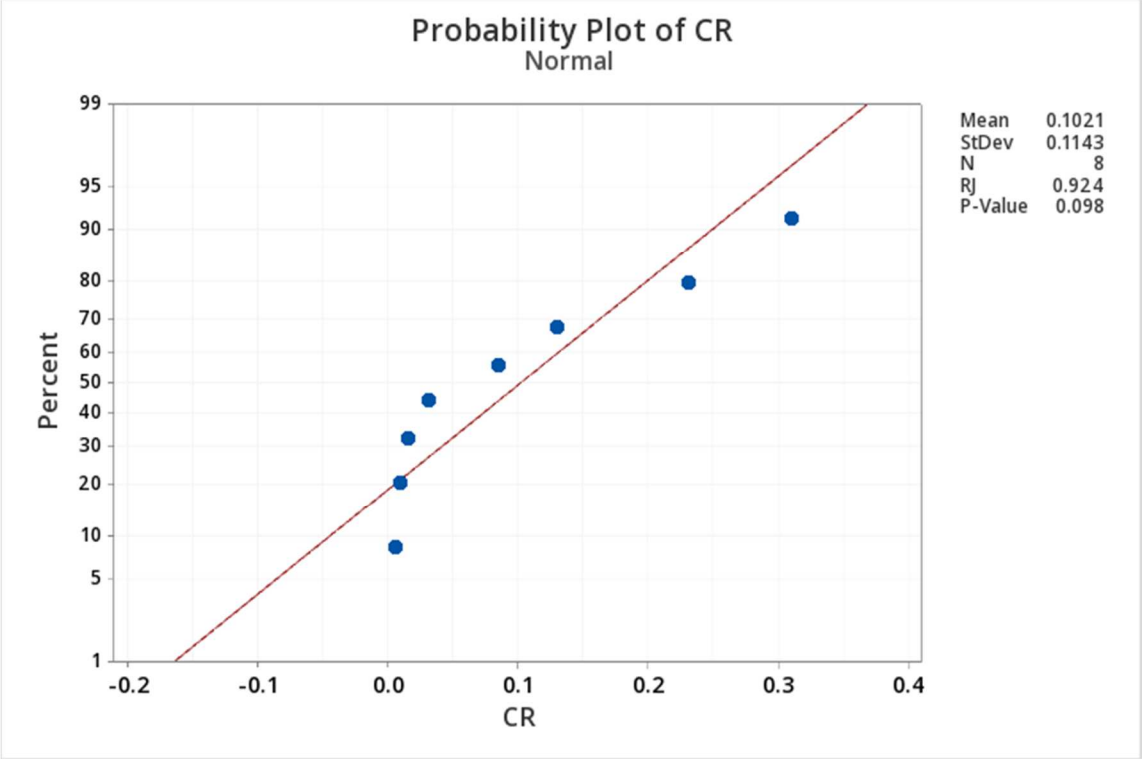

Figure 26. Normality Test for Consistency Ratios

Equal variance was not assumed for the sample groups, as the overall sample size is relatively low. A total of 8 datapoints were utilized for the analysis. Table 30 provides a summary of the dataset being analyzed per two sample T-test.

A mean consistency ratio of 0.189 is reported for the negative outcome sample group, while a mean of 0.0152 is reported for the positive outcome sample group.

Table 30. Summary Statistics for Consistency Ratios

Outcome	**N**	**Mean**	**StDev**	**SE Mean**
Negative	4	0.189	0.101	0.051
Positive	4	0.0152	0.0111	0.0056

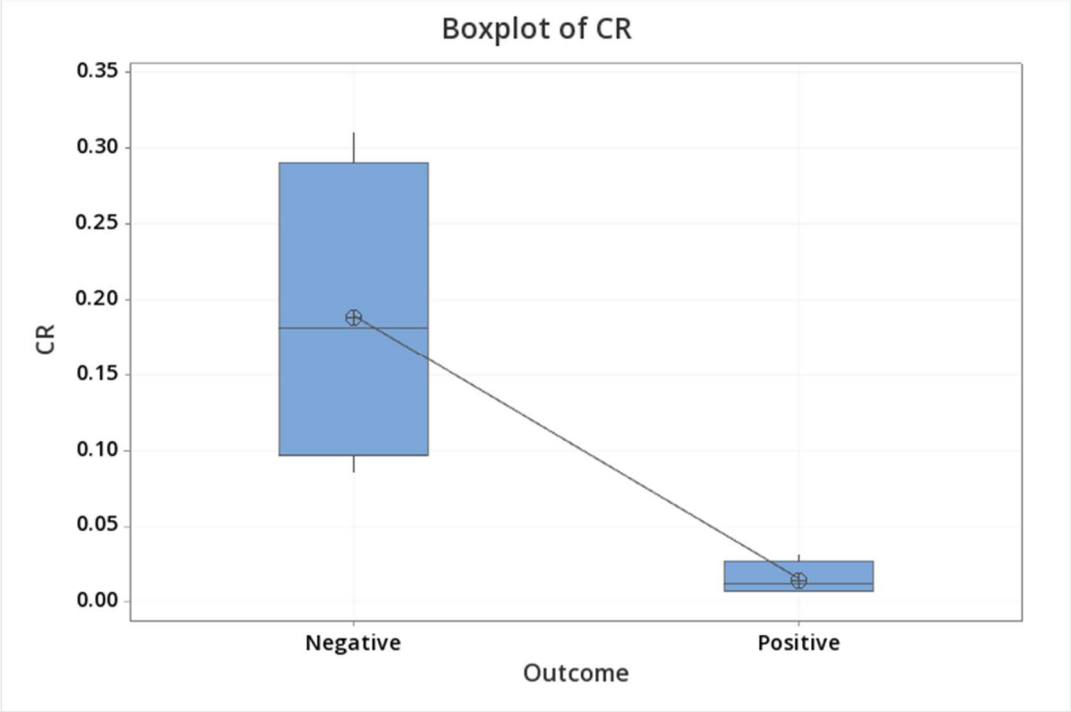

Figure 27. Boxplot Results of the Two Sample T-test Comparing Consistency Ratios

Figure 28 presented the boxplot comparison of consistency ratios between the negative and positive outcome sample groups. A higher standard deviation of 0.101 is reported for the negative outcome sample group, implying that the variance in outcomes is higher when negative.

Table 31. Estimation of Differences for Consistency Ratios

Difference	**Pooled StDev**	**90% CI for Difference**
0.1738	0.0720	(0.0749, 0.2726)

Table 31 presented a summary for the estimation of differences between consistency ratios of the AHP and CRW AHP methodologies. A difference of 0.1738 is reported with a 90% confidence interval ranging from 0.0749 to 0.2726. Figure 29 presents the individual value plots of consistency ratios for both the negative and positive outcome sample groups. To further reiterate the results from Figure 28, the overall variance is significantly higher for the negative outcome sample group.

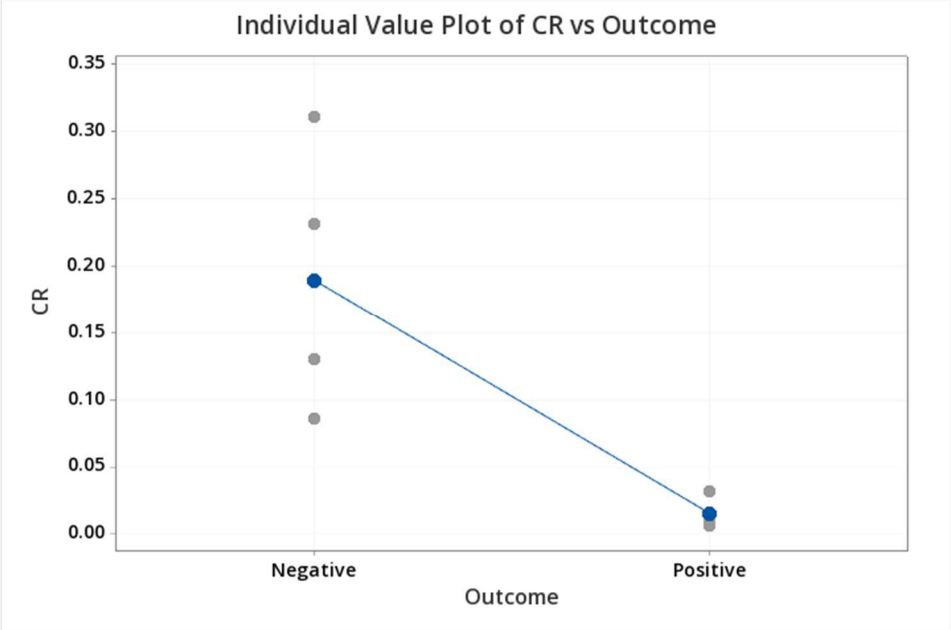

Figure 28. Individual Value Plots for Comparison of Consistency Ratios

The key results of the T-test are provided in Table 32. Based on the comparison of consistency ratios with a binary positive or negative outcome as categorical sample groups, a p-value of 0.014 was measured. At an alpha of 0.05, a statistically significant difference in means between the sample group is implied.

Table 32. Results of Two Sample T-test Comparing Consistency Ratios

T-Value	DF	P-Value
3.41	6	0.014

The overall results of the research provide the following high-level deductions for the dataset utilized in the analysis:

- The CRW AHP methodology provides a statistically higher level of aggregated consistency when compared to AHP
- The CRW AHP methodology does not increase the standard error for the calculated criteria weights when compared to AHP, in a statistically significant manner
- The CRW AHP results in a non-equivalent report of criteria weights when compared to AHP
- A statistically significant binary correlation is found between the consistency ratios of negative and positive outcomes when implementing AHP

Chapter 5—Discussion and Conclusions

5.1 Discussion

The presented research focuses on a novel methodology of improving the AHP by aiming to leverage variations in consistency ratios in multi-expert settings. As proposed in Chapter 1, the concept of transitivity and reciprocity must be sufficiently satisfied in order to accept the logical basis of any given expert's pairwise comparison. The Consistency Ratio-Based AHP methodology increases the aggregated consistency of any multi-expert AHP analysis by generating a transitivity-based expert weight prior to analysis of the collective pairwise comparison. As part of the research, several analytical conditions were interrogated as part of assessing differences between the AHP and CRW AHP methodologies:

- RQ1: Will implementing a Consistency-Ratio Based Weight prior to judgement aggregation increase overall analytical consistency?
- RQ2: Will including all expert feedback per Consistency-Ratio Based Weights increase the standard error for a comparison matrix?
- RQ3: Will implementing a Consistency-Ratio Based Weight prior to judgement aggregation augment the results of comparison with respect to the classical approach?
- RQ4: Can a correlation between AHP consistency ratios and reported survey success from literature case studies be confirmed?

Raw data from literature sources were collected and analyzed to answer the research questions, and relevant analytical methods were selected to understand the significance.

5.2 Conclusions

The following conclusions summarize the results of the research:

- RQ1: With a p-value of 0.047 per paired t-test, the aggregated consistency ratio resulting from the CRW AHP methodology is significantly lower than the AHP methodology
    - α: 0.05
    - H_0: $\mu_{difference} = 0$
- RQ2: With a p-value of 0.167 per paired t-test, the standard error for the comparison matrix derived from the CRW AHP methodology is not significantly different from the AHP methodology
    - α: 0.05
    - H_0: $\mu_{difference} = 0$
- RQ3: With a p-value of 0.156 per equivalence test, equivalence cannot be claimed for the criteria weights resulting from the CRW AHP methodology and AHP methodology
    - α: 0.10, 5% range of equivalence
    - H_0: $\mu_{difference} >$ range of equivalence
- RQ4: With a p-value of 0.014 per two-sample t-test, the consistency ratio is significantly higher for AHP analysis with favorable outcomes versus rejected outcomes
    - α: 0.05
    - H_0: $\mu_{difference} = 0$

Taken together, these conclusions mean that the CRW AHP method enables the centralized normalization of expert feedback prior to aggregation by the practitioner of the methodology. This implies that experts do not have to be retrained and re-surveyed in order to achieve higher consistency, and feedback from experts that achieve higher transitivity in their pairwise comparisons are amplified. Revisiting an example from Chapter 3, Lee's effort in surveying 250 experts for a homeownership decision resulted in only 133 usable datasets due to issues with consistency (Lee et al., 2013). In this case, colocation of the experts could not have been possible for retraining and social normalization. The CRW AHP methodology could have been utilized to process all 250 datapoints from the survey while facilitating a higher aggregated consistency ratio.

5.3 Contributions to Body of Knowledge

1. A novel methodology for utilizing an expert's consistency ratio to calculate weights for proportionally adjusted aggregation of max eigenvalues in an AHP analysis.
2. A novel methodology for utilizing an expert's consistency ratio to calculate weights and aggregate multi-expert feedback without the need for discarding pairwise comparisons based on poor consistency ratios.
3. A novel methodology for utilizing an expert's consistency ratio to calculate weights and aggregate multi-expert feedback without the need for reevaluating pairwise comparisons based on poor consistency ratios.
4. A novel methodology for intaking non-normalized expert judgement without requiring training or modulation of pairwise comparisons during the survey process

5.4 Recommendations for Future Research

Several areas of additional research may benefit the proposed CRW AHP methodology. The opportunities for future research can be identified within three successive analytical stages of the methodology.

- Stage 1: Data collection & processing
    - *Opportunity:* Impact on consistency ratios for both CRW AHP and AHP when experts are retrained and reevaluated based on low consistency
    - *Description*: Although retraining is not required for the CRW AHP methodology, studying the impact on the aggregation consistency ratio between both AHP and CRW AHP may further validate the benefit of the research product. There may be scenarios where inconsistency is too severe to correct through CRW AHP, and thus re-surveying is required. In this case, the results should be compared again to AHP.
- Stage 2: Expert feedback aggregation
    - *Opportunity*: Comparing CRW AHP to AHP when using aggregation methods beyond geometric mean
    - *Description*: For this body of work, only geometric means were used to aggregate multi-expert judgement for AHP. Testing additional methods of data aggregation, such as harmonic or arithmetic, may further validate the benefits of CRW AHP when compared to AHP.

- Stage 3: AHP analysis for calculating weights
    - *Opportunity*: Implementing a tolerance limit to void expert feedback below some level of predefined inconsistency prior to aggregation
    - *Description*: In circumstances of severe inconsistency, defining a threshold to filter out expert feedback may be favorable. This is especially true when the number of experts and pairwise comparisons are high. Understanding the impact of a consistency threshold on both CRW AHP and AHP should be investigated to understand how far the research product can be challenged by inconsistent feedback.

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Appendix A

Table 33. Wicaksono 2020, 1 - Calculation of CRW

EXPERT	CR WEIGHT	WEIGHT INV	EX WEIGHT
E1	0.062	16.23	0.103
E2	0.081	12.40	0.079
E3	0.074	13.53	0.086
E4	0.032	30.78	0.195
E5	0.033	30.53	0.194
E6	0.053	18.84	0.120
E7	0.332	3.01	0.019
E8	0.061	16.28	0.103
E9	0.172	5.80	0.037
E10	0.099	10.08	0.064

Table 34. Wicaksono 2020, 2 - Calculation of CRW

EXPERT	CR WEIGHT	WEIGHT INV	EX WEIGHT
E1	0.033	30.25	0.228
E2	0.094	10.67	0.080
E3	0.123	8.10	0.061
E4	0.117	8.54	0.064
E5	0.080	12.52	0.094
E6	0.049	20.22	0.153
E7	0.180	5.56	0.042
E8	0.099	10.14	0.077
E9	0.048	20.92	0.158
E10	0.177	5.65	0.043

Table 35. Tung 1998 - Calculation of CRW

EXPERT	CR WEIGHT	WEIGHT INV	EX WEIGHT
E1	0.313	3.19	0.134
E2	0.052	19.05	0.800
E3	0.634	1.58	0.066

Table 36. Pedrycz 2011 - Calculation of CRW

EXPERT	CR WEIGHT	WEIGHT INV	EX WEIGHT
E1	0.213	4.69	0.181
E2	0.227	4.41	0.170
E3	0.135	7.39	0.285
E4	0.220	4.55	0.176
E5	0.205	4.88	0.188
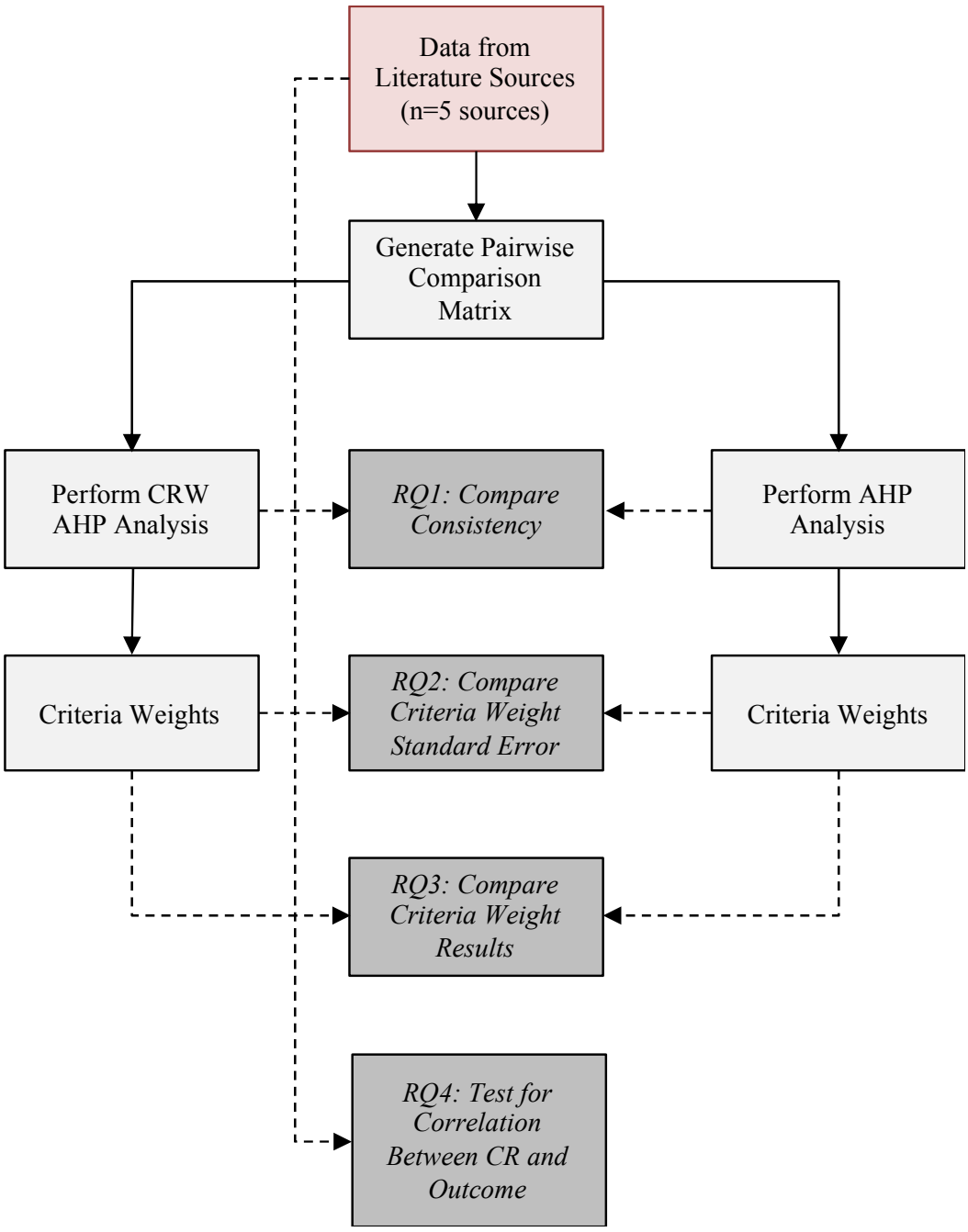
Figure 29. High-Level Overview of DOE