Essays on the Effects of Macroeconomic Volatility

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Dedication

This dissertation is dedicated to my mother, *Sonia María Magdalena*. Her unconditional support and love have sustained me throughout my life. She was my first Math teacher and has always been an inspiration to pursue my goals. At the end of my trip, I will bring no honors or degrees from this world, only my mother's unlimited love. *RLM*

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Abstract of Dissertation

Essays on the Effects of Macroeconomic Volatility

This dissertation consists of three essays of how macroeconomic fluctuations affect the welfare and financial deepening, evaluating the cyclical response of fiscal policy from the expenditure side. The first chapter analyzes the welfare cost of macroeconomic (real) volatility in term of the equivalent variation of per-capita annual consumption for developed countries and one representative developing region (Latin America). In an influential set of lectures, Lucas argues that the welfare cost associated with the business cycle in United States has been negligible in the post-WWII era. As a corollary of his finding, policies more countercyclical, than those already applied would be unnecessary. Although most developing countries register higher macroeconomic volatility in terms of GDP and consumption per capita, there are few studies that estimate and compare the welfare consequences of real volatility in these economies. Starting with the Lucas framework, I provide a quantitative assessment of the welfare cost of macroeconomic fluctuations for developed and Latin American countries. Two alternative models are evaluated across countries: the first one allows for a stochastic trend in the consumption process, while the second one is a general equilibrium model with uninsurable idiosyncratic labor risk. Through the second alternative specification, I find non-negligible welfare costs of more than two-orders of magnitude higher than those estimated with the Lucas model.

Since the high macroeconomic volatility is a well-known phenomenon in most developing countries, many empirical and theoretical works identify the shallow financial system as the source of output volatility, resulting in inefficient risk-sharing and credit misallocation. Other studies, however, emphasize that patterns of production and the lack of technological diversification are responsible for the high and persistent volatility, but without considering that volatility originating in the real sector may

hinder the development of financial system. To explore this possible interaction, in Chapter 2, I develop a general equilibrium framework to explain the channels through which high and persistent macroeconomic volatility can affect financial deepening, by focusing on the exogenous shocks from the real sector. I find that the volatility of total factor productivity (TFP) shocks does not translate one-to-one to a change in output volatility, because of adjustments to investment in capital accumulation induced by changes in the risk premium. This response of the risk premium to TFP shocks explains why real volatility can hinder financial deepening. Consequently, a reduction in the aggregate volatility may increase the country's financial deepening. From this perspective, a shallow financial system can thus be a by-product of the high and persistent macroeconomic volatility.

Promoting more resilient output sectors requires time and resources, in the meantime, policy makers should focus on reducing macroeconomic fluctuations to trigger its beneficial effects on financial development. Given the policy challenges, in Chapter 3, the cyclical properties of government expenditures are assessed in both developing and industrial countries for the period 1970-2014. In most developing regions, fiscal policy, from the expenditure side, has been mainly procyclical during the period, while acylical or countercyclical in industrial countries. However, it is possible to identify a breaking point (year) after which the fiscal policy has become more acyclical, or even countercyclical. For developing Asia this year is 1999, while 2002 and 2005 for Sub-Saharan-African and Latin American countries respectively. Asymmetric behaviors of fiscal policy are also registered during the business cycle in all regions. Finally, there is evidence that fiscal policy tends to be more procyclical with low financial openness and acyclical the higher is the access to external borrowing.

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List of Abbreviations

ECLAC: United Nations Economic Commission for Latin America and the Caribbean.

GDP: Gross Domestic Product.

HAC: Heteroskedastic and Autocorrelation Consistent.

HP: Hodrick-Prescott.

IV/2SLS: Instrumental Variables and Two Stage Least Squares.

LAC: Latin American Countries.

PWT: Penn World Table.

SSA: Sub-Saharan Africa.

UN-COMTRADE: United Nations Database - International Trade Statistics.

WDI: World Development Indicators, The World Bank Group.

WWII: Second World War.

Introduction

As is well documented, most emerging and developing countries exhibit higher and more persistent output volatility than developed (industrial) economies. In this dissertation, I explore the different consequences of high and persistent macroeconomic volatility: from its welfare cost to its relatively unexplored link to shallow financial systems, including an assessment of the fiscal policy response from the expenditure side. Throughout this dissertation, the terms business cycle, macroeconomic volatility, and aggregate risk are used interchangeably.

Lucas (1987) argues that the welfare gain from eliminating macroeconomic fluctuations are negligible, thus policies more countercyclical, than those already applied would not bring additional benefits. Since his seminal work, a large literature has analyzed the welfare effect of macroeconomic (real) fluctuations for the US economy with different specifications and assumptions. Although most developing countries register higher macroeconomic volatility in terms of GDP and consumption per capita, there are few studies that estimate and compare the welfare consequences of business cycles in these economies.

In **Chapter 1**, I resume the discussion of how important is the welfare effect of reducing, or eliminating, the macroeconomic volatility. Different frameworks are calibrated not only for developed countries, but also for Latin American economies, as a representative developing region. I find that the welfare cost of real volatility is always small in the Lucas framework, no matter how volatile the economy is. Therefore, two alternative models are evaluated across countries: the first one allows for a stochastic trend in the consumption process, while the second one is a general equilibrium model with an uninsurable idiosyncratic risk that follows the Krebs (2003) model. As a result, welfare cost of more of more than two-order of magnitude higher than in Lucas model are estimated with the general equilibrium framework.

Typically, the relationship between financial deepening and macroeconomic volatility is often analyzed as a unidirectional phenomenon. Most empirical works that study the causal effects of financial deepening on output volatility are based on theoretical models that point to the underdeveloped financial system as the main source of high macroeconomic volatility. Instead of absorbing the shocks, credit market imperfections amplify them, with the result that firms are unable to pool the risk of high-return investments, and respond by choosing low-return projects. However, a second line of theoretical and empirical studies argues that the macroeconomic volatility in many emerging countries comes from their production pattern, which is highly concentrated in more volatile industries with nonflexible technologies. Therefore, if the degree of financial development is not the primary source of volatility, it remains the question of how high and persistent macroeconomic fluctuations can affect the depth of financial system. Although some intuitive answers can be proposed with partial equilibrium frameworks, the question requires a general equilibrium approach to explain the causal relationship between the macroeconomic volatility and financial deepening.

In **Chapter 2**, I study the channels through which persistent output volatility can affect the financial deepening, for economies where aggregate volatility is an intrinsic phenomenon originating in the real sector. To this aim, I propose a general equilibrium framework where investors (capital producers) are subject to an uninsurable idiosyncratic risk, whose distribution is affected by the aggregate risk. Financial intermediaries diversify their portfolios with two possible investment options: lending to the capital producer or buying a risk-free government bond. The model is an extension of Labadie (1998), with an optimal contract scheme similar to the one proposed by Bernanke, Gertler, and Gilchrist (1999). The main finding is that a reduction (increase) in aggregate volatility increases (reduces) the optimal level of private loans and investments by lowering (increasing) the risk premium of lending

to the capital producers, which may increase (reduce) the financial depth indicator. At the same time, the aggregate risk is not translated one-to-one to output (GDP) volatility, given its indirect effect on capital accumulation.

In addition to structural problems, inconsistent macroeconomic policies have played an important role in increasing the macro volatility in developing countries. If the welfare cost of real volatility is non-negligible, then more countercyclical policies might reduce macroeconomic cycles. However, if the shallow financial system is a by-product of high and persistent aggregate risk, the transmission mechanisms of monetary policy in these economies might be affected. In **Chapter 3**, following Calderón, Chuhan-Pole, and López-Monti (2017), the cyclical properties of government consumption in industrial and developing countries are analyzed. Although most developing regions have conducted procyclical fiscal policies (from the expenditure side) during the period 1970-2014, some progress has been made towards more countercyclical (or acyclical) policies since the end of last century. Evidence of an asymmetric behavior during the cycle are found, while financial openness seems to play a role to explain the degree of fiscal policy cyclicality.

Chapter 1

The Welfare Cost of Real Volatility: A Comparative Analysis[1]

In a seminal work, Lucas (1987, 2003) argues that the business cycle in the US economy after the WWII has generated negligible welfare cost, i.e. equivalent to less than one-hundredth of one percent of annual per capita consumption. Thus, he suggests that additional countercyclical policies beyond those already applied would be unnecessary. Since most of the related literature estimates the welfare cost generated by the US business cycle, in this article I extend the study not only to all developed countries, but also to the Latin American economies. I cover the period from 1960 to 2006, before the international financial crisis, to make my results comparable with other works previous to the crisis. I assess the welfare cost through different specifications, taking the representative agent model suggested by Lucas (1987) as the baseline case. In addition, I consider two alternative frameworks, which do not intent to match the consumption process, but to compare the welfare cost through other specifications. The first one is an extension of the baseline case by allowing for a stochastic trend in the consumption process, while the second one is a general equilibrium analysis with uninsurable idiosyncratic human capital risk following Krebs (2003). With the latter model, I find non-negligible welfare costs of more than two-order of magnitude higher than those estimated with the Lucas model. In fact, the annual increase of per capita consumption that would compensate households for living with the real volatility can reach 9.1 percent in Latin America and 7.7 percent in the US economy, compared to 0.06 percent and 0.006 percent respectively estimated with the baseline model.

The Lucas results have been tested using different assumptions and specifications. Obstfeld (1994), Dolmas (1998), and Tallarini (2000) assume Epstein-Zin preferences

[1]I started working on this study as part of my M.Sc. Dissertation at the University of Warwick. Preliminary versions have been presented in international conferences and meetings during the last years

which better fit asset prices data by separating the elasticity of intertemporal substitution from the coefficient of relative risk aversion. This alternative specification suggests that consumption volatility is not very costly unless fluctuations are highly persistent: the welfare cost in terms of per capita consumption ranges between 0.01 percent and a maximum value of 12.6 percent, with serially correlated consumption fluctuations and higher risk-aversion.

On the other hand, there are several studies that consider consumer heterogeneity. Thus, volatility faced by some fraction of the population might be underestimated. Based on the empirical evidence, Imrohoroğlu (1989) finds that stabilization affects earning risk by avoiding long periods of unemployment, given the fact that unemployment spells are short in "good times" but longer during a recession. As a result, she estimates a cost of business cycle in the US economy of about 0.3 percent of household consumption, ruling out the interest rate risk. Krusell and Smith (1999) follow the idea that stabilization reduces the period of unemployment, but they allow for the interest rate to vary over the cycle and introduce an asymmetric wealth distribution. Under this framework, the cost of aggregate fluctuations would be 3.68 percent for those individuals who are unemployed, but smaller and even negative for households with savings. As a result, in the Krusell-Smith's framework the real volatility might be on average beneficial, depending on the fraction of the population who are unemployed and with borrowing constraints. Thus, in countries with better wealth distribution and extended access to the financial markets, stabilization policies could make the majority of people worse off. However, this is not the case of most emerging economies, which are characterized by an unequal wealth distribution and a less-developed financial system.

By looking at the US reports of household earnings, Storesletten et al. (2001) find that income shocks are highly persistent. When an idiosyncratic labor income risk is introduced and household's income falls, the earnings will be low for a longer

time than in Krusell-Smith's specification. They estimate a gain from eliminating the aggregate fluctuations in the US economy of 2.5 percent of life-time consumption as a whole, while for those household without any savings the gain reach 7.4 percent. On the other hand, Krebs (2003) extends Storesletten et al. (2001) by adding permanent idiosyncratic income shocks that households are not able to offset. The welfare cost of real volatility in his calibration for the US economy is 7.4 percent of the life-time household consumption.

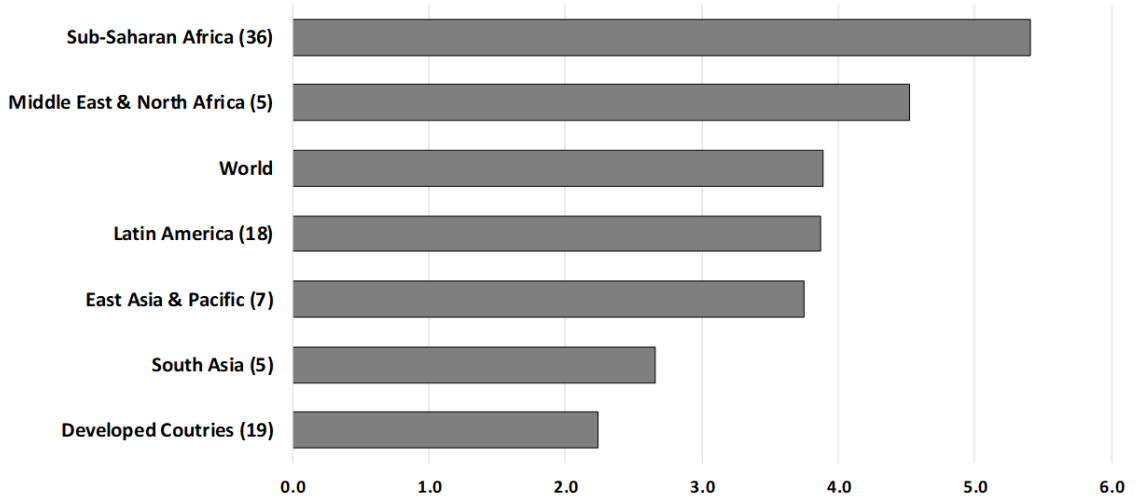
Other papers suggest that the average consumption level might change in response to stabilization policies. Instead of reverting to its trend, as in Lucas model, consumption might increase with stabilization. In other words, the consumption path in a stable economy might exceed the expected level with high macroeconomic fluctuations. Following this assumption, De Long et al. (1988) estimate a cost between 1.6 percent and 1.9 percent of US household consumption. Finally, stabilization might affect the consumption growth rates instead of the levels. Indeed, Barlevy (2004) shows that the welfare effects of real volatility might be substantial, reaching 8 percent of life-time consumption. Alternatively, Pallage and Robe (2003) is one of the few studies addressing the welfare cost of consumption volatility in poor countries. They find that the potential welfare gain from eliminating the macro volatility in poor countries exceeds the gain from one percent increase in the growth path.

In the next section, I characterize the macroeconomic fluctuations in both developing and developed countries. In the following section, the welfare costs associated with real volatility are studied through the different specifications, taking the Lucas model as the baseline case.

1.1 The Macroeconomic Volatility in Perspective

The standard deviation of per capita GDP growth rates is the most common indicator of macroeconomic fluctuations to capture the volatility of the whole economy. However, the second moment is also used to study the behavior of different GDP components, such as consumption, investments, exports, and so on. In general, the "volatility" of a certain variable refers to its deviation from a reference value, which gives the idea of a permanent state or trend. Since most economic variables (in levels) are non-stationary, i.e. the fluctuation is around a non-constant trend, different statistical techniques can be used to separate the permanent (trend) and transitory components (cycle).

Figure 1.1: Standard Deviation of per Capita GDP Growth, 1961-2016 (median by region)

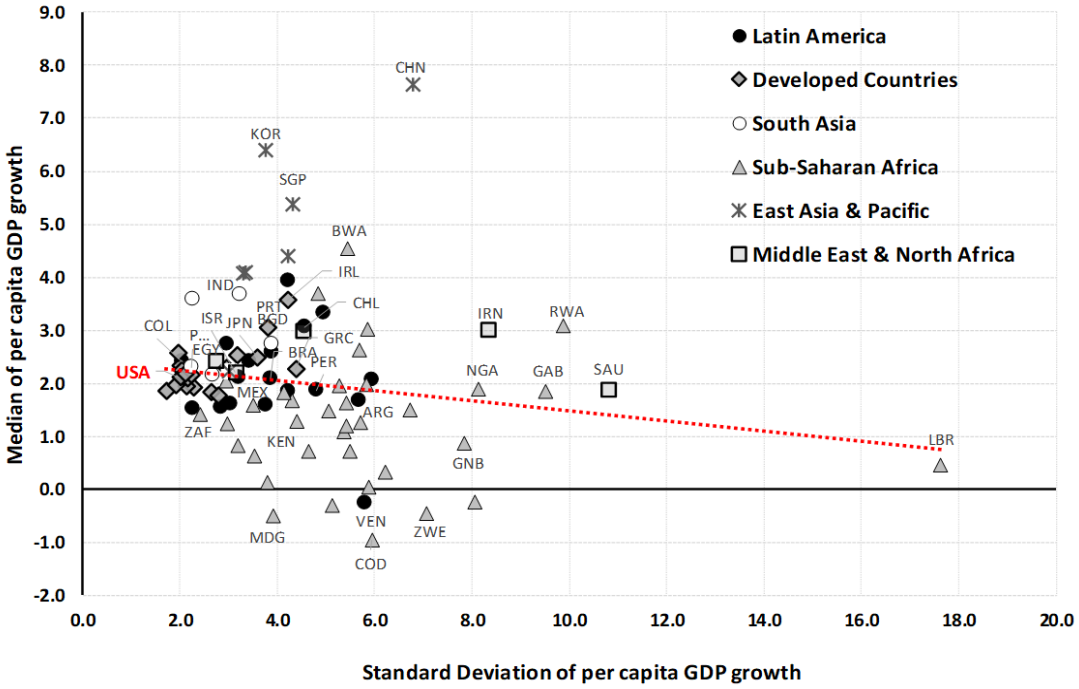


Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: GDP per capita is at constant 2010 dollars. The number of countries in each group is given in parenthesis. The World comprises a total of 90 countries.

Although macroeconomic volatility is a worldwide phenomenon, most developing countries have registered the highest levels of GDP per capita volatility over the last five decades (Figure 1.1). At a country level, the real volatility is not only higher in most developing countries but it seems to have a negative effect on growth

(Figure 1.2).This phenomenon was first documented by Ramey and Ramey (1995) and motivated an extensive literature on the negative relationship between volatility and growth.[2] Figure 1.2 shows that most developed countries are grouped with relatively low level of macroeconomic fluctuations and growth, while the developing countries have different patters. Therefore, the study of the welfare effects of real volatility should not only focus in the US economy, but also in the developing countries.

Figure 1.2: Volatility and Growth, 1961-2016



Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: GDP per capita is at constant 2010 dollars.

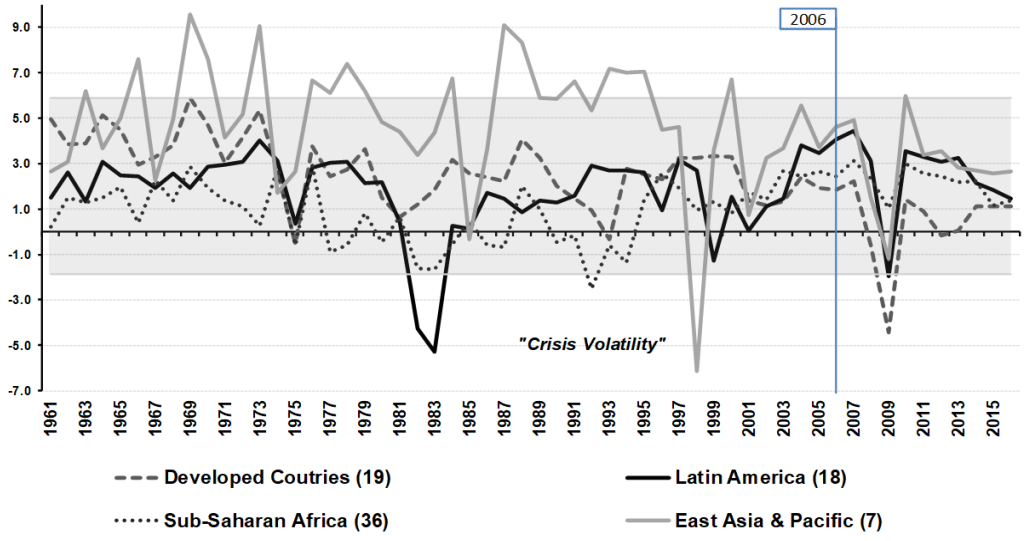
Hnatkovska and Loayza (2005) use an ad-hoc definition of "crisis" volatility, specifically when the per capita GDP growth lies below one standard deviation of the worldwide average of per capita GPD growth. Following this definition from 1961 to 2006, only the developing regions (i.e. Latin America, East Asia, and Sub-Saharan Africa) registered "crisis" levels of per capita GDP growth (Figure 1.3). However, the US economy and other developed countries became the epicenter for the international

[2]Among others Fatás (2002), Acemoglu et al. (2003), and Hnatkovska and Loayza (2005)

financial crisis of 2008-2009, that pulled the rest of world. The year 2006 will be the end of the sample period (1960-2006) for which the welfare cost of macroeconomic (real) volatility will be estimated in the following section. As was mentioned before, the financial crisis could easily be included in this assessment, but the resulting estimates would not be comparable to previous literature.

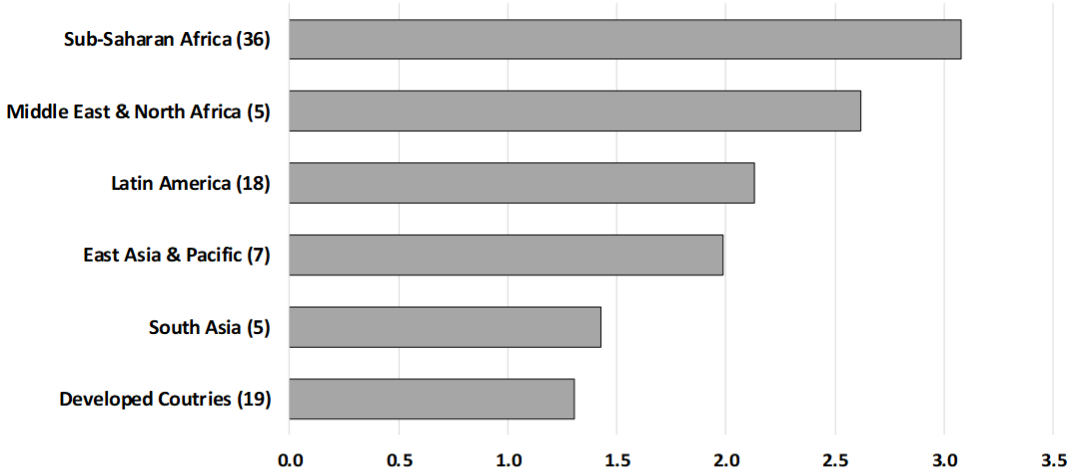
Macroeconomic volatility is also measured through the standard deviation of the cyclical component of the output per capita. Throughout this chapter the Hodrick-Prescott (HP) filter is applied to isolate the transitory components of output and consumption per capita, with a smoothing parameter $\lambda = 6.25$ (Ravn and Uhlig 2002). When looking at volatility of the business cycle for different regions, the resulted pattern replicates the ranking obtained through the standard deviation of growth rates (Figure 1.1). Indeed, Figure 1.4 shows that the developing regions lead the ranking of cycle volatility during the period.

Figure 1.3: Normal and Crisis Volatility, 1961-2016
(GDP per capita growth, median by region)



Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: As in Hnatkovska and Loayza (2005), the shaded area defines the "normal" volatility as one standard deviation of the worldwide median of per capita GPD growth rates (in constant 2010 dollars) during the period. The number of countries in each group is given in parenthesis. The World comprises a total of 90 countries.

Figure 1.4: Volatility of the Business Cycle, 1961-2016
(Percentage difference from trend, median by region)



Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: The HP filter (with $\lambda = 6.25$) is applied to the natural logarithm of GDP per capita (in constant 2010 dollars) by country and the resulted cyclical component is then multiplied by 100. The number of countries in each group is given in parenthesis. The World comprises a total of 90 countries.

Although explaining the structural reasons behind the higher macroeconomic (real) volatility in most developing regions goes beyond is not the aim of this work, the literature has basically identified three main reasons to explain the persistence of high volatility on these economies:

(i) Some developing countries face larger external shocks than developed economies. For instance, in the last four decades, Latin America has suffered important external shocks, nominal and real, as a result of fluctuations in both international capital flows and terms of trade.

(ii) Production patterns are highly concentrated in more volatile industries that are intensive in non-flexible technologies and unskilled workers, whereas the structure in developed countries is based on flexible technologies and skilled workers. Kraay and Ventura (2007) argue that the cross-country differences in real volatility can be explained by different patterns in industrial specialization between developed and developing economies. Alternatively, Koren and Tenreyro (2007)

show that poorer countries are specialized in fewer and more volatile sectors that explain higher macroeconomic fluctuations. In general, the previous findings support the idea that more flexible and resilient to shock technologies are chosen as the economic development increases.

(iii) Finally, institutional instabilities and inconsistent macroeconomic policies have also played an important role in increasing the real volatility in several developing countries. Macroeconomic policies have been characterized by abrupt changes in rules with deep socio-economic consequences. Most developing regions have weaker "shock absorbers" or filtering mechanisms. They have had inefficient market filters as a result of a shallow financial systems, insufficient export diversification, and endogenous rigidities associated with the political economy. Acemoglu et al. (2003) argue that most of these problems are associated with a weak institutional framework.

Given the fact that macroeconomic volatility is relatively higher in developing countries, in the next section, I analyze the welfare cost of real fluctuations through three different frameworks, not only for the US economy and other developed countries, but also for Latin American economies as a representative developing region. Since this work focuses on the welfare effect of macroeconomic fluctuations, what ultimately matters is the household consumption volatility. In a complete market scheme, households could diversify portfolios and risk, shielding its consumption from income volatility. In this context, households are able to avoid the "non-permanent" income volatility through the precautionary savings and insurance mechanisms. However, incomplete markets provide a limited protection in the context of permanent shocks. The *excess volatility of consumption* is a well known feature that characterizes many small-open economies. Indeed, Aguiar and Gopinath (2007) find that consumption is 40 percent more volatile than GDP in emerging economies, while slightly less volatility than income for developed countries.

1.2 Measuring the Welfare Cost of Real Volatility

The "welfare" is a subjective idea which not only depends on individual priorities but also on the pleasure or happiness that consumption generates. For this reason, the utility-function approach seems to be a natural way to measure satisfaction and to rank different consumption plans, assuming rationality and completeness. In order to measure the welfare effects of macroeconomic fluctuation, the main question throughout this section is how large the increase in annual household consumption is when the total volatility is eliminated. Formally,

$$U\left[\{(1+\Delta)\,C_t\}\right] = U\left[\{C_t^*\}\right], \tag{1.1}$$

where actual household consumption (C_t) deviates from the trend (C_t^*) by an assumed random variable. Thus, the cost of real volatility is represented by the fraction Δ, which makes the consumer indifferent to the economic fluctuations.

1.2.1 The Baseline (Lucas) Model

Following Lucas (1987), the one-period preferences are defined by a constant relative risk aversion utility function over the consumption plan:

$$U\left[\{C_t\}\right] = E_0\left[\sum_{t=1}^{\infty}\beta^t\frac{C_t^{1-\gamma}-1}{1-\gamma}\right], \text{with } 0<\beta<1 \text{ and } \gamma\geq 0. \tag{1.2}$$

As usual β is the rate at which utility is discounted and γ is equal to the coefficient of relative risk-aversion, so the higher γ the more reluctant is the consumer to face a volatile consumption path. Given an initial value C_0, the per capita consumption function is defined as follows:

$$C_t = (1+g)^t\,C_0 e^{\varepsilon_t-\sigma_\varepsilon^2/2}, \tag{1.3}$$

where g is the consumption growth rate and $\varepsilon(0, \sigma_\varepsilon)$ is an i.i.d. normal random variable. In this framework, the consumption level in the economy without real fluctuations (i.e. $\sigma_\varepsilon^2 = 0$) is equal to the expected value of C_t. After some algebra, the percentage increase across all dates in annual consumption from eliminating the consumption uncertainty is given by

$$U\left[\{(1+\Delta)\, C_t\}\right] = \begin{cases} (1+\Delta)^{1-\gamma}\, e^{-\frac{\sigma_\varepsilon^2}{2}\gamma(1-\gamma)} = 1 & \text{if } \gamma \neq 1 \\ ln\,(1+\Delta) = \dfrac{\sigma_\varepsilon^2}{2} & \text{if } \gamma = 1. \end{cases} \tag{1.4}$$

Thus, the equivalent variation as a function of the consumption variance is

$$\Delta(\sigma_\varepsilon^2, \gamma) = e^{\frac{\sigma_\varepsilon^2}{2}\gamma} - 1, \;\; \forall\gamma. \tag{1.5}$$

Therefore, the first-order Taylor approximation of Equation (1.5) in the neighborhood of $\sigma_\varepsilon^2 = 0$ yields:

$$\Delta(\sigma_\varepsilon^2, \gamma) \approx \frac{\sigma_\varepsilon^2}{2}\gamma, ; \; \forall\gamma. \tag{1.6}$$

This represents Lucas's approximation of the cost of consumption volatility, which depends on the degree of risk aversion and the variance of the shock. Therefore, real volatility is more costly the more volatile the economy becomes and the more averse the individual is. However, by using US data for consumption volatility after the WWII and assuming a log utility ($\gamma = 1$), Lucas (1987) shows that the cost of consumption fluctuation is negligible. In other words, US consumers would be willing to give up less than 0.01 percent of their consumption, uniform across all dates, in order to get macro stability. In light of these results, Lucas concludes that more aggressive countercyclical policies than those applied in the post-WWII period are not needed, since they would bring almost no benefit in terms of welfare. Before going on with the analysis it is important to clarify the scope of this finding.

Lucas does not argue that countercyclical policies are unnecessary at all from the welfare point of view. On the contrary, what he points out is that *further countercyclical policies* than those already applied in the US since 1945 would have a negligible welfare gain for consumers. Real consumption volatility would have been larger if stabilization policies had not been applied at all in US during the post-WWII. Indeed, monetary and fiscal policies have played an important role in stabilizing the US economy in the last decades, the question is whether *additional countercyclical policies would be useful*. Since the welfare effects of real volatility are analyzed in both developed and developing countries, Table 1.1 summarizes the welfare cost of consumption volatility using the Lucas approximation, i.e. Equation (1.6), not only for the United States and other developed economies, but also for Latin American countries, that represent the developing world.

In general, under the Lucas framework and reasonable values of risk aversion, eliminating the real volatility would entail negligible welfare cost across the countries. Although the cost of real fluctuations in Latin America is on average six times higher than the average cost in developed countries (ten times higher than in US), it is still lower than 0.6 percent of per capita consumption. It is straightforward to show that these findings are due to the higher consumption volatility in Latin American economies, where the standard deviation of the cyclical component of per capita consumption is on average more than two times higher than that in developed countries (see column 2).

Table 1.1: Welfare Effects of Consumption Volatility with the Lucas Model
(Equivalent variation of per capita annual household consumption in percentage, 1961-2006)

	STDEV of the cyclical component	γ=1.0	γ=1.5	γ=2	γ=5	γ=10
United States	**1.1**	**0.006**	**0.009**	**0.011**	**0.028**	**0.057**
Latin America (simple average)	**3.2**	**0.059**	**0.088**	**0.118**	**0.294**	**0.588**
Argentina	**4.3**	0.094	0.142	0.189	0.472	0.945
Bolivia	**1.7**	0.015	0.022	0.030	0.074	0.148
Brazil	**2.8**	0.040	0.061	0.081	0.202	0.404
Chile	**4.8**	0.114	0.170	0.227	0.568	1.136
Colombia	**1.4**	0.010	0.015	0.020	0.050	0.100
Costa Rica	**3.1**	0.047	0.071	0.095	0.237	0.474
Ecuador	**1.5**	0.011	0.017	0.022	0.055	0.110
El Salvador	**3.5**	0.061	0.092	0.123	0.307	0.613
Guatemala	**1.0**	0.005	0.007	0.010	0.025	0.049
Honduras	**2.1**	0.022	0.033	0.044	0.110	0.219
Mexico	**2.3**	0.026	0.039	0.052	0.130	0.260
Nicaragua	**5.8**	0.171	0.256	0.341	0.853	1.707
Panama	**4.4**	0.097	0.145	0.194	0.484	0.969
Paraguay	**2.5**	0.031	0.046	0.061	0.154	0.307
Peru	**3.8**	0.073	0.110	0.146	0.366	0.731
Dominican Republic	**4.2**	0.087	0.131	0.174	0.435	0.870
Uruguay	**4.5**	0.101	0.151	0.202	0.505	1.009
Venezuela	**3.3**	0.053	0.079	0.106	0.264	0.528
Other Developed Countries (simple average)	**1.4**	**0.011**	**0.016**	**0.021**	**0.053**	**0.106**
Australia	**0.8**	0.003	0.005	0.007	0.016	0.033
Austria	**1.0**	0.005	0.008	0.010	0.025	0.051
Belgium	**0.9**	0.004	0.006	0.008	0.019	0.038
Finland	**1.8**	0.016	0.024	0.032	0.080	0.161
France	**0.6**	0.002	0.003	0.004	0.010	0.020
Greece	**1.4**	0.010	0.015	0.020	0.050	0.099
Ireland	**1.9**	0.018	0.028	0.037	0.092	0.184
Italy	**1.2**	0.007	0.011	0.015	0.037	0.074
Japan	**1.0**	0.005	0.008	0.011	0.027	0.053
Netherlands	**1.3**	0.009	0.014	0.018	0.045	0.091
New Zealand	**1.7**	0.015	0.022	0.029	0.074	0.147
Norway	**1.4**	0.010	0.016	0.021	0.052	0.103
Portugal	**2.6**	0.034	0.051	0.068	0.170	0.340
United Kingdom	**1.4**	0.009	0.014	0.018	0.046	0.092

Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: To isolate the cyclical component of per capita consumption (in constant 2010 dollars), the HP filter is applied to the natural logarithm values with $\lambda = 6.25$.

On the other hand, instead of eliminating the macroeconomic volatility, it is possible to compute the welfare benefits of a one-percentage point increase in household consumption trend (g) in this framework by computing the following equivalent variation:

$$U\left[\left\{(1+\Delta)C(g,\sigma_\varepsilon^2)\right\}\right] = U\left[\left\{C(g',\sigma_\varepsilon^2)\right\}\right], \text{whith } g' > g, \tag{1.7}$$

where g and g' are the average variation of the trend component of per-capita consumption (in logs), with $g' = g(1.01)$. By using the expected life-time utility, the benefit of increasing the consumption trend is

$$\Delta(g, g', \beta, \gamma) = \begin{cases} \left[\frac{1-\beta(1+g)^{1-\gamma}}{1-\beta(1+g')^{1-\gamma}}\right]^{\frac{1}{1-\gamma}} - 1 & \text{if } \gamma \neq 1 \\ e^{\left(\frac{\beta}{1-\beta}\right) ln\left(\frac{1+g'}{1+g}\right)} - 1 & \text{if } \gamma = 1. \end{cases} \tag{1.8}$$

Table 1.2 reports the aggregate results for an increase in one-percentage point of the trend consumption growth rate (g'). Notice that the benefits rises as the discount factor (β) increases, but falls as the risk aversion (γ) increases; this is because γ negatively affects the effective discount factor applied to future consumption. The benefits are considerably higher in both developed and developing countries. The extra percent point of trend growth is equivalent to a significant increase of more than 20 percent in consumption per capita across all dates, reaching more than 37 percent for $\beta = 0.97$. As a result, the conclusion in Lucas (1987) seems to be applicable for all countries, that is, the policy makers should focus on increasing the trend growth rates, rather than reducing the macroeconomic fluctuations. In the Lucas representative agent model, the only way to get considerable welfare effects is by increasing the trend growth rate of per capita consumption instead of reducing the volatility. Nonetheless, two alternative specifications will be analyzed in the following sections.

1.2.2 Alternative (I): *A representative agent framework with a stochastic trend in the consumption process*

Lucas (1987) assumes that consumption is generated by a trend-stationary process as in Equation (1.3), however, Hall (1978) suggests that the consumption process contains a unit root under the permanent income hypothesis with rational expectations. Table 1.3 shows the results from different unit root tests (Augmented Dickey-Fuller,

Table 1.2: Benefit of One-Percent Increase of the Trend of per Capita Consumption
(Equivalent variation of per capita annual household consumption in percentage, 1961-2006)

		Discount Factor (β) =0.95					Discount Factor (β) =0.97				
	g	γ=1.0	γ=1.5	γ=2	γ=5	γ=10	γ=1.0	γ=1.5	γ=2	γ=5	γ=10
United States	2.4	20.3	15.5	12.5	5.6	2.8	36.9	23.8	17.5	6.6	3.0
Latin America	1.5	20.5	17.0	14.5	7.9	4.6	37.3	27.3	21.8	10.1	5.5
Other Developed Countries	2.5	20.3	15.3	12.3	5.5	2.7	36.9	23.4	17.2	6.4	3.0

Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: To isolate the cyclical component of per capita consumption (in constant 2010 dollars), the HP filter is applied to the natural logarithm values with $\lambda = 6.25$. Simple average by country group.

ADF and the Phillips-Parron, PP) with the null hypothesis that the per capita consumption path (in natural logarithms) has a stochastic trend. The null is not rejected in almost all countries in the sample. Even though the ADF test rejects the null for the US and Netherlands, the PP test does not. Reis (2009) conducts a variety of statistical tests to investigate whether the consumption in United States has a unit root (including the Elliot-Rothenberg-Stock and Ng-Perron tests), finding that null hypothesis is never rejected at significance level of 5 percent.

Since the martingale is the simplest specification that describes a process with a unit root, let the natural logarithms of per capita consumption (c_t) follow:

$$c_t = c_0 + \left(g - \frac{1}{2}\sigma_\varepsilon^2\right) t + \sum_{i=1}^{t} \varepsilon_i. \tag{1.9}$$

As before g is the trend annual growth rate, $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, and the initial value c_0 is $ln(C_0)$. A similar analysis is found in Obstfeld (1994), including the term $-\frac{1}{2}\sigma_\varepsilon^2$ to ensure that the increase in the variance of ε_t does not affect the mean. Although Lucas (1987) does not completely rule out the possibility of a unit root in the consumption process, he would prefer an intermediate case between (1.3) and (1.9).[3] Under (1.9), the level of per capita consumption function becomes

[3]See Lucas (1987), pp. 22-23, footnote 1

$$C_t = C_0 e^{\left(g-\frac{1}{2}\sigma_\varepsilon^2\right)t} e^{\sum_{i=1}^{t}\varepsilon_i}. \tag{1.10}$$

Table 1.3: Unit Root Tests on Consumption per capita, 1961-2006

	Augmented Dickey Fuller Test		Phillips-Perron Test	
	Test Statistics	Decision at 5% critical value (-3.513)	Test Statistics	Decision at 5% critical value (-3.511)
Latin America				
Argentina	**-3.31**	**NR**	**-2.83**	**NR**
Bolivia	**-1.97**	**NR**	**-2.07**	**NR**
Brazil	**-1.06**	**NR**	**-1.18**	**NR**
Chile	**-1.83**	**NR**	**-1.24**	**NR**
Colombia	**-1.78**	**NR**	**-1.78**	**NR**
Costa Rica	**-2.30**	**NR**	**-2.77**	**NR**
Ecuador	**-1.66**	**NR**	**-1.64**	**NR**
El Salvador	**-1.92**	**NR**	**-1.56**	**NR**
Guatemala	**-2.72**	**NR**	**-1.95**	**NR**
Honduras	**-1.95**	**NR**	**-1.95**	**NR**
Mexico	**-2.38**	**NR**	**-1.79**	**NR**
Nicaragua	**-1.44**	**NR**	**-1.70**	**NR**
Paraguay	**-0.74**	**NR**	**-1.13**	**NR**
Peru	**-3.02**	**NR**	**-2.45**	**NR**
Uruguay	**-2.75**	**NR**	**-2.39**	**NR**
Venezuela	**-2.06**	**NR**	**-1.50**	**NR**

	Augmented Dickey Fuller Test		Phillips-Perron Test	
	Test Statistics	Decision at 5% critical value (-3.513)	Test Statistics	Decision at 5% critical value (-3.511)
Developed Countries				
United States	**-3.66**	**R**	**-2.26**	**NR**
Australia	**-1.71**	**NR**	**-1.79**	**NR**
Austria	**-0.74**	**NR**	**-0.46**	**NR**
Belgium	**-0.48**	**NR**	**-0.72**	**NR**
Denmark	**-3.49**	**NR**	**-3.49**	**NR**
Finland	**-1.97**	**NR**	**-2.25**	**NR**
France	**-3.16**	**NR**	**-2.76**	**NR**
Greece	**-2.27**	**NR**	**-2.25**	**NR**
Ireland	**-1.97**	**NR**	**-1.42**	**NR**
Italy	**-2.24**	**NR**	**-1.48**	**NR**
Japan	**-2.89**	**NR**	**-2.67**	**NR**
Netherlands	**-3.83**	**R**	**-2.06**	**NR**
New Zealand	**-1.96**	**NR**	**-1.22**	**NR**
Norway	**-2.06**	**NR**	**-2.20**	**NR**
Portugal	**-3.18**	**NR**	**-1.79**	**NR**
Spain	**-2.68**	**NR**	**-3.39**	**NR**
Sweden	**-2.68**	**NR**	**-2.75**	**NR**
United Kingdom	**-2.97**	**NR**	**-2.22**	**NR**

Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: ADF test includes a constant and a time trend. "NR" stands for "Not-Rejected" and "R" for "Rejected".

Following Obstfeld (1994), the equivalent variation to keep the agent indifferent if consumption process changes from $(C_0, g, \sigma_\varepsilon^2)$ to $(C_0, g', \sigma_\varepsilon'^2)$ in (1.9) is defined by

$$U\left[\left\{C((1+\Delta)C_0, g, \sigma_\varepsilon^2)\right\}\right] = U\left[\left\{C(C_0, g', \sigma_\varepsilon'^2)\right\}\right]$$

If consumption follows a martingale process, then the equivalent variations (Δ) is

$$\Delta(g, g', \sigma_\varepsilon^2, \sigma_\varepsilon'^2, \beta, \gamma) = \begin{cases} \left[\dfrac{1-\beta e^{(1-\gamma)\left(g-\frac{\gamma\sigma_\varepsilon^2}{2}\right)}}{1-\beta e^{(1-\gamma)\left(g'-\frac{\gamma\sigma_\varepsilon'^2}{2}\right)}}\right]^{\left(\frac{1}{1-\gamma}\right)} - 1 & \text{if } \gamma \neq 1 \\ e^{\left[g'-g-\frac{(\sigma_\varepsilon'^2-\sigma_\varepsilon^2)}{2}\right]\frac{\beta}{1-\beta}} - 1 & \text{if } \gamma = 1. \end{cases} \tag{1.11}$$

To estimate the case when $g = g'$ and $\sigma_\varepsilon'^2 = 0$, that is, a change to an economy with

the same consumption growth but without volatility, (1.11) becomes

$$\Delta(\hat{g}, \hat{\sigma}_{\varepsilon}^2, \beta, \gamma) = \begin{cases} \left[\dfrac{1 - \beta e^{(1-\gamma)\left(\hat{g} - \frac{\gamma \hat{\sigma}_{\varepsilon}^2}{2}\right)}}{1 - \beta e^{(1-\gamma)\hat{g}}} \right]^{\left(\frac{1}{1-\gamma}\right)} - 1 & \text{if } \gamma \neq 1 \\ e^{\frac{\hat{\sigma}_{\varepsilon}^2}{2}\left(\frac{\beta}{1-\beta}\right)} - 1 & \text{if } \gamma = 1. \end{cases} \tag{1.12}$$

To calibrate this alternative model, I regress the first-difference of per capita consumption in logs (Δc_t) on a constant by ordinary least squareto obtain an estimate of standard errors ($\hat{\sigma}_{\varepsilon}$). After that, point estimates for $\hat{g}$ in specification (1.9) is the result of running the previous regression again, but with a constant and adding $\frac{1}{2}\hat{\sigma}_{\varepsilon}^2$ to the dependent variable Δc_t.

Table 1.4 reports the welfare cost of consumption fluctuations estimated by (1.12) for different values of risk aversion (γ) and discount factors (β). The first two columns show estimate of $\hat{g}$ and $\hat{\sigma}_{\varepsilon}$.[4] By allowing for a unit root in the consumption process, the average welfare loss in developed countries (for $\gamma = 1$ and $\beta = 0.95$) is more than sixty times the average cost estimated by the baseline (Lucas) model, and it is more than forty times for Latin American countries. Even more, for $\beta = 0.97$ the average welfare effects in both developed countries and Latin America might be more than one hundred times and sixty times the baseline calculations respectively. This result is explained by the larger standard deviations under (1.9) than those calculated after applying the HP filter in the Lucas model.

Again the Latin American region registers the highest welfare cost associated with the macroeconomic volatility. While the welfare cost in US is under 0.5 percent of per capita consumption for any value of γ and β, the average cost in Latin America ranges between 2.4 percent and 11.9 percent of per capita consumption, with countries like Argentina and Chile where the estimate welfare cost exceeds the 10 percent for intermediate values of γ and β (say 5 and 0.96 respectively). These larger welfare

[4]Countries are not reported when $\hat{g}$ is not statistically significant at the 10 percent level.

costs suggest that further countercyclical macro policies than those applied since 1960 are needed in most of Latin American countries. However, before drawing any conclusion an alternative model will be analyzed in the next section, which incorporates idiosyncratic risk and incomplete markets.

Table 1.4: Welfare Effects When the Consumption Process has a Unit Root
(Equivalent variation of per capita annual household consumption in percentage, 1961-2006)

	$\bar{g}$	$\bar{\sigma}_\varepsilon$	Discount Factor (β) =0.95					Discount Factor (β) =0.97				
			γ=1.0	γ=1.5	γ=2	γ=5	γ=10	γ=1.0	γ=1.5	γ=2	γ=5	γ=10
United States	**2.39** (0.00)	**1.6**	**0.23**	**0.28**	**0.31**	**0.38**	**0.40**	**0.39**	**0.42**	**0.43**	**0.45**	**0.44**
Latin America	**1.90**	**4.6**	**2.4**	**3.0**	**3.4**	**5.1**	**9.5**	**4.1**	**4.6**	**4.9**	**6.5**	**9.7**
Argentina	**1.68** (0.08)	**6.5**	4.1	5.3	6.3	10.9	33.0	7.0	8.4	9.5	14.4	-
Bolivia	**0.83** (0.07)	**3.0**	0.8	1.2	1.5	2.7	4.1	1.4	1.9	2.3	3.7	5.2
Brazil	**2.27** (0.01)	**5.2**	2.6	3.2	3.7	5.1	6.8	4.5	5.0	5.3	6.2	7.8
Chile	**2.29** (0.03)	**7.0**	4.8	5.9	6.8	10.4	22.9	8.3	9.2	9.9	13.1	34.0
Colombia	**1.75** (0.00)	**2.5**	0.6	0.8	0.9	1.3	1.5	1.0	1.2	1.3	1.6	1.7
Costa Rica	**1.77** (0.02)	**4.8**	2.2	2.8	3.3	5.0	7.1	3.7	4.4	4.8	6.3	8.4
Ecuador	**1.88** (0.00)	**2.9**	0.8	1.0	1.2	1.6	1.9	1.4	1.6	1.7	2.0	2.1
Guatemala	**1.31** (0.00)	**2.0**	0.4	0.5	0.6	0.9	1.1	0.6	0.8	0.9	1.1	1.3
Honduras	**1.26** (0.02)	**3.5**	1.2	1.6	1.9	3.2	4.4	2.0	2.5	2.9	4.1	5.3
Mexico	**2.00** (0.00)	**3.5**	1.2	1.5	1.7	2.4	2.8	2.0	2.3	2.5	2.9	3.2
Panama	**3.00** (0.01)	**7.1**	4.9	5.8	6.4	8.5	13.4	8.5	8.7	8.9	10.1	15.7
Dominican Republic	**3.13** (0.00)	**6.6**	4.2	4.9	5.3	6.8	9.2	7.3	7.3	7.3	8.0	10.4
Venezuela, RB	**1.53** (0.07)	**5.6**	3.0	4.0	4.7	8.0	15.3	5.1	6.3	7.1	10.5	21.1
Developed countries	**2.6**	**2.5**	**0.6**	**0.7**	**0.8**	**1.0**	**1.1**	**1.0**	**1.1**	**1.1**	**1.2**	**1.2**
Selected Countries:												
France	**2.48** (0.00)	**1.5**	0.23	0.27	0.30	0.37	0.38	0.38	0.41	0.42	0.43	0.42
Italy	**2.88** (0.00)	**2.4**	0.53	0.61	0.67	0.78	0.80	0.90	0.91	0.91	0.90	0.87
Japan	**3.52** (0.00)	**2.9**	0.80	0.89	0.94	1.02	1.01	1.37	1.30	1.26	1.16	1.09
United Kingdom	**2.35** (0.00)	**2.0**	0.37	0.45	0.51	0.64	0.68	0.64	0.69	0.71	0.76	0.75

Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: Simple average by country group.

1.2.3 Alternative (II): *A general equilibrium model with an uninsurable idiosyncratic labor risk*

The baseline model assumes a representative agent and complete markets where the macroeconomic volatility does not affect the economic growth. In this section, I deviate from the representative agent environment by studying a general equilibrium model following Krebs (2003) that incorporates an uninsurable idiosyncratic labor risk. Since the labor risk is uninsurable, there is no asset in the financial market with payoffs tied to the realizations of the idiosyncratic shocks. Although it is difficult to state from the data whether income shocks are permanent or just highly persistent, in this model the income shock is taken as permanent.[5]

The Economy. There is an aggregate productivity shock (S_t) that affects the returns of both physical and human capital investments. In addition, let s_t be a human capital shock that only affects the human capital accumulation. Both shocks are unpredictable, so they have an i.i.d. distribution over time. There are $i = 1, .., I$ ex-ante identical households, meaning that the idiosyncratic shocks are identically distributed across them. There is one non-perishable good that can be consumed or invested and only one firm that produces it. This firm produces Y_t units of the all-proposed good by employing physical (K_t) and human capital (H_t) with a standard neoclassical production function with constant-returns-to-scale. Formally, $Y_t = A(S_t)F(K_t, H_t)$, where A is a total factor productivity function $A\colon S \to \mathbb{R}^+$ that assigns a productivity level for each aggregate state.[6] The gross physical and human capital returns are denoted by $\hat{r}_k$ and $\hat{r}_h$ respectively. Hence, the firm faces the following static maximization problem:

$$\max_{K_t, H_t} \quad A(S_t)F(K_t, H_t) - \hat{r}_{kt}K_t - \hat{r}_{ht}H_t. \tag{1.13}$$

[5] Imrohoroğlu (1989) and Krusell et al. (2009) consider shocks with some degree of persistence.

[6] All standard neoclassical properties hold, in particular F is twice-continuously differentiable, strictly concave and $\lim_{K\to 0} F'_K = \lim_{H\to 0} F'_H = +\infty$ and $\lim_{K\to\infty} F'_K = \lim_{H\to\infty} F'_H = 0$

Let's define some useful variables: the capital-to-labor ratio $\tilde{k}_{it} = \frac{k_{it}}{hit}$, the total wealth $w_{it} = k_{it} + h_{it}$, and the net returns of physical and human capital as $r_{kt} = \hat{r}_{kt} - \delta_k$ and $r_{ht} = \hat{r}_{ht} - \delta_h$. For convenience, the average depreciation rates of physical and human capital (δ_k and δ_h respectively) are constant and independent of the aggregate risk.[7]

Households have identical time-additive preference over the consumption plan $\{c_{it}\}$. Consequently, each household i solves the following maximization problem:

$$\max_{\{c_{it}, w_{it+1}, \tilde{k}_{it+1}\}} \quad \sum_{t=0}^{\infty} E\left[\beta^t u(c_{it})\right], \tag{1.14a}$$

$$\text{s.t.} \quad w_{it+1} = w_{it}\left[1 + \frac{\tilde{k}_{it}}{1+\tilde{k}_{it}} r_{kt} + \frac{1}{1+\tilde{k}_{it}}\left(r_{ht} + \eta_{it}\right)\right] - c_{it}, \tag{1.14b}$$

$$c_{it} \geq 0, w_{it+1} \geq 0, k_{it+1} \geq 0, \forall t, \text{ where } w_{i0} \text{ and } \tilde{k}_{i0} \text{ are given.} \tag{1.14c}$$

The term η_{it} is the household-specific shock that only affects the stock of human capital with $E(\eta_{it}|S_t) = 0$. The function $\eta : s \times S \to \mathbb{R}$ assigns to each pair (s, S) a realization η. The idiosyncratic shock is taken as another source of human capital depreciation, and is related to the specific-skill losses or skills no longer used in the event of a job displacement. Consequently, there is some part of the accumulated human capital that is either destroyed or made obsolete when an agent loses his job, in that case $\eta_{it} < 0$. Of course, this cost is more severe in a context of an economic downturn, where the aggregate risk affects the variance of the idiosyncratic shock. This model focuses on the permanent wage loss instead of taking into account the missed wage during the unemployment period.

The Equilibrium.The recursive equilibrium can be found by solving a decision problem of a household who lives in autarky and faces the same problem every period with different sets of information. Focusing on the stationary recursive equilibria with a constant physical-to-human capital ratio, $\tilde{K}_t = \tilde{K} = K_t/H_t$, the equilibrium will be

[7] Krebs (2003) assumes that the depreciation rate of physical and human capital are equal and depend on the aggregate shocks, i.e. $\delta_{kt} = \delta_{ht} = \delta(S_t)$.

the sequences of prices and actions,$\{r_{kt}, r_{ht}\}$, $\{k_t, h_t\}$ and $\left\{c_{it}, w_{it+1}, \tilde{k}_{it+1}\right\}$, where:

(i) $\{k_t, h_t\}$ solves the firm maximization problem (1.13).

(ii) $\left\{c_{it}, w_{it+1}, \tilde{k}_{it+1}\right\}$ maximizes (1.14a) subject to the sequential budget constraint (1.14b).

(iii) Markets clear.

The FOCs associated with the firm's maximization problem gives the return of physical and human capital

$$r_k(\tilde{K}, S_t) = A(S_t)F'_K - \delta_k, \tag{1.15a}$$

$$r_h(\tilde{K}, S_t) = A(S_t)F(\tilde{K}, 1) - \tilde{K}F'_K - \delta_h. \tag{1.15b}$$

However, the market clearing condition implies that the aggregate capital-labor ratio must equal to the firm's capital-to-labor ratio

$$\frac{\sum_i^I \frac{\tilde{k}_{it}}{1+\tilde{k}_{it}} w_{it}}{\sum_i^I \frac{1}{1+\tilde{k}_{it}} w_{it}} = \frac{\sum_i^I k_{it}}{\sum_i^I h_{it}} = \frac{K_t}{H_t} = \tilde{K}. \tag{1.16}$$

If the households's capital-labor ratios are symmetric, $\tilde{k}_{it} = \tilde{k}$, then (1.16) is satisfied if only if $\tilde{K} = \tilde{k}$. Consequently, the returns defined in (1.15a) and (1.15b) become $r_k(\tilde{k}, S_t)$ and $r_h(\tilde{k}, S_t)$ respectively. Assuming a constant degree of relative risk aversion, the one-period utility function is $u(c_{it}) = c_{it}^{1-\gamma}/(1-\gamma)$ if $\gamma \neq 1$, or $u(c_{it}) = ln(c_{it})$ if $\gamma = 1$. Thus, the two Euler equations associated with the maximization problem (1.14) for a constant and symmetric capital-labor ratio are as follows

$$c_{it}^{-\gamma} = \beta E_t \left\{ c_{it+1}^{-\gamma} \left[1 + \frac{\tilde{k}}{1+\tilde{k}} r_k(\tilde{k}, S_{t+1}) + \frac{1}{1+\tilde{k}} \left(r_h(\tilde{k}, S_{t+1}) + \eta(s_{it+1}, S_{t+1}) \right) \right] \right\}, \tag{1.17}$$

and

$$E_t\left\{c_{it+1}^{-\gamma}\left[r_h(\tilde{k},S_{t+1})+\eta(s_{it+1},S_{t+1})-r_k(\tilde{k},S_{t+1})\right]\right\}=0. \tag{1.18}$$

The first Euler condition basically states that the marginal utility loss (utility cost) of investing (saving) one more unit of good must be equal to the expected utility gain of doing so. The second Euler equation states the equality of expected returns on the two investment opportunities (marginal utility weighted). As noted by Krebs (2003), the solution exits if

$$\sup_{\tilde{k}}\beta E\left[\left(1+\frac{\tilde{k}}{1+\tilde{k}}r_k+\frac{1}{1+\tilde{k}}(r_h+\eta_i)\right)^{1-\gamma}\right]<1. \tag{1.19}$$

Direct calculation shows that the plan $c_{it}=a(1+r_{it})w_{it}$ solves the Euler equations together with the budget constraint. Defining $w_{it+1}=(1-a)(1+r_{it})w_{it}$, the Euler conditions (1.17) and (1.18) become

$$a=1-\left[\beta E_t\left[(1+r_{t+1})^{1-\gamma}\right]\right]^{1/\gamma}, \text{if } \gamma\neq 1, \text{ or } a=1-\beta, \text{ if } \gamma=1, \tag{1.20a}$$

$$E_t\left[\frac{r_h(\tilde{k},S_{t+1})+\eta(s_{it+1},S_{t+1})-r_k(\tilde{k},S_{t+1})}{(1+r_{t+1})^{\gamma}}\right]=0. \tag{1.20b}$$

Condition (1.19) ensures that $0<a<1$. Additionally, (1.20a) and (1.20b) define not only the equilibrium values of a and $\tilde{k}$, but also that transversality condition holds, $\lim_{t\to\infty}\beta^t E_t\left[c_{it}^{-\gamma}w_{it+1}\right]\to 0$, so in this case the Euler equations are necessary and sufficient. Therefore, the allocations of the simple recursive equilibrium are given by

$$\begin{aligned}\tilde{k}_{it}&=\tilde{k};\\ c_{it}&=a\left[1+r(\tilde{k},s_{it},S_t)\right]w_{it};\\ w_{it+1}&=(1-a)\left[1+r(\tilde{k},s_{it},S_t)\right]w_{it};\end{aligned}\qquad\begin{aligned}k_{it}&=\frac{\tilde{k}}{1+\tilde{k}}w_{it};\\ h_{it}&=\frac{1}{1+\tilde{k}}w_{it};\end{aligned} \tag{1.21}$$

where $r(\tilde{k}, s_{it}, S_t) = \frac{\tilde{k}}{1+\tilde{k}} r_{kt} + \frac{1}{1+\tilde{k}}(r_{ht} + \eta_{it})$ is the total return on investments and a is the consumption-to-wealth ratio. Finally, the aggregate returns are

$$r_{kt} = r_k(\tilde{k}, S_t) \quad \text{and} \quad r_{ht} = r_h(\tilde{k}, S_t). \tag{1.22}$$

Model Calibration. Following Krebs (2003), a Cobb-Douglas production function with constant returns-to-scale is considered, $f(\tilde{k}_t) = A_t \tilde{k}_t^{\alpha}$, as well as logarithmic preferences ($\gamma = 1$). Thus, (1.20a) and (1.20b) read

$$a = 1 - \beta, \tag{1.23a}$$

$$E_t \left[\frac{r_h(\tilde{k}, S_{t+1}) + \eta(s_{it+1}, S_{t+1}) - r_k(\tilde{k}, S_{t+1})}{(1 + r_{t+1})} \right] = 0. \tag{1.23b}$$

The aggregate per capita consumption growth rate in this economy is obtained by the individual (gross) consumption growth in expression (1.21) and the law of large numbers

$$\mu(S) = \frac{C_{t+1}}{C_t} = E\left[\frac{c_{it+1}}{c_{it}} | S_{t+1} \right] = (1 - a)\left[1 + E\left[r(\tilde{k}, s_{it+1}, S_{t+1}) \right] \right]. \tag{1.24}$$

Then, $\mu(S)$ is an i.i.d. process.[8] Considering the value of a and the assumption of a Cobb-Douglas production function, the per-capita consumption growth (1.24) become

$$\mu(S) = \beta \left[1 + \frac{\tilde{k}}{1+\tilde{k}} A_t \alpha \tilde{k}^{\alpha - 1} + \frac{1}{1+\tilde{k}} A_t (1 - \alpha) \tilde{k}^{\alpha} - \tilde{\delta} \right], \tag{1.25}$$

where $\tilde{\delta}$ is the total depreciation rate of both physical and human capital. The quantitative analysis is based on annual data (1960-2006) for developed and Latin American countries, with parameter values extensively used in RBC models. The share of capi-

[8] See Campbell and Cochrane (1999)

tal in total income (α) is equal to 0.36.[9] Furthermore, the common discount factor (β) equals 0.96 to ensure that all households have the same intertemporal preferences.[10] As in Krebs (2003) and Jones et al. (1999), the total depreciation rate $\tilde{\delta}$ is set in 0.06. If a common choice for the depreciation rate of physical capital is $\delta_k = 0.05$ (see Cooley and Prescott (1995)), then the depreciation rate of human capital would be $\delta_h = 0.01$.

I assume two-state shocks in the aggregate economy: the low level (L) and a high level (H) of the economic cycle. Thus, S has two possible realizations $\{L, H\}$ with equal probabilities $\pi(L) = \pi(H) = 0.5$. In addition, the human capital shock (s_{it}) follows a standard normal distribution, which is independent of the aggregate shock S. The household-loss of human capital is defined as follows

$$\eta(s_i, S) = \sigma_\eta(S) s_i, \tag{1.26}$$

therefore $\eta_{it} \sim \mathcal{N}(0, \sigma_\eta^2(S_t))$. The *integration principle*, which was first proposed by Krusell and Smith (1999), is applied to eliminate the aggregate risk (business cycle).[11] Thus, the function (1.26) is replaced by

$$\eta^*(s_i) = 0.5 \left[\sigma_\eta(L) + \sigma_\eta(H)\right] s_i. \tag{1.27}$$

The idiosyncratic human capital risk is transformed from the heteroscedastic to homoscedastic process: $\eta_{it} \sim \mathcal{N}(0, \sigma_\eta^2(S_t))$ to $\eta_{it}^* \sim \mathcal{N}\left(0, E\left[\sigma_\eta^2(S_t)\right]\right)$, where $E\left[\sigma_\eta^2(S_t)\right] = 0.5\left[\sigma_\eta^2(L) + \sigma_\eta^2(H)\right]$.

The expected value of per-capita consumption growth for each economy matches

[9] Although this value is commonly used in the calibration of RBC models, it might not match the real income share in Latin American countries.

[10] This β value is suggested by Cooley and Prescott (1995). Even though it is possible to derivate β from (1.25), this would stand for different preferences in each country with unrealistic values for some countries.

[11] Also see Krusell et al. (2009)

the per-capita household consumption data in each country for the period 1960-2006, obtaining the total factor productivity for the two states: $A(L)$ and $A(H)$. In order to take into account the consumption volatility, the growth rates for the two possible states depend on the standard deviation of the per capita consumption growth rate in each country:

$$E\left[g(S_t)\right] = 0.5\left[\mu(L) + \mu(H)\right], \tag{1.28}$$
$$\text{with } \mu(L) = \mu - \sigma_\mu, \text{ and } \mu(L) = \mu - \sigma_\mu.$$

Next, I calculate the equivalent (or adjusted) average growth rate ($\hat{\mu}$) of the per capita consumption, which is defined as the certain growth rate for which expected life-time utility is equal to that associated with the uncertain growth rate $g(S) \sim \mathcal{N}(\mu(S), \sigma_g(S))$ with $\sigma_g(S) = \beta\sigma_y(S)$ and keeping the initial consumption constant:

$$\begin{aligned} E\left[\sum_{t=0}^{\infty} \beta^t log(c_{it})\right] &= \frac{1}{1-\beta} log(c_{i0}) + \frac{\beta}{(1-\beta)^2} E\left[log(1+g(S))\right] \\ &= \frac{1}{1-\beta} log(c_{i0}) + \frac{\beta}{(1-\beta)^2} log(1+\hat{\mu}) \\ &= E\left[\sum_{t=0}^{\infty} \beta^t log(c^*_{it})\right], \end{aligned} \tag{1.29}$$

where $c^*_{it} = c_{i0}(1+\hat{\mu}(S))^t$. Solving for $\hat{\mu}(S)$ yields

$$\hat{\mu}(S) = e^{E[log(1+g(S))]} - 1. \tag{1.30}$$

The risk-adjusted consumption growth rate is independent of the initial level (c_{i0}) and is also equal for all households and the aggregate economy. The expression $E[log(1+g(S)]$ is calculated through the Gauss-Hermite quadrature rule for expectations of

functions of a normal random variable with two nodes.[12] The per capita consumption growth rate changes to a homoscedastic process, i.e. $g' \sim \mathcal{N}(E[\mu(S)], E[\sigma_g(S)])$, when the aggregate risk is eliminated. Then, the change in the risk-adjusted growth rate can be estimated by

$$\Delta\hat{\mu} = \hat{\mu}' - E[\hat{\mu}(S)] = e^{E[log(1+g')]} - 0.5\left[\hat{\mu}(L) + \hat{\mu}(H)\right]. \tag{1.31}$$

This expression approximates the welfare change expressed in risk-adjusted growth rates. However, to compare this calibration procedure with the baseline and first-alternative models, I need to get the equivalent change in consumption levels that has the same positive effect of eliminating the business cycle. By direct calculation

$$\begin{aligned}
&log(c_{i0}(1+\Delta c)) + \frac{\beta}{1-\beta}E\left[log(1+g(S))\right] = log(c_{i0}) + \frac{\beta}{1-\beta}E\left[log(1+g')\right], \\
&\Delta c = e^{\frac{\beta}{1-\beta}\{E[log(1+g')] - E[log(1+g(S))]\}} - 1, \\
&\Delta c \approx \frac{\beta}{1-\beta}\Delta\hat{\mu}.
\end{aligned} \tag{1.32}$$

Given the expression of individual labor income ($y_{iht} = (r_h + \delta_h)h_{it}$) and (1.21), the law of motion of human capital is

$$h_{it+1} = (1-\theta)w_{it+1} = \beta\left[1 + \theta r_k + (1-\theta)(r_h + \eta_{it})\right]h_{it}, \tag{1.33}$$

where $\theta = \tilde{k}/(1+\tilde{k})$ is the total wealth invested in physical capital. To follow the empirical literature, the first difference of the labor income in logs approximately

[12] Algebraically: $E[log(1+g(S)] \approx \pi^{-1/2}\sum_{i=1}^{2}\omega_i log\left[1 + \left(\sqrt{2}\sigma_g(S)x_i + \mu(S)\right)\right]$, where x_i are the quadrature-nodes with weights ω_i. See Judd (1998) for nodes values and weights.

follows a random walk with drift:[13]

$$\begin{aligned} log(y_{it+1}) - log(y_{it}) &= log\left[(r_h + \delta_h)h_{it+1}\right] - log\left[(r_h + \delta_h)h_{it}\right] \\ &= log(h_{it+1}) - log(h_{it}) \\ &= log(\beta) + log\left[1 + \theta r_k + (1-\theta)(r_h + \eta_{it})\right] \quad \text{by(1.33)} \\ &\approx \underbrace{log(\beta) + \theta r_k + (1-\theta)r_h}_{\textbf{Drift } (Z)} + \underbrace{(1-\theta)\eta_{it}}_{\tilde{\eta}_{it}}, \text{ since } log(1+r) \approx r, \end{aligned} \tag{1.34}$$

with an error term $\tilde{\eta}_{it} \sim \mathcal{N}\left(0, \sigma_y^2(S_t)\right)$, where the income variance will depend on aggregate risk (i.e. the business cycle)

$$\sigma_y^2(S_t) = (1-\theta)^2\sigma_\eta^2(S_t) = \frac{\sigma_\eta^2(S_t)}{(1+\tilde{k})^2}. \tag{1.35}$$

Perhaps the most difficult step in the calibration process is to find a suitable approximation of the standard deviation defined in (1.35), because there is not enough empirical literature that estimates the income labor with an idiosyncratic risk component for Latin American countries. As a benchmark, I take an average of values reported by Krebs (2003), based on different empirical studies that estimate the standard deviation of the error term for a random walk specification of the labor income (σ_y) in the US economy:[14]

$$\sigma\left[{}^{y_{iht+1}}/_{y_{iht}} | S_t = L\right] = 0.26 \quad \text{and} \quad \sigma\left[{}^{y_{iht+1}}/_{y_{iht}} | S_t = H\right] = 0.10. \tag{1.36}$$

Although the estimated labor income volatility based on the US data (mainly PSID data) might be generalized for some developed countries, it can underestimate the

[13] See Krebs (2007) and Storesletten et al. (2001)

[14] For some empirical studies see Carroll and Samwick (1997), Meghir and Pistaferri (2004), and Storesletten et al. (2007).

actual labor income volatility in Latin American countries, since these economies have higher per capita income volatility (as discussed in the Section 1.1) and an extended informal labor market.[15] Therefore, it would be reasonable to assume that the higher macroeconomic volatility and more informal workers in these economies might translate into higher labor-income volatility. As a result, if the labor income volatility based on the US data (1.36) is used to estimate the welfare effects for developing countries (in this case for Latin America), the results should be taken as a lower-bound estimate.

Table 1.5 shows the expected value of the per capita consumption growth rate ($E[\mu(S)]$), the change in the risk-adjusted growth rate ($\Delta\hat{\mu}$) from eliminating the real fluctuations, and the welfare effects in terms of equivalent changes in consumption level (Δc). Even though the US labor income volatility is used across the countries, the welfare costs associated with the business cycle are considerably higher than those estimated in the previous sections. Indeed, an average welfare cost of 8.0 percent of per capita consumption in developed economies is estimated, while the average welfare cost in Latin America is more than 9 percent. As was mentioned before, the latest value for Latin American countries should be taken as a lower-bound estimate since the labor income volatility in most developing countries might be higher than in US.

[15] On average the informal economy in Latin America represents 42 percent of GNP but 18 percent in the high-income OECD countries (Schneider (2002)).

Table 1.5: Welfare Effects of Eliminating the Business Cycle with Idiosyncratic Risk

(Equivalent variation of per capita annual household consumption in percentage, 1961-2006)

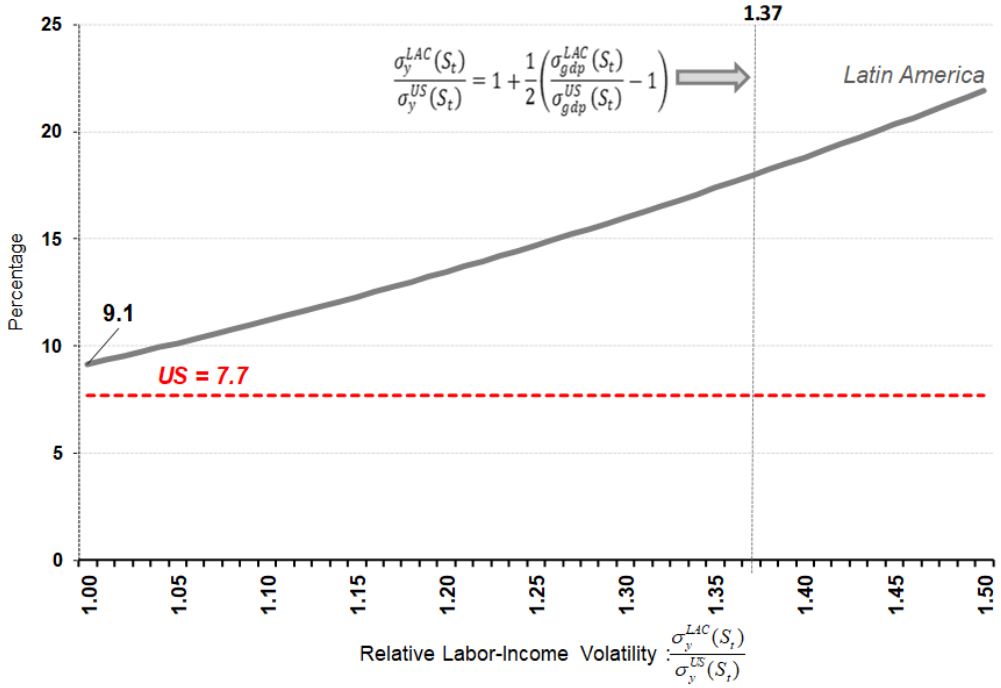
		(E1) With Benchmark values: σ_y (L)=0.26 and σ_y (H)=0.10				(E1) With Benchmark values: σ_y (L)=0.26 and σ_y (H)=0.10	
Country	**E[u(S)]**	**$\Delta\hat{u}$**	**Welfare Effects Δc**	**Country**	**E[u(S)]**	**$\Delta\hat{u}$**	**Welfare Effects Δc**
United States	2.4	0.32	**7.7**	Argentina	1.5	0.40	**9.6**
Australia	2.1	0.32	**7.7**	Bolivia	0.8	0.35	**8.4**
Austria	2.5	0.33	**7.9**	Brazil	2.1	0.38	**9.1**
Belgium	2.3	0.33	**7.8**	Chile	2.0	0.41	**9.7**
Denmark	2.0	0.34	**8.2**	Colombia	1.7	0.34	**8.1**
Finland	2.8	0.34	**8.2**	Costa Rica	1.7	0.37	**8.9**
France	2.5	0.32	**7.7**	Dominican Republic	2.9	0.39	**9.5**
Greece	3.3	0.33	**8.0**	Ecuador	1.8	0.34	**8.2**
Ireland	2.8	0.34	**8.2**	El Salvador	1.1	0.39	**9.5**
Italy	2.9	0.33	**8.0**	Guatemala	1.3	0.33	**8.0**
Japan	3.5	0.34	**8.1**	Honduras	1.2	0.36	**8.5**
Netherlands	2.2	0.34	**8.1**	Mexico	1.9	0.35	**8.5**
New Zealand	1.3	0.34	**8.2**	Nicaragua	(0.1)	0.49	**11.7**
Norway	2.6	0.33	**7.9**	Panama	2.7	0.40	**9.7**
Portugal	3.2	0.37	**8.8**	Paraguay	1.1	0.37	**9.0**
Spain	3.1	0.34	**8.1**	Peru	0.8	0.39	**9.4**
Sweden	1.6	0.33	**8.0**	Uruguay	1.2	0.40	**9.7**
United Kingdom	2.3	0.33	**7.9**	Venezuela, RB	1.4	0.34	**8.3**
Developed Countries	**2.5**	**0.33**	**8.0**	**Latin America**	**1.5**	**0.38**	**9.1**

Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: Basic Assumptions: $\alpha = 0.36$,$\gamma = 1$ (log-utility),$\beta = 0.96$, and total depreciation rate is 0.06. The welfare effects (Δc) is calculated with (1.32) and $\hat{\mu}$ with (1.31).Simple average by country group.

Even though there are not reliable estimates of labor income volatility in Latin American countries relative to the US values ($\sigma_y^{LAC}(S)/\sigma_y^{US}(S)$), a sensitivity exercise captures the equivalent variation in per capita consumption for different values of the labor income volatility in Latin America relative to the US benchmark (Figure 1.5). As an extreme case, the cost of real fluctuation could represent more than 20 percent of per capita consumption in Latin America, if its labor income volatility were 50 percent higher than in US. Let's, just for a comparison purpose, assume that the ratio of the labor-income volatility between Latin America and US is half of the difference in their relative per-capita GDP volatility. This would result in an average

welfare cost of almost 18 percent in Latin America, if the labor-income volatility in this region were about 37 percent higher than the one in US. Although, there does not exit an aggregate estimate of the relative magnitude, the welfare cost is increasing in the labor-income volatility, crossing the 10 percent of per-capita annual consumption as soon as labor-income volatility is just 5 percent higher than the one in US.

Figure 1.5: Sensitivity Analysis Assuming that the Ratio Between the Labor Income Volatility in Latin American Region and US Ranges from 1 to a Factor of 1.5 *(Equivalent variation of per capita annual household consumption in percentage, 1961-2006)*



Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: Basic Assumptions: $\alpha = 0.36$,$\gamma = 1$ (log-utility),$\beta = 0.96$, and total depreciation rate is 0.06. The welfare effects (Δc) is calculated with (1.32) and $\hat{\mu}$ with (1.31).Simple average by country group. "LAC" stands for "Latin America".

1.3 Conclusions

Table 1.6 compares the equivalent variation in consumption level that has the same positive effects of eliminating the real volatility for the three models analyzed (based on the same parameter values). The baseline (Lucas) specification shows the lowest results for the welfare cost by country, while the greatest change in consumption is

obtained in the general equilibrium model. The main reason behind this finding is that the elimination of business cycle in the second alternative model is growth-enhancing by increasing the adjusted consumption growth rate ($\hat{\mu}$).

Table 1.6: Comparative Results on the Welfare Effects of Eliminating the Macroeconomic Volatility
(Equivalent variation of per capita annual household consumption in percentage, 1961-2006)

	Welfare Effects Δc				*Order of magnitud higher w.r.t. the baseline model*		
	Baseline (Lucas) Model	*Alternative (I): Stochastic trend in the consumption process*	*Alternative (II): GE model with an uninsurable idiosyncratic labor risk*		*Alternative I*	*Alternative (II)*	
			$\sigma_y^{US} = \sigma_y^{DC} = \sigma_y^{LAC}$	$\frac{\sigma_y^{LAC}}{\sigma_y^{US}} = 1.37$		*Same labor income volatility*	$\frac{\sigma_y^{LAC}}{\sigma_y^{US}} = 1.37$
United States	**0.006**	**0.3**	**7.7**	**n/a**	**1.7**	**3.1**	**n/a**
Latin America	***0.059***	***3.0***	***9.1***	***17.9***	**1.7**	**2.2**	**2.5**
Argentina	**0.094**	**5.1**	**9.6**	**18.8**	**1.7**	**2.0**	**2.3**
Bolivia	**0.015**	**1.1**	**8.4**	**16.4**	**1.9**	**2.8**	**3.0**
Brazil	**0.040**	**3.3**	**9.1**	**17.7**	**1.9**	**2.4**	**2.6**
Chile	**0.114**	**6.1**	**9.7**	**19.1**	**1.7**	**1.9**	**2.2**
Colombia	**0.010**	**0.8**	**8.1**	**15.9**	**1.9**	**2.9**	**3.2**
Costa Rica	**0.047**	**2.8**	**8.9**	**17.5**	**1.8**	**2.3**	**2.6**
Dominican Republic	**0.087**	**5.3**	**9.5**	**18.6**	**1.8**	**2.0**	**2.3**
Ecuador	**0.011**	**1.0**	**8.2**	**16.1**	**2.0**	**2.9**	**3.2**
El Salvador	**0.061**	**-**	**9.5**	**18.5**	**-**	**2.2**	**2.5**
Guatemala	**0.005**	**0.5**	**8.0**	**15.6**	**2.0**	**3.2**	**3.5**
Honduras	**0.022**	**1.5**	**8.5**	**16.7**	**1.8**	**2.6**	**2.9**
Mexico	**0.026**	**1.5**	**8.5**	**16.6**	**1.8**	**2.5**	**2.8**
Nicaragua	**0.171**	**-**	**11.7**	**23.0**	**-**	**1.8**	**2.1**
Panama	**0.097**	**6.3**	**9.7**	**19.0**	**1.8**	**2.0**	**2.3**
Paraguay	**0.031**	**-**	**9.0**	**17.6**	**-**	**2.5**	**2.8**
Peru	**0.073**	**-**	**9.4**	**18.3**	**-**	**2.1**	**2.4**
Uruguay	**0.101**	**-**	**9.7**	**18.9**	**-**	**2.0**	**2.3**
Venezuela, RB	**0.053**	**3.8**	**8.3**	**18.2**	**1.9**	**2.2**	**2.5**
Other DC	***0.010***	***0.8***	***8.0***	***n/a***	**1.9**	**2.9**	***n/a***

Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: Basic Assumptions: $\alpha = 0.36$,$\gamma = 1$ (log-utility),$\beta = 0.96$, and total depreciation rate is 0.06. Simple average by country group. In Alternative (I): Countries where the estimate of $\hat{g}$ is not statistically significant at the 10 percent level are not listed. "DC" stands for "developed countries" and "LAC" for "Latin America".

Under the Lucas framework, real volatility is not relevant in terms of welfare for all developed countries and Latin American economies. However, the welfare effects in both Latin America and developed countries are on average more than two-orders of magnitude higher those estimated with the baseline model, even without taking into account the most recent international financial crisis. In sum, the magnitude of the welfare cost of macroeconomic volatility is framework-dependent, leaving unanswered

the question of whether more countercyclical policies would be needed to reduce the macroeconomic fluctuations in most developing countries.

Chapter 2

The Effect of Macroeconomic Volatility on Financial Deepening: A Missing Link?

Most emerging economies have exhibited higher and more persistent macroeconomic volatility than developed countries, a stylized fact that has been well documented by the literature.[1] The debate has concentrated on the sources of such volatility. An important strand of the theoretical and empirical literature argues that the shallow financial system in less developed countries is one of the main factors explaining their output volatility.[2] The theoretical arguments rely on two transmission mechanisms. First, instead of absorbing the shocks, credit market imperfections can amplify a small exogenous shock.[3] In addition, a lack of opportunities for financial diversification means risk-averse agents avoid investing in riskier projects with higher productivity (returns), and instead invest in projects with low risk.[4] An alternative view is that the macroeconomic volatility observed in many emerging countries originates in their production patterns, which are highly concentrated in more volatile industries with non-flexible and less diversified technologies.[5] The question arises whether these production patterns and the related volatility hinder the development of the financial sector, possibly explaining the shallow financial system.

In this chapter, I follow the second strand of literature where the macroeconomic volatility is an *intrinsic phenomenon* originating in the real sector with independence

[1] Among other results, Aguiar and Gopinath (2007) find the income level and growth in emerging markets are twice as volatile as those in developed economies for the period 1980-2003.

[2] The empirical studies have found mixed evidence. Among others, Braun and Larrain (2005) and Raddatz (2006), show that the financial development can reduce output volatility, while Wang et al. (2018) find that its effectiveness is decreasing. However, Acemoglu et al. (2003) argue that institutions (instead of financial development) can explain not only volatility but also growth and crisis. Beck et al. (2006) do not find a robust relationship between financial intermediary development and growth volatility, when they control with terms of trade shocks and inflation.

[3] See Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Aghion et al. (2010).

[4] See Greenwood and Jovanovic (1990), Acemoglu and Zilibotti (1997), and Aghion et al. (1999)

[5] See Koren and Tenreyro (2007),Kraay and Ventura (2007), and Koren and Tenreyro (2013).

of the initial level of financial development (deepening).[6] Therefore, if the financial deepening is not the primary source of excess volatility, the question remains of how the high and persistent macroeconomic fluctuations might affect the development of a financial system. Although some intuitive answers could be given with a partial equilibrium framework, the question requires a general equilibrium approach to explain the causal relationship between the macroeconomic volatility and the financial system. To this aim, I propose a general equilibrium framework to study the channels through which the persistent aggregate volatility can affect the financial deepening, proxied by the private credit to GDP ratio.[7]

The model is an extension of Labadie (1998) with an optimal contract scheme like the one in Bernanke, Gertler, and Gilchrist (1999). There are two types of firms, the producers of consumption goods and the producers of capital goods (investors). Consumption producers are directly exposed to total factor productivity (TFP) shocks, while capital producers are subject to an uninsurable idiosyncratic risk whose distribution is affected by the aggregate volatility. Throughout this chapter, the standard deviation of TFP shocks is referred as *aggregate risk*, since it cannot be diversified away. In addition, the financial intermediary diversifies its portfolio with two possible investment options: lending to the capital producer or buying a risk-free government bond. The realization of the idiosyncratic shock is private information known only by capital producers, but observable to the financial intermediary if he pays a monitoring cost, i.e. a costly state verification (CSV) approach.

Although the interdependence between macroeconomic fluctuations and producer-specific shocks has been studied to explain investment decisions and leverage, the

[6]Koren and Tenreyro (2013) show that the decline in aggregate volatility is independent of the country's financial development.

[7]The level of financial development (or deepening) is generally proxied by the ratio of private credit to GDP, while most of the theoretical models narrow even further the concept, focusing on investment in fixed capital as the main purpose of borrowing. Having a single indicator to represent the complexity of the financial development can generate misleading measures of the actual financial depth and access (Čihák et al., 2012; Sahay et al., 2015).

novelty of my approach is to study how this interdependence affects not only the investment decisions, but also the output volatility and financial deepening. Another important feature of the model is that the aggregate risk (TFP volatility) does not have a one-to-one effect on the output volatility (variance of the aggregate output), since it is partially offset by the response of investment to changes in the aggregate uncertainty. This finding, however, depends on the elasticity of the risk premium with respect to the aggregate risk.

The main results of the chapter can be summarized as follows: (i) A reduction (increase) in the aggregate volatility increases (reduces) the optimal level of private loans and investment by lowering (increasing) the risk premium of lending to the capital producers; (ii) A corollary is that the amount of domestic loanable funds available to the government is positively correlated with the aggregate volatility, that is, the government can sell more bonds to the domestic financial system as real volatility increases; (iii) The reduction in the output volatility is lower than the decrease in aggregate risk, given the impact of the latter on investment, depending on the risk-premium elasticity with respect to aggregate uncertainty; (iv) Finally, as the aggregate volatility decreases the financial depth indicator (FD) may increase. While most empirical and theoretical works discuss the causal effects of FD on the output volatility, this is one of the first studies that addresses the channels to uncover the reverse causality through a simplified general equilibrium framework. In sum, if a country faces intrinsic volatility from the real sector, then the underdeveloped financial system can be a by-product of that volatility.

In countries with this type of volatility, the challenges from the policy makers' perspective would be how to reduce the aggregate fluctuations. To attack the source of this problem, moving to technologies with greater resilience to shocks and diversifying production would be a reasonable solution in the long-run. However, a change in the production patterns with technological diversification requires time and re-

sources, particularly in countries where the financial system is endogenously shallow. Additionally, economic agents must deal with the intrinsic volatility in the short and medium run. Therefore, in the meantime, policy makers should focus in reducing the aggregate volatility, for example by applying more countercyclical macroeconomic policies, to lower the risk of investment projects and increase the amount of credits to the private sector.[8]

The chapter is organized as follows. Section 2 provides the main features of the benchmark economy without aggregate volatility, but with idiosyncratic risk. In the following sections, the aggregate uncertainty is introduced in the form of TFP shocks to study how different levels of aggregate risk can affect the macroeconomic fluctuations and the financial depth indicator. Finally, some conclusions and possible extensions are summarized.

2.1 The Benchmark Model: Deterministic Case

The economy has two goods, a multi-purpose consumption good and a capital good. There are five types of agents: two-period lived households, producers of consumption goods, two-period lived producers of capital goods, and infinitely lived financial intermediaries and government.

In each period, there are N_t young households and N_{t-1} old households. For simplicity, I assume no population growth,i.e. $N_{t-1} = N_t = N$. Young households are endowed with L units of labor, *normalized to unity*, that is inelastically supply to the consumption-good producers at a wage rate. With the labor income, young agents consume, pay a lump-sum tax, and save in the form of deposits with the financial intermediary. Old agents only consume by using their savings when young. Producers of consumption goods use both capital goods and labor, renting capital from the capital-producers, who have access to the investment technology. In order

[8]See Calderón, Chuhan-Pole, and López-Monti (2017) for an assessment of the cyclical properties of fiscal policies from the expenditure side in Sub-Saharan Africa and other developing regions

to invest, they borrow from the financial intermediary, who decides the composition of its portfolio between lending to the capital producer and buying a risk-free government bond. In this section, there is only an idiosyncratic shock that affects the return of capital-producers investment. The realization of this specific shock is only known by each producer in the second period of its life. However, this information is observable to the financial intermediaries at a monitoring cost, in other words, a costly state verification (CSV) assumption.[9] The government simply spends all the tax revenue and the funds offered by the financial intermediaries net of debt repayment. Next, the benchmark model is explained in detail.

2.1.1 Consumption-Good Producers

There is a unit mass of producers of consumption goods, each of whom rents capital (K_t) from the capital producers, which completely depreciates in one period, and hires the young agents (N) to produce Y_t units each period. The Cobb-Douglas function is given by $Y(K_t, N) = AK_t^{\delta} N^{(1-\delta)}$. There are no aggregate shocks in this benchmark case, thus, without a loss of generality, total factor productivity (TFP) is assumed to be constant over time (A).

Given competitive markets, each factor is paid its marginal product, so the per-worker returns $w(k_t)$ and $\rho(k_t)$ are the wage rate and the return of capital respectively:

$$w(k_t) = (1-\delta) A k_t^{\delta}, \tag{2.1}$$

$$\rho(k_t) = \delta A k_t^{(\delta-1)}, \tag{2.2}$$

[9] Having two production sectors is not only a strategy to separate the investment decision, but also to take into account that an optimal standard-debt-contract (SDC) requires the borrowers be risk neutral (Gale and Hellwig 1985) In addition, if the producer of consumption goods were also the borrower and subject to both the aggregate and idiosyncratic shocks, two problems may arise: (i) Since households provide labor services and get paid the same wage, then the question would be why both the households and financial intermediaries cannot infer what the output and then the actual shocks are from the wage rate and the return to capital; (ii) If the consumption firm had a very low shock and is unable to pay its workers, some households would end up without income to consume.

where $k_t = K_t/N_t$. In the first period ($t = 1$), each producer has an initial stock of capital k_0.

2.1.2 Households

In the first period, new-born agents (*subscript 1*) inelastically supply labor at wage w_t. They decide how much to consume $c_{1,t}$ and save in the form of deposits (d_t^s) after paying a lump-sum tax to the government (τ_t). Individual deposits can be thought as equity shares and the return as dividends. When agents are old (*subscript 2*), they consume their deposits including a gross deposit rate $r^d(k_{t+1})$. Since the young generations are the ultimate owners of all financial institutions, the return on deposits can be thought as dividends paid on equity shares d_t. Taking as given prices $(w(k_t), r^d(k_{t+1}))$ and the lump-sum tax, the representative young agent solves the following problem

$$\max_{\{c_{1,t}, c_{2,t+1}, d_t\}} \quad [log\,(c_{1,t}) + \beta log\,(c_{2,t+1})]\,, \tag{2.3a}$$

$$\text{s.t.} \quad c_{1,t} + d_t \leq w(k_t) - \tau_t, \tag{2.3b}$$

$$c_{2,t+1} \leq d_t^s r^d(k_{t+1}), \tag{2.3c}$$

$$c_{1,t} \geq 0, \quad c_{2,t+1} \geq 0, \quad d_t \geq 0. \tag{2.3d}$$

After substituting $c_{1,t}$ and $c_{2,t+1}$ and solving for the first-order conditions (FOCs), the optimal choice and supply of deposits is

$$d^s(k_t) = \beta \frac{w(k_t) - \tau_t}{(1+\beta)}. \tag{2.4}$$

In this environment, the income and substitution effects cancel out, while the stochastic discount factor (SDF), or intertemporal marginal rate of substitution (IMRS), is given by

$$m_{t+1} = \beta \frac{c_{1,t}}{c_{2,t+1}} = \frac{1}{r^d(k_{t+1})} = m(k_{t+1}). \tag{2.5}$$

The initial old generation is endowed with an amount of consumption goods a^0 that is consumed ($c_{2,1} = a^0$).

2.1.3 Financial Intermediary

At the beginning of each period the financial intermediaries receive deposits from young households $d^s(k_t)$, which are taken as equity shares, and decide how much to invest in a one-period risk-free government bond (b_t^d), which pays a risk-free gross return R_{t+1}^f, and to lend to the young capital producers (l_t). Define $r^l : \mathbb{R}^+ \to \mathbb{R}^+$ as the gross return to lending that is a function of the capital stock. Before defining the problem faced by a typical financial intermediary, the following assumption ensures a minimum demand of government bonds:[10]

Assumption 1. *The financial intermediary is required to hold a fraction $\kappa \in (0, 1)$ of its deposits as minimum reserves, $RR_t = \kappa d(k_t)$, which are invested in the risk-free government bonds.*

Therefore, the total demand of government bonds is the sum of the required minimum reserves over deposits and the excess demand, $B_t^d = RR_t + b_t^d$.

Let define the net cash-flow in period t:

$$W_t = B_{t-1} R_t^f + l_{t-1} r^l(k_t) - d_{t-1} r^d(k_t) + d_t^d - l_t^s - B_t^d \geq 0, \tag{2.6}$$

where the nonnegative condition is imposed to guarantee solvency.

Each typical financial intermediary maximizes the discounted present value of the net cash flow, taking as given prices $(R_{t+1}^f, r^l(k_{t+1}), r^d(k_{t+1}))$ and the deterministic

[10] This is useful in case of a corner solution. The reserve requirement can also introduce an additional policy parameter (κ).

SDF[11]

$$\max_{\left\{d_t^d, b_t^d, l_t^s\right\}} \quad \left[W_t + m(k_{t+1})W_{t+1}\right], \tag{2.7a}$$

$$\text{s.t.} \quad B_t^d \geq \kappa.d_t^d, \tag{2.7b}$$

$$k_{t+1} = f(l_t) \text{ , the law of motion of capital,} \tag{2.7c}$$

and subject to the solvency condition, with $B_t^d = \kappa.d_t^d + b_t^d$. Let μ_t be the Lagrange multiplier for constraint (2.7b). The FOCs are:

$$d_t^d : \quad 1 - m(k_{t+1})r^d(k_{t+1}) - \kappa\mu_t = 0, \tag{2.8a}$$

$$b_t^d : \quad -1 + m(k_{t+1})R_{t+1}^f + \mu_t = 0, \tag{2.8b}$$

$$l_t : \quad -1 + m(k_{t+1})r^l(k_{t+1}) = 0. \tag{2.8c}$$

Two cases can be analyzed:

(i) A positive excess demand of goverment bond ($b_t^d > 0$) implies $\mu_t = 0$ and $B_t^d > \kappa.d_t^d$. In this case, from combining the FOCs we get the equality of returns:

$$R_{t+1}^f = r^d(k_{t+1}) = r^l(k_{t+1}). \tag{2.9}$$

(ii) If (2.7b) holds with equality, then $\mu_t > 0$ and $b_t^d = 0$. From (2.8a) and (2.8b) we get the relationship between the return on deposits and the risk-free rate:

$$\mu_t(1-\kappa) = \left[r^d(k_{t+1}) - R_{t+1}^f\right]. \tag{2.10}$$

[11]When the free-entry condition is imposed, the financial intermediary problem would be equivalent to maximize the household utility subject to the financial constraint $W_t + m(k_{t+1})W_{t+1} = 0$. Although this is a common practice, as is discussed in the second part, in this model the intermediary will be able to diversify the capital-producer risk and cover itself from the aggregate volatility. Therefore, it is not reasonable to assume that the financial intermediary will behave as a risk-averse agent.

Since left-hand side and the SDF are strictly positive, it must be the case that $r^d(k_{t+1}) > R^f_{t+1}$. But from (2.8c) and (2.5) $r^l(k_{t+1}) = r^d(k_{t+1})$. Therefore, in the corner solution, the return to lending must be greater than the risk-free return:

$$r^d(k_{t+1}) = r^l(k_{t+1}) > R^f_{t+1}. \tag{2.11}$$

This is an intuitive result, that is, if the return from lending is always greater than the return of government bonds, the financial intermediary only keeps the required reserves in bonds ($B^d_t = \kappa d^d_t$), lending the remaining funds to capital-producers, $l^s_t = (1-\kappa)d^d_t$.

However, only the first case will survive in equilibrium. Since $m(k_{t+1})$ is the inverse of $r^d(k_{t+1})$, FOC (2.8a) implies that μ_t must be zero. Additionally, the second case would generate a loss equivalent to $\kappa d_t(R^f_{t+1} - r^d(k_{t+1}))$, thus, it is ruled out. Therefore, only (2.9) will be considered in equilibrium.

2.1.4 Capital Producers

Each period, a *unit mass* of two-period lived capital producers has access to the technology for producing capital goods. They only care about the second-period consumption. As in Labadie (1998) and Gale and Hellwig (1985), since all producers are ex-ante identical and there is a rationing in the size of loans, in period one they demand the same amount of funds (l_t) to invest and produce capital goods, which are available to rent next period. The production is subject to the producer-specific shock θ_i (idiosyncratic shock) that is only known by each producer. The technology $\theta_i l^\alpha_t$ defines the individual production of capital goods, which is rented to the producers of consumption goods, receiving a payment of $\rho(k_{t+1})\theta_i l^\alpha_t$. The following assumption on the idiosyncratic shock is based on the simplified environment proposed by Bernanke et al. (1999).

Assumption 2. *Let the producer-specific shock* (θ_i) *be an i.i.d random variable with a continuous twice-differentiable cumulative distribution function* $G(\tilde{\theta}) = prob(\theta \leq \tilde{\theta})$ *over a non-negative support* $\left[\underline{\theta}, \bar{\theta}\right]$*, with a probability density function* $g(\tilde{\theta}) = \dfrac{dG}{d\tilde{\theta}}$*, and* $E(\theta_i) = 1$*.*

Although the financial intermediary does not know the realization of each producer's shock in $t+1$, they have the technology to monitor at any time, optimally used only when the capital producer claims bankruptcy. Monitoring is assumed to be deterministic, a capital producer announcing insolvency will be monitored with certainty.[12]

To observe the actual return, the financial intermediary must pay a fraction $\gamma \in (0,1)$ of the producer's return in the second period, $\gamma\rho(k_{t+1})\theta_i l_t^{\alpha}$. When $\rho(k_{t+1})\theta_i l_t^{\alpha} > l_t r^l(k_{t+1})$, the producer might falsely claim bankruptcy, however, he could consume at most $(1-\gamma)\rho(k_{t+1})\theta_i l_t^{\alpha} - l_t r^l(k_{t+1})$, which is always lower than the consumption from telling the truth: $\rho(k_{t+1})\theta_i l_t^{\alpha} - l_t r^l(k_{t+1}) \geq 0$. Therefore, there is no need of an incentive-compatibility constraint.

As all capital producers are ex-ante identical and receive the same credit from the financial intermediary, the subscript i can be dropped to work with the representative capital producer. The break-even producer's shock $(\hat{\theta})$ is defined when the rental revenue only covers the lending cost:

$$\rho(k_{t+1})\hat{\theta} l_t^{\alpha} = l_t r^l(k_{t+1}). \tag{2.12}$$

This threshold characterizes the payment scheme: the producer announces default when $\theta < \hat{\theta}$, or repays the loan when $\theta \geq \hat{\theta}$. Given the realization of the idiosyncratic

[12]Stochastic monitoring is studied by Bernanke and Gertler (1989)

shock, the financial intermediary receives the following payoff:

$$\phi(k_{t+1}, l_t, \gamma, \theta, \hat{\theta}) = \begin{cases} (1-\gamma)\,\rho(k_{t+1})\theta l_t^{\alpha} & \text{if } \theta < \hat{\theta} \\ \rho(k_{t+1})\hat{\theta} l_t^{\alpha} & \text{if } \theta \geq \hat{\theta}. \end{cases} \tag{2.13}$$

Therefore, the producer's payoff is

$$\omega(k_{t+1}, l_t, \theta, \hat{\theta}) = \begin{cases} 0 & \text{if } \theta < \hat{\theta} \\ (\theta - \hat{\theta})\rho(k_{t+1}) l_t^{\alpha} & \text{if } \theta \geq \hat{\theta}. \end{cases} \tag{2.14}$$

When the capital producer is in default, all its revenues (net of monitoring costs) are paid to the lender, leaving nothing to consume, and the economy suffers a loss of $\gamma\rho(k_{t+1})\theta l_t^{\alpha}$. Taking into account the *optimal contract* defined in (2.13), the capital-producer's expected payment (Φ) is

$$\Phi(k_{t+1}, l_t, \gamma, \hat{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}} \phi(k_{t+1}, l_t, \gamma, \theta, \hat{\theta}) dG = \rho(k_{t+1}) l_t^{\alpha} \left[\int_{\underline{\theta}}^{\hat{\theta}} (1-\gamma)\,\theta dG + \int_{\hat{\theta}}^{\bar{\theta}} \hat{\theta} dG \right]. \tag{2.15}$$

Following Bernanke et al. (1999), I define the expression in brackets on the right hand side as the expected net-payment relative to the expected revenue, which can be divided into two different terms:

(i) the expected gross payment

$$\Lambda(\hat{\theta}) \equiv \int_{\underline{\theta}}^{\hat{\theta}} \theta dG + \int_{\hat{\theta}}^{\bar{\theta}} \hat{\theta} dG,$$

which is strictly increasing and concave since $\Lambda'(\hat{\theta}) = 1 - G(\hat{\theta}) > 0$ and $\Lambda''(\hat{\theta}) = -g(\hat{\theta}) < 0$.

(ii) and the expected monitoring cost

$$\gamma\Omega(\hat{\theta}) \equiv \gamma \int_{\underline{\theta}}^{\hat{\theta}} \theta dG,$$

which is strictly increasing and convex, assuming $\hat{\theta}g(\hat{\theta})$ is increasing, a property that is satisfied for most common distributions (for example the uniform distribution and any monotonically increasing transformation of a normal distribution). Thus, $\gamma\Omega'(\hat{\theta}) = \gamma\hat{\theta}g(\hat{\theta}) > 0$ and $\gamma\Omega''(\hat{\theta}) = \gamma\left[g(\hat{\theta}) + \hat{\theta}g'(\hat{\theta})\right] > 0$.

Using this notation, the expected payment (2.15) can be rewritten as

$$\Phi(k_{t+1}, l_t, \gamma, \hat{\theta}) = \rho(k_{t+1}) l_t^{\alpha} \left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]. \tag{2.16}$$

Given the properties of $\Lambda(\hat{\theta})$ and $\Omega(\hat{\theta})$, the producer's expected payment is a non-monotonic function with respect to the cutoff point $\hat{\theta}$

$$\frac{d\Phi(\cdot)}{d\hat{\theta}} = \rho(k_{t+1}) l_t^{\alpha} \left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right] \gtreqless 0, \tag{2.17}$$

reaching a global maximum when $\Lambda'(\theta^m) = \gamma\Omega'(\theta^m)$. The expected payment is increasing on the interval $(\underline{\theta}, \theta^m)$ and decreasing after $(\theta^m, \bar{\theta})$.

Since there is a unit mass of capital producers, as is pointed out by Uhlig (1996), integration over individual producers is the same as integrating over all possible realizations of θ. Let capital producers be indexed by $j \in [0, 1]$, and their shocks $(\theta(j))$ be identically distributed and pairwise uncorrelated with a common mean $E[\theta] = 1$ (Assumption 2), then

$$\int_0^1 \theta(j) dj = 1. \tag{2.18}$$

As a consequence, from the perspective of each financial intermediary, the idiosyncratic risk is eliminated by the expected payment defined in (2.16).

The Optimal Choice of Investment. The capital producer chooses the cutoff point $\hat{\theta}$ and the investment level l_t, taking as given prices $(R^f_{t+1}, r^l(k_{t+1}), r^d(k_{t+1}))$ and the expected payoff that results from the optimal contract (2.13)-(2.14). To simplify steps, the repayment amount $r^l(k_{t+1})\, l_t$ is equivalent to $R^f_{t+1} l_t$, given the equality of returns in equilibrium. From this perspective, each financial intermediary must receive at least the opportunity cost of lending in expectation. Let's define the set of feasible loanable funds: $L(k_t) = \{l \in y(k_t) : 0 < l \leq (1-\kappa)d(k_t)\}$. Thus, the capital-producer's decision problem is as follows

$$\max_{\hat{\theta}, l_t \in L(k_t)} \left\{ \max \left\{ \left[E(\theta) - \Lambda(\hat{\theta}) \right] \rho(k_{t+1}) l_t^{\alpha}, 0 \right\} \right\}, \tag{2.19a}$$

$$\text{s.t.} \quad \rho(k_{t+1}) l_t^{\alpha} \left[\Lambda(\hat{\theta}) - \gamma \Omega(\hat{\theta}) \right] \geq R^f_{t+1} l_t, \tag{2.19b}$$

$$k_{t+1} = f(l_t), \text{ the law of motion of capital.} \tag{2.19c}$$

Let λ be the Lagrange multiplier for constraint (2.19b). The Lagrangian function for a non-zero expected payoff is

$$\mathcal{L} = \left[1 - \Lambda(\hat{\theta}) \right] \rho(k_{t+1}) l_t^{\alpha} + \lambda \left[\rho(k_{t+1}) l_t^{\alpha} \left[\Lambda(\hat{\theta}) - \gamma \Omega(\hat{\theta}) \right] - R^f_{t+1} l_t \right]. \tag{2.20}$$

Note that $E(\theta) = 1$ by Assumption (2), so $0 < 1 - \Lambda(\hat{\theta}) < 1$.[13] The case when $\lambda = 0$ is ruled out because the optimizing capital-producer will never pay more than the repayment agreement, thus constraint (2.19b) holds with equality.

[13] As a result, $\Lambda(\hat{\theta}) = \int_{\underline{\theta}}^{\hat{\theta}} \theta dG - \int_{\hat{\theta}}^{\bar{\theta}} \hat{\theta} dG < 1$.

The FOCs for an *interior solution* ($\lambda > 0$) are

$$\hat{\theta}: \qquad \lambda(\hat{\theta}) = \frac{\Lambda'(\hat{\theta})}{\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})}, \tag{2.21a}$$

$$l_t: \qquad \alpha\rho(k_{t+1})l_t^{\alpha-1} = \frac{\lambda(\hat{\theta})R_{t+1}^f}{\left[1-\Lambda(\hat{\theta})\right] + \lambda(\hat{\theta})\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]}, \tag{2.21b}$$

$$\lambda: \qquad \rho(k_{t+1})l_t^{\alpha}\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right] = R_{t+1}^f l_t. \tag{2.21c}$$

The second-order condition (SOC):

$$[\Lambda''(\hat{\theta}) - \gamma\Omega''(\hat{\theta})]\lambda - \Lambda''(\hat{\theta}) < 0,$$

is always satisfied, since $\Lambda'' < 0$, $\gamma\Omega'' > 0$, and $\lambda > 1$ (see Appendix B).

To analyze possible interior solutions, let $\varepsilon(\hat{\theta})$ be the *excess return* from lending to the capital producer, from the FOC (2.21b): [14]

$$\varepsilon(\hat{\theta}) = \frac{\lambda(\hat{\theta})}{\left[1-\Lambda(\hat{\theta})\right] + \lambda(\hat{\theta})\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]}, \tag{2.22}$$

and using (2.21a) to substitute $\lambda(\hat{\theta})$ we get

$$\varepsilon(\hat{\theta}) = \frac{\Lambda'(\hat{\theta})}{\left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right]\left[1-\Lambda(\hat{\theta})\right] + \Lambda'(\hat{\theta})\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]}, \tag{2.23}$$

which is greater than 1 and increasing in $\hat{\theta}$ (see Appendix C).

Proposition 2.1.1. *Let $\hat{\theta}^*$ and l_t^* be the optimal cutoff point and investment respectively that solve the problem (2.19a) subject to (2.19b)-(2.19c). To get an interior solution, $\hat{\theta}^*$ must be chosen on the increasing part of the expected return function (2.16), this is when $\hat{\theta}^* \in (\underline{\theta}, \theta^m)$. Then, both $\hat{\theta}^*$ and l_t^* are unique, defining the*

[14] When aggregate risk is introduced in the next section, the excess return will represent the risk premium.

optimal demand of government bonds by the financial intermediary.

Proof. For the first part, as $\hat{\theta}^*$ gets closer to the maximizer of the expected payment (θ^m), where $\Lambda'(\theta^m) = \gamma\Omega'(\theta^m)$, the $\lim_{\hat{\theta}\to\theta^m} \lambda(\hat{\theta}) = +\infty$. This implies that

$$\lim_{\hat{\theta}\to\theta^m} \frac{\lambda(\hat{\theta})}{\left[1-\Lambda(\hat{\theta})\right] + \lambda(\hat{\theta})\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]} = \frac{1}{\Lambda(\theta^m) - \gamma\Omega(\theta^m)}. \tag{2.24}$$

Substitute (2.24) in (2.21b) to get

$$\rho(k_{t+1}) l_t^{\alpha-1} \left[\Lambda(\theta^m) - \gamma\Omega(\theta^m)\right] = \frac{R_{t+1}^f}{\alpha}. \tag{2.25}$$

Then, it is possible to show that if (2.25) holds, then (2.21c) will never hold with equality and the expected return of lending is always greater than its opportunity cost for any value of l:

$$\frac{R_{t+1}^f}{\alpha} > R_{t+1}^f, \quad \text{or} \quad R_{t+1}^f(1-\alpha) > 0. \tag{2.26}$$

In this case the financial intermediary has an incentive to lend out all the available resources (net of the reserve requirement) to capital producers, reaching the upper bound of the set of feasible loanable funds: $l_t^* = (1-\kappa)\, d(k_t)$. If $\hat{\theta}^*$ is chosen in the decreasing part of the expected return function (2.16), when the cutoff $\hat{\theta} \in (\theta^m, \bar{\theta})$, then $\lambda(\hat{\theta})$ would be negative.

Once the interior solution is guaranteed by $\hat{\theta}^* \in (\underline{\theta}, \theta^m)$, the optimal values of both the break-even point and investment are the solution to the system of equations formed by the FOCs (2.21a)-(2.21c). The equilibrium break-even point ($\hat{\theta}^*$) is obtained by substituting (2.21c) in (2.21b) and using the excess return defined in (2.23)

to get

$$\frac{\Lambda'(\hat{\theta}^*)}{\left[\Lambda'(\hat{\theta}^*) - \gamma\Omega'(\hat{\theta}^*)\right]\left[1 - \Lambda(\hat{\theta}^*)\right] + \Lambda'(\hat{\theta}^*)\left[\Lambda(\hat{\theta}^*) - \gamma\Omega(\hat{\theta}^*)\right]} = \frac{\alpha}{\Lambda(\hat{\theta}^*) - \gamma\Omega(\hat{\theta}^*)}. \tag{2.27}$$

The right hand side of (2.27) is the gross-return on investment (GRI), that is, the ratio of the expected marginal revenue of capital producer (EMR) to return of loans ($r^l(k_{t+1})$):

$$\begin{aligned} GRI(\hat{\theta}) &= \frac{EMR(k_{t+1}, l_t)}{r^l(k_{t+1})} \\ &= \frac{\alpha\rho(k_{t+1})l_t^{\alpha-1}}{\rho(k_{t+1})l_t^{\alpha-1}\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]} \\ &= \frac{\alpha}{\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})}. \end{aligned}$$

Since GRI is strictly decreasing in $\hat{\theta}$ and the excess return is strictly increasing in $\hat{\theta}$, there exits a unique solution $\hat{\theta}^*$ to (2.27). All capital-producers face the same distribution of indiosyncratic risk, thus $\hat{\theta}_i^* = \hat{\theta}^*$ for all i.

Given the optimal choice $\hat{\theta}^*$, the optimal level of investment (l_t^*) is the solution of the FOC (2.21b),

$$\alpha\rho(k_{t+1})(l_t^*)^{\alpha-1} = \varepsilon(\hat{\theta}^*)R_{t+1}^f. \tag{2.28}$$

Again, the left-hand side of (2.28) is the expected marginal return of capital producer ($EMR(k_{t+1}, l_t)$) that has the following properties:

(i) $\lim_{l\to 0} EMR(k_{t+1}, l_t) = +\infty$.

(ii) The lowest value is given by the corner solution: $EMR(k_{t+1}, (1-\kappa)d_t) = \dfrac{\alpha\rho(k_{t+1})}{[(1-\kappa)d_t]^{1-\alpha}}$.

(iii) It is decreasing in l_t as $\dfrac{\partial EMR(\cdot)}{\partial l_t} = \alpha(\alpha-1)\rho(k_{t+1})l_t^{\alpha-2} < 0$.

(iv) And convex since $\dfrac{\partial^2 EMR(\cdot)}{\partial l_t^2} = \alpha(\alpha-1)(\alpha-2)\rho(k_{t+1})l_t^{\alpha-3} > 0.$

By solving (2.28), any capital producer i equalizes his EMR_i, which is convex and strictly decreasing in l, with $\varepsilon(\hat{\theta}^*)R_{t+1}^f$, which does not depend on l. Then, the capital-producer i chooses a unique optimal level of investment given by

$$l_i^*(\varepsilon(\hat{\theta}^*), R_{t+1}^f, \rho(k_{t+1})) = \left[\frac{\alpha\rho(k_{t+1})}{\varepsilon(\hat{\theta}^*)R_{t+1}^f}\right]^{\frac{1}{1-\alpha}}. \tag{2.29}$$

□

2.1.5 Government

At the beginning of each period, the government announces the risk-free rate (R_{t+1}^f) at which will offer the one-period bonds and choses the lump-sum taxes (τ_t). Once these policy variables are chosen, the total government spending per worker (g_t) is endogenously determined by the total demand of government bonds (B_t^d), net of previous debt repayment, to satisfy

$$g_t = \tau_t + B_t^d - R_t^f B_{t-1}, \tag{2.30}$$

$$\lim_{t\to\infty} \frac{B_t}{R^{f^{t-1}}} = 0 \text{ (no-Ponzi constraint)}, \tag{2.31}$$

$$\text{and } B_0 \text{ is given.}$$

In other words, the government issues bonds to cover the financial sector's demand by assuming a perfectly elastic supply of bonds at the announced rate. The risk-free rate (R_{t+1}^f) can be thought as a policy rate that affects the entire economy through (2.28).[15] In addition, the reserve requirement (κ) could also be used as another policy instrument to reduce the loanable funds available to the private sector and increase the total demand of goverment bonds.

[15] In the last section, I propose an extension of this model, where the government can choose the level of spending but without the perfectly-elastic supply of bonds.

2.1.6 Equilibrium and the Financial Depth Indicator

In this section, I examine the steady state taking as given the risk-free return on government bonds ($R^f_{t+1} = R^f$). Any change in R^f will result in a transition to a new steady state.

The law of motion of capital stock for a given k_0 and R^f is defined by

$$k(l^*, E(\theta)) = E(\theta){l^*}^{\alpha} \tag{2.32}$$

$$= {l^*}^{\alpha} \quad \text{(by Assumption 2).} \tag{2.33}$$

Then, the total output per worker in steady state is

$$y(l^*, A) = A{l^*}^{\alpha\delta}, \tag{2.34}$$

and the factor payments are

$$w(l^*, A) = (1-\delta)A{l^*}^{\alpha\delta}, \tag{2.35}$$

$$\rho(l^*, A) = \delta A{l^*}^{\alpha(\delta-1)}. \tag{2.36}$$

From (2.28) and (2.36), the steady state level of investment is defined as follows

$$l^*(\varepsilon(\hat{\theta}^*), R^f, A) = \left[\frac{\alpha\delta A}{\varepsilon(\hat{\theta}^*)R^f}\right]^{\frac{1}{1-\alpha\delta}}, \tag{2.37}$$

which is decreasing in both the excess return and the risk-free return, but increasing

in the total factor productivity.[16]

The steady state level of deposits (d^*) is given by $d^s = \beta \dfrac{w(l^*, A) - \tau_t}{(1+\beta)} = d^d = d^*$. At the given R^f, the government supplies bonds to absorb its demand in steady state ($B^{d^*} = d^* - l^*$), validating a spending level g^* that satisfies (2.30).

The resource constraint in the economy is

$$y(l^*, A) = c_1^* + c_2^* + g^* + l^* + \int_{\underline{\theta}}^{\bar{\theta}} \omega(k, l, \theta, \hat{\theta}) g(\theta) d\theta \ + DWL, \tag{2.38}$$

where DWL is the deadweight loss from monitoring, which is defined as follows

$$DWL = \rho(l^*, A) {l^*}^{\alpha} \gamma \Omega(\hat{\theta}) \tag{2.39}$$

$$= \delta y(l^*, A) \gamma \Omega(\hat{\theta}). \tag{2.40}$$

To get a graphical representation of the optimal decision and steady- state values in the entire economy, let EMR^E be aggregate expected marginal-return function. Since all capital producers are ex-ante identical, we get $EMR^E = \alpha \rho(k_{t+1}) l_t^{\alpha - 1}$, which is a convex and strictly decreasing function on l_t. Given R^f and $k_0 > 0$, Figure 2.1 depicts the optimal decision (2.28) for the economy, resulting in a unique steady-state level of investment and demand of government bonds.

The effect of changes in R^f, the excess return, and productivity (A) are graphically analyzed in Appendix A. For instance, if the government announces an increase in the

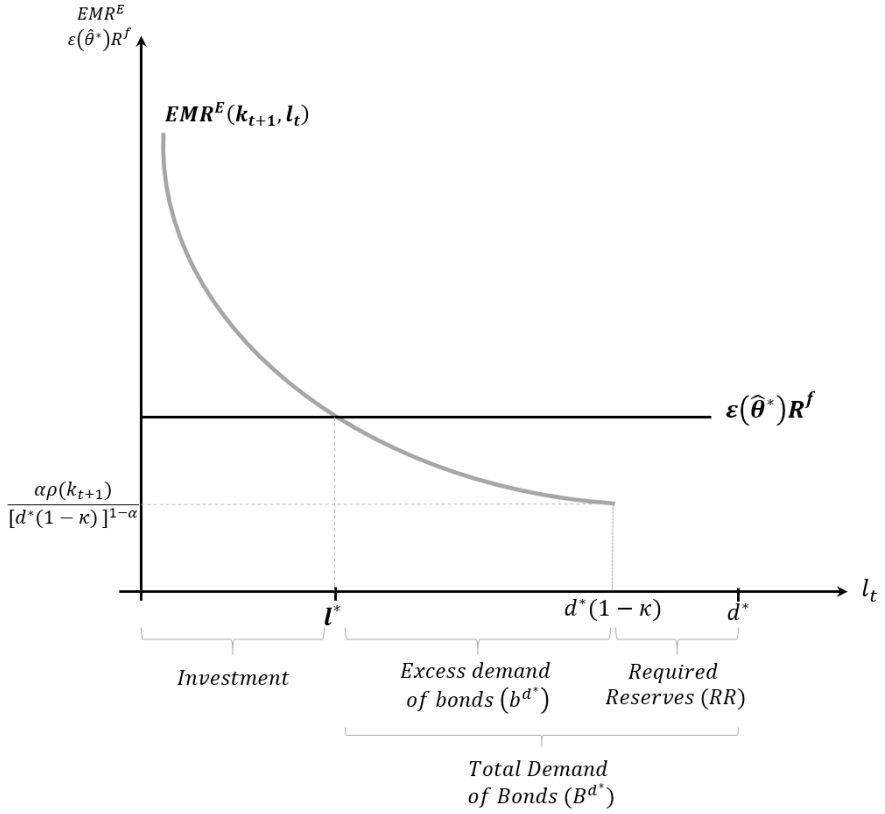
[16] Taking the partial derivatives:

$$\frac{\partial l^*}{\partial \varepsilon(\hat{\theta}^*)} = -\frac{1}{\varepsilon(\hat{\theta}^*)} \left(\frac{1}{1 - \alpha\delta} \right) \left[\frac{\alpha\delta A}{\varepsilon(\hat{\theta}^*) R^f} \right]^{\frac{1}{1-\alpha\delta}} < 0,$$

$$\frac{\partial l^*}{\partial R^f} = -\frac{1}{R^f} \left(\frac{1}{1 - \alpha\delta} \right) \left[\frac{\alpha\delta A}{\varepsilon(\hat{\theta}^*) R^f} \right]^{\frac{1}{1-\alpha\delta}} < 0,$$

$$\frac{\partial l^*}{\partial A} = A^{\frac{\alpha\delta}{1-\alpha\delta}} \left(\frac{1}{1 - \alpha\delta} \right) \left[\frac{\alpha\delta}{\varepsilon(\hat{\theta}^*) R^f} \right]^{\frac{1}{1-\alpha\delta}} > 0.$$

Figure 2.1: Aggregate Investment and Demand of Government Bonds in Steady State



risk-free rate from R^f to $R^{f'}$ to finance more spending (g). Everything else equal, this policy would generate a reduction in the private investment, a phenomenon known as *crowding out effect*. In this framework, the increase of government expenditures is equivalent to the reduction in private investment as is depicted in Figure A1.

Finally, if the financial depth (FD) is proxied by the level of private credit extended by the financial intermediaries as a share of total output, the capital producers are the only private agents who borrows from the financial intermediary.[17] Therefore,

[17]This might be a limitation of the model, since the total credit to the private sector typically includes a variety of loans to households, financial institutions, and funds used as working capital. From the national account perspective, the FD indicator in this model would be equivalent to the country's investment rate.

given R^f and k_0, the financial depth indicator in steady state (FD^*) is

$$\begin{aligned} FD^*(\varepsilon(\hat{\theta}^*), R^f) &= \frac{l^*(\varepsilon(\hat{\theta}^*), R^f, A)}{y(l^*, A)} \\ &= \frac{{l^*}^{1-\alpha\delta}}{A} \\ &= \frac{\alpha\delta}{\varepsilon(\hat{\theta}^*)R^f} \quad \text{by (2.37).} \end{aligned} \tag{2.41}$$

The degree of financial deepening in equilibrium is decreasing in both the excess return and the risk-free return. However, unlike the investment level, FD^* does not depend on the level of total factor productivity.

2.2 Introducing Aggregate Risk into the Benchmark Model

The preceding section provides a benchmark model with equilibrium levels of investment and financial depth in an economy without aggregate risk, and only subject to the idiosyncratic risk in the production of capital goods. In this section, an aggregate productivity shock (S_t) is introduced that will affect both the TFP (A_t) and the distribution of the producer-specific shock (G). For simplicity, there are two possible states of the world with equal probability, low ($A(L)$) and high ($A(H)$) productivity with $\pi(L) = \pi(H) = 0.5$, and let σ_p be the standard deviation of productivity shocks. Hereafter the subscripts "V" and "v" will identify the functions and chosen variables, respectively, associated with the stochastic environment.

Some empirical studies find that permanent changes in macroeconomic risk can explain structural changes in the equity price trends and risk premium.[18] In line with this literature, I propose a link between the TFP volatility and the distribution of idiosyncratic risk, which will have consequences on the risk premium. The following

[18]By using postwar data, Lettau et al. (2007) show that there is a strong correlation between the macroeconomic volatility and asset prices, so with the equity risk premium as well, not only in United States but also at international level. Bansal et al. (2005) also find that GARCH volatility measures can explain quarterly price-dividend ratios in United Sates, United Kingdom, Germany, and Japan.

assumption summarizes this key link between the macroeconomic risk and individual risk:

Assumption 3. *A change in the aggregate risk or TFP volatility generates a mean-preserving spread (MPS) change in the original distribution of the idiosyncratic shock over the same non-negative support* $\left[\underline{\theta}, \bar{\theta}\right]$. *The standard deviation of the new distribution depends on the volatility of the productivity shock* (σ_p), *that is, the more volatile is the economy, the riskier is the production of capital goods.*

These properties are formalized next.

Definition 2.2.1. *Let* $\theta_{vi} \sim G_V$ *and* $\theta_i \sim G$ *be random variables associated with the realizations of the producer-specific shock for an economy with and without aggregate volatility respectively.* G_V *is a mean-preserving spread* G *if and only if*

$$\theta_{vi} \overset{d}{=} \theta_i + z,$$

for some noise z *such that* $E(z|\theta_i) = 0 \ \forall \theta_i$. *In addition, the variance of the noise* z *is assumed to be an increasing function of the aggregate volatility:* $\partial Var(z)/\partial \sigma_p > 0$.

Since the MPS is a special case of the Second Order Stochastic Dominance (SOSD), G second-order stochastically dominates G_V, therefore the following conditions hold[19]:

(i) The two distributions have the same mean:

$$\int_{\underline{\theta}}^{\bar{\theta}} \theta dG_V = \int_{\underline{\theta}}^{\bar{\theta}} \theta dG. \tag{2.42}$$

(ii) Integrating over the difference in the distributions is a non-negative function of

[19]For discussions and economic applications of the mean-preserving increase in risk see Rothschild and Stiglitz (1970, 1972) and Diamond and Stiglitz (1974)

the aggregate volatility:

$$\Gamma(t, \sigma_p) = \int_{\underline{\theta}}^{t} \left[G_V(\theta, \sigma_p) - G(\theta) \right] d\theta \geq 0, \quad \text{for all} \quad \underline{\theta} \leq t \leq \bar{\theta}. \tag{2.43}$$

From Definition 2.2.1 and Assumption 3, an increase in the aggregate risk will generate a mean preserving increase in the capital-producer's specific risk. Thus, it is reasonable to impose the following working assumption:[20]

Assumption 4. *A mean-preserving spread increase in the idiosyncratic risk will raise the default probability for a given cut off point:* $G_V(\hat{\theta}_v, \sigma_p') > G_V(\hat{\theta}_v, \sigma_p)$ *when* $\sigma_p' > \sigma_p$. *By moving weight from the center of the probability density function to the tails, without affecting the mean, the pdf is assumed to shift up over the interval* $[\underline{\theta}, \hat{\theta}_v]$.

By introducing aggregate volatility, the factor payments depend on the current state,

$$w(k_t, S_t) = (1 - \delta) A(S_t) k_t^{\delta}, \tag{2.44}$$

$$\rho(k_t, S_t) = \delta A(S_t) k_t^{(\delta - 1)}, \tag{2.45}$$

as well as the optimal supply of deposits:

$$d_{v,t}^{s} = \beta \frac{w(k_t, S_t) - \tau_t}{(1 + \beta)}. \tag{2.46}$$

As in previous section, the financial intermediary maximizes the discounted present value of the net cash flow, but now the next period cash flow (W_{t+1}) and the SDF depend on the next period aggregate shock (S_{t+1}). Then, FOCs for the stochastic

[20]The assumption of equal means requires that the distribution functions G and G_V have to cross at least once, therefore, there exits the possibility that $G_V(\hat{\theta}, \sigma_p) \leq G(\hat{\theta})$. However, that situation should be ruled out, because it would result in a counterintuitive prediction, that is, a relatively lower default probability when the aggregate volatility increases.

version of (2.7) are

$$d^d_{v,t}: \quad 1 - E_t\left[m(k_{t+1}, S_{t+1}) r^d(k_{t+1}, S_{t+1})\right] - \kappa\mu_{v,t} = 0, \tag{2.47a}$$

$$b^d_{v,t}: \quad -1 + E_t\left[m(k_{t+1}, S_{t+1}) R^f_{t+1}\right] + \mu_{v,t} = 0, \tag{2.47b}$$

$$l_{v,t}: \quad -1 + E_t\left[m(k_{t+1}, S_{t+1}) r^l(k_{t+1}, S_{t+1})\right] = 0, \tag{2.47c}$$

where $\mu_{v,t}$ is the Lagrangian multiplier for the required-reserve constraint.

For the same reason as explained in the deterministic benchmark, only the case when $\mu_{v,t} = 0$ and $b^d_{v,t} > 0$ is considered. Combining the FOCs and defining the $m(k_{t+1}, S_{t+1}) = 1/r^d(k_{t+1}, S_{t+1})$, we get the standard relationships

$$E_t\left[\frac{r^l(k_{t+1}, S_{t+1})}{r^d(k_{t+1}, S_{t+1})}\right] = E_t\left[\frac{1}{r^d(k_{t+1}, S_{t+1})}\right] R^f_{t+1} = 1. \tag{2.48}$$

2.2.1 The Capital-Producer Problem with Idiosyncratic and Aggregate Risk

When aggregate risk is introduced, the optimal decision of capital producer faces two major changes. First, there is a riskier distribution for his specific shock that increases the default probability at a given break-even point. Second, his income from renting the new capital is subject to the next period state, i.e. $\rho(k_{t+1}, S_{t+1})\theta_v l^{\alpha}_{v,t}$.

Since the standard debt contract (SDC) is characterized by a fixed repayment agreement (Gale and Hellwig, 1985), let $\hat{r}$ be the fixed payment per unit of loan. Thus, the capital producer is breakeven when

$$\rho(k_{t+1}, S_{t+1})\hat{\theta}_v l^{\alpha}_{v,t} = \hat{r} l_{v,t}, \tag{2.49}$$

where $\hat{\theta}_v$ is the break-even point for the stochastic problem. Given $(k_{t+1}, S_{t+1}, l_{v,t})$,

the financial intermediary receives the following payoff:

$$\phi_V(k_{t+1}, S_{t+1}, l_{v,t}, \gamma, \theta_v, \hat{\theta}_v) = \begin{cases} (1-\gamma)\,\rho(k_{t+1}, S_{t+1})\theta_v l_{v,t}^{\alpha} & \text{if } \theta_v < \hat{\theta}_v \\ \hat{r} l_{v,t} & \text{if } \theta_v \geq \hat{\theta}_v. \end{cases} \tag{2.50}$$

Using (2.49) to substitute $\hat{r} l_{v,t}$, the producer's expected payment from the benchmark model (2.16) is replaced by:

$$\Phi_V(k_{t+1}, S_{t+1}, l_{v,t}, \gamma, \hat{\theta}_v, \sigma_p) = \rho(k_{t+1}, S_{t+1}) l_{v,t}^{\alpha} \left[\Lambda_V(\hat{\theta}_v, \sigma_p) - \gamma \Omega_V(\hat{\theta}_v, \sigma_p)\right], \tag{2.51}$$

where the expected gross payment and monitoring cost relative to the income are respectively,

$$\Lambda_V(\hat{\theta}_v, \sigma_p) \equiv \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v dG_V + \int_{\hat{\theta}_v}^{\bar{\theta}} \hat{\theta}_v dG_V \tag{2.52}$$

and

$$\gamma \Omega_V(\hat{\theta}_v, \sigma_p) \equiv \gamma \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v dG_V. \tag{2.53}$$

Once the aggregate risk is defined (σ_p), the realization of the aggregate state (S_{t+1}) will affect the return on capital, but not $\left[\Lambda_V(\hat{\theta}_{,}\sigma_p) - \gamma \Omega_V(\hat{\theta}_v, \sigma_p)\right]$.

The expected payment function has the same properties defined in the benchmark case: it is nonmonotonic in the cutoff point, reaching a global maximum at θ_v^m, where it is increasing on the interval $(\underline{\theta}, \theta_v^m)$ and decreasing after.

In sum, the capital producer chooses the break-even point and investment (loans) from the available funds $L(k_t, S_t) = \{l_{v,t} \in y(k_t, S_t) : 0 \leq l_{v,t} \leq (1-\kappa) d(k_t, S_t)\}$, taking as given prices $(R_{t+1}^f, r^l(k_{t+1}, S_{t+1}), r^d(k_{t+1}, S_{t+1}))$, to maximize his expected payoff over the two possible states $S_{t+1} \in \{L, H\}$ with equal probability. Each producer faces the financial intermediary's participation constraint, and the law of motion of capital. The intermediary will voluntarily participate in lending if the expected payment covers at least $r^l(k_{t+1}, S_{t+1})\, l_{v,t}$ or its equivalent in equilibrium $R_{t+1}^f l_{v,t}$. For-

mally, the representative capital producer solves

$$\max_{\hat{\theta}_v, l_{v,t} \in L(k_t, S_t)} \left\{ \max \left\{ \frac{l_{v,t}^{\alpha}}{2} \left[E(\theta_v) - \Lambda_V(\hat{\theta}_v, \sigma_p) \right] \left[\rho(k_{t+1}, L) + \rho(k_{t+1}, H) \right], 0 \right\} \right\}, \tag{2.54a}$$

$$\text{s.t.} \quad \frac{l_{v,t}^{\alpha}}{2} \left[\Lambda_V(\hat{\theta}_v, \sigma_p) - \gamma \Omega_V(\hat{\theta}_v, \sigma_p) \right] \left[\rho(k_{t+1}, L) + \rho(k_{t+1}, H) \right] \geq R_{t+1}^{f} l_{v,t}, \tag{2.54b}$$

$$k_{t+1} = f(l_{v,t}), \text{ the law of motion of capital.} \tag{2.54c}$$

To simplify notation, hereafter I omit the arguments of functions Λ_V and Ω_V, while the return on capital $\rho(k_{t+1}, S_{t+1})$ is abbreviated as $\rho(S_{t+1})$. Since the distribution of the idiosyncratic shock θ_v is a MPS of that for θ, then $E(\theta_v) = E(\theta) = 1$.

Let λ_v be the Lagrange multiplier for the constraint (2.54b), the FOCs for an interior solution ($\lambda_v > 0$) are quite similar to those associated with the benchmark model:

$$\hat{\theta}_v : \qquad \lambda_v = \frac{\Lambda_V'}{\Lambda_V' - \gamma \Omega_V'}, \tag{2.55a}$$

$$l_{v,t} : \qquad \alpha E_t \left[\rho(S_{t+1}) \right] l_{v,t}^{\alpha - 1} = \frac{\lambda_v R_{t+1}^{f}}{\left[1 - \Lambda_V \right] + \lambda_v \left[\Lambda_V - \gamma \Omega_V \right]}, \tag{2.55b}$$

$$\lambda_v : \qquad l_{v,t}^{\alpha} \left[\Lambda_V - \gamma \Omega_V \right] E_t \left[\rho(S_{t+1}) \right] = R_{t+1}^{f} l_{v,t}. \tag{2.55c}$$

The SOC is satisfied when

$$\left[\Lambda_V'' - \gamma \Omega_V'' \right] \Lambda_V' - \left[\Lambda_V' - \gamma \Omega_V' \right] \Lambda_V'' < 0,$$

which is similar to the condition found in the deterministic case, but substituting out λ_v with (2.55a). As before, an optimizing producer will pay no more than $r^l(k_{t+1}, S_{t+1})\, l_{v,t}$ or, equivalently, $R_{t+1}^{f} l_{v,t}$, thus, constraint (2.54b) holds with equality.

In the stochastic framework, the excess return is the *risk premium* from lending

to the capital producer, since it does not only depend on the cutoff point, but also on aggregate volatility.

$$\varepsilon_V(\hat{\theta}_v, \sigma_p) = \frac{\lambda_v}{[1-\Lambda_V] + \lambda_v [\Lambda_V - \gamma\Omega_V]}. \tag{2.56}$$

The procedures followed in Preposition (2.1.1) are still valid in the stochastic environment in order to get the equilibrium break-even point and investment. The former is obtained when the risk premium is equal to the expected gross return on investment ($GRI_V(\hat{\theta}_v, \sigma_p)$). Substituting λ_v with (2.55a) we obtain:[21]

$$\frac{\Lambda'_V}{[1-\Lambda_V][\Lambda'_V - \gamma\Omega'_V] + \Lambda'_V [\Lambda_V - \gamma\Omega_V]} = \frac{\alpha}{\Lambda_V - \gamma\Omega_V}. \tag{2.57}$$

Recall that uniqueness is guaranteed because the left-hand side is increasing, while the right-hand side is decreasing in the cutoff point ($\hat{\theta}_v$). As was proved for the benchmark case, the optimal cutoff point is chosen in the increasing part of the expected net payment function, i.e. $\hat{\theta}_v^* \in (\underline{\theta}, \theta_v^m)$.

Following the analysis of Section 2.1.6, we can derive the steady state investment (l_v^*) for given values of the risk-free return ($R_{t+1}^f = R^f$) and initial capital ($k_0 > 0$). After substituting (2.56) in (2.55b) at the optimal cutoff point $\hat{\theta}_v^*$ obtained from (2.57), and replacing $E_t[\rho(S_{t+1})]$ for its value in equilibrium, we find

$$l_v^*\left(\varepsilon_V(\hat{\theta}_v^*, \sigma_p), R^f, E[A(S)]\right) = \left[\frac{\alpha\delta E[A(S)]}{\varepsilon_V(\hat{\theta}_v^*, \sigma_p) R^f}\right]^{\frac{1}{1-\alpha\delta}}. \tag{2.58}$$

When the capital producer chooses the amount of borrowing, he considers the ex-

[21] The expected gross return on investment with aggregate risk is

$$GRI_V(\hat{\theta}_v, \sigma_p) = \frac{\alpha l_{v,t}^{\alpha-1} E_t[\rho(S_{t+1})]}{R_{t+1}^f} = \frac{\alpha l_{v,t}^{\alpha-1} E_t[\rho(S_{t+1})]}{l_{v,t}^{\alpha-1}[\Lambda_V - \gamma\Omega_V] E_t[\rho(S_{t+1})]}, \quad \text{using (2.55c)}$$
$$= \frac{\alpha}{\Lambda_V - \gamma\Omega_V}.$$

pected future return from renting capital. Therefore, the steady state investment in (2.58) not only depends on the risk premium and the given R^f, but also on the expected value of productivity shocks. As before, l_v^* is decreasing in the risk premium and the return of government bonds, but increasing in $E[A(S)]$. There is a trade-off between increasing the risk premium and the expected value of the productivity shocks. If there were an increase in both the volatility of productivity shock (σ_p) and its expected value ($E[A(S)]$), the optimal level of investment (2.58) would depend of the relative effect of the aggregate shock on the risk premium. This interaction is studied in the following section.

Since the financial intermediary lends out funds to a countably infinite number of risk-neutral producers, the latter bear both the aggregate and individual risks associated with the contract. Even though when the aggregate volatility is in place, the intermediary can diversify the aggregate risk among borrowers.

***But what about the output volatility (σ_y)?*:** After modifying the equilibrium condition for output (2.34) to account for the aggregate risk, and substituting the steady state investment (2.58), the output becomes $y(l^*, A(S_t)) = A(S_t)l^{*\alpha\delta}$. Therefore, the standard deviation of total output is given by

$$\sigma_y(\sigma_p, l_v^*) = \sigma_p l_v^{*\alpha\delta}, \tag{2.59}$$

which means that the output volatility is endogenous, and in equilibrium, depends not only on the volatility of the productivity shock, but also on the equilibrium level of investment. Therefore, the following is true:

Result 1. *It follows from (2.59) and (2.58) that any change in the aggregate risk (σ_p) has two effects on output volatility (σ_y): a one-to-one direct effect and an indirect one, since the optimal level of investment l_v^* is also a function of σ_p.*

Once the effect of aggregate risk on the optimal level of investment is defined in

the next section, Result 1 will be replaced by Result 4.

The steady state level of financial deepening with aggregate risk is

$$FD_v^*\left(\varepsilon_V(\hat{\theta}_v^*,\sigma_p),R^f,E\left[A(S)\right],A(S)\right)=\frac{l_v^{*1-\alpha\delta}}{A(S)}$$
$$=\left[\frac{\alpha\delta}{\varepsilon_V(\hat{\theta}_v^*,\sigma_p)R^f}\right]\left[\frac{E\left[A(S)\right]}{A(S)}\right]. \qquad (2.60)$$

The first factor is equivalent to the measure found in benchmark case, while the second one is the direct impact of the aggregate productivity shock. The latter factor represents the first link between the macroeconomic volatility and the financial deepening, which can be summarized as follows

Result 2. *The financial depth indicator in steady state is not only affected by the risk premium and the return of government bond, but it also fluctuates with the current productivity shock.*

This is an intuitive result, since the FD is normalized by the output, thus its dynamic is also a function of the current productivity shock. [22]

Results (1) and (2) reflect the complexity faced by any empirical work that studies relationship between the financial depth (FD) and output volatility (σ_y), because both are endogenous! When the FD is used as the explanatory variable, it is not always easy to find the proper instrumental variable to isolate it from the aggregate risk.

The optimal investment (2.58) and the financial depth indicator defined in (2.60) show the importance of having a link between the aggregate risk and the idiosyncratic shock. If this interaction were omitted, the risk premium would not be affected by the aggregate volatility. Then, if there were an increase in both the volatility of the

[22]However, if the financial depth is measured in terms of the expected output, the second factor in (2.60) would be simplified. The resulting indicator would match the form of the FD for the benchmark economy (2.41), but now with the risk premium being a function of the aggregate risk.

productivity shock (σ_p) and its expected value ($E\left[A(S)\right]$), the following results would hold:

(i) From (2.58), there would be an increase in the level of investment, amplifying the output volatility in (2.59).

(ii) At the same time, the level of financial deepening (2.60) would solely depend on the expected productivity shock relative to the actual state.

(iii) An increase in the aggregate risk and its expected value would result in an increase in the optimal level of investment, amplifying the output volatility, and with the possibility of having a deeper financial system.

2.3 In Search of the Missing Links

In this part, the effect of aggregate risk or TFP volatility is studied in steps. First, the analysis is concentrated on the investment level, or total credit to the private sector, and consequently on the output fluctuations. In a second step, the results are introduced in the financial depth indicator (FD) in order to check the effects on financial deepening.

To isolate the impact of the aggregate risk, let's consider an economy that starts with a certain level of variability on its total factor productivity, say σ_p^B. Now assume a one-time *exogenous* reduction in its fluctuation to σ_p^A, but keeping constant all other features associated with this model, including the mean of productivity shocks, i.e. $E_t\left[A(S_{t+1}^a)\right] = E_t\left[A(S_{t+1}^b)\right] = E_t\left[A(S_{t+1})\right]$ for all periods. If $\sigma_p^A = 0$, the economy is back to the benchmark case. It is also possible to think of this exercise as two economies with different aggregate volatility. Next, this change is assessed in the context of the investment/lending decisions to finally reach some conclusions about its impact on financial depth.

2.3.1 The Effect of Aggregate Risk on Lending and Output Volatility

Let's start from the optimal decision (2.57) associated with the initial aggregate risk σ_y^B, but without substituting λ_v from the risk premium for the scope of the analysis:

$$\frac{\lambda_v(\hat{\theta}_v^{*^B}, \sigma_p^B)}{\left[1 - \Lambda_V(\hat{\theta}_v^{*^B}, \sigma_p^B)\right] + \lambda_v(\hat{\theta}_v^{*^B}, \sigma_p^B)\left[\Lambda_V(\hat{\theta}_v^{*^B}, \sigma_p^B) - \gamma\Omega_V(\hat{\theta}_v^{*^B}, \sigma_p^B)\right]} = \frac{\alpha}{\Lambda_V(\hat{\theta}_v^{*^B}, \sigma_p^B) - \gamma\Omega_V(\hat{\theta}_v^{*^B}, \sigma_p^B)}. \tag{2.61}$$

To obtain the effect of reducing the aggregate risk on lending, let's analyze how both the risk premium and the expected gross return on investment change. As noted before, the gross return on investment in the right-hand side is decreasing on the break-even point ($\hat{\theta}_v$), while the risk premium (left-hand side) is increasing. Figure 2.2 depicts these facts, considering that the $GRI(\cdot)$ is a convex function (since $\partial^2 GRI(\cdot)/\partial\hat{\theta}_v^2 > 0$) and assuming a concave $\varepsilon_V(\cdot)$, as long as $\partial^2\varepsilon_V(\cdot)/\partial\hat{\theta}_v^2 < 0$. The convex and concave shapes of these functions will be irrelevant for the final results. At the intersection of the curves, a unique optimal break-even point is found ($\hat{\theta}_v^{*^B}$) for the initial situation with σ_p^B.

The following proposition summarizes the individual effect on the GRI function.

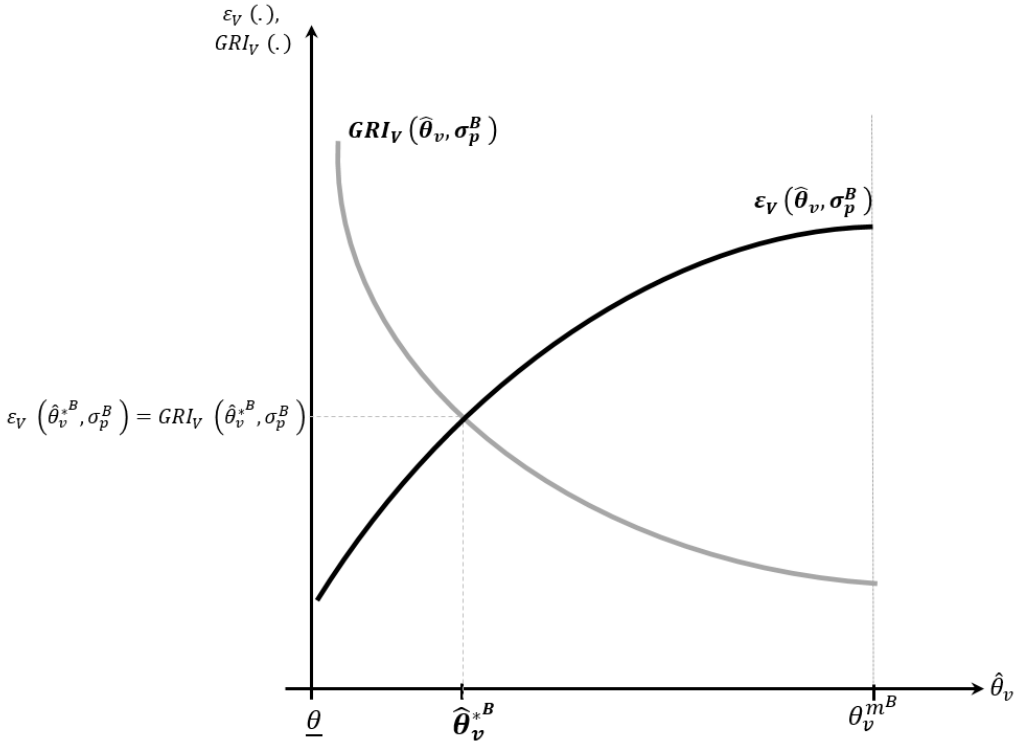
Proposition 2.3.1. *The lower is the aggregate risk (σ_p), the higher is the expected net-payment relative to the expected income in each state, $\left[\Lambda_V(\hat{\theta}_v, \sigma_p) - \gamma\Omega_V(\hat{\theta}_v, \sigma_p)\right]$, and the lower is the expected gross return on investment at the cutoff point. Given $\sigma_p^B > \sigma_p^A$, this implies that $GRI_V(\hat{\theta}_v, \sigma_p^B) > GRI_V(\hat{\theta}_v, \sigma_p^A)$.*

Proof. Let $Z(\hat{\theta}_v, \sigma_p^A, \sigma_p^B)$ be the difference in the expected net-payment after reducing the aggregate risk, relative to the expected income in each state, for all possible values of $\hat{\theta}_v$:

$$Z(\hat{\theta}_v, \sigma_p^A, \sigma_p^B) = \left[\Lambda_V(\hat{\theta}_v, \sigma_p^B) - \gamma\Omega_V(\hat{\theta}_v, \sigma_p^B)\right] - \left[\Lambda_V(\hat{\theta}_v, \sigma_p^A) - \gamma\Omega_V(\hat{\theta}_v, \sigma_p^A)\right]. \tag{2.62}$$

To simplify the analysis, let's apply integration by parts in (2.52) and (2.53), and the

Figure 2.2: Risk Premium and the Expected Gross Return on Investment



fact that $\int_{\hat{\theta}_v}^{\bar{\theta}} \hat{\theta}_v dG_V = \hat{\theta}_v \left(1 - G_V(\hat{\theta}_v)\right)$. Therefore, the gross payment becomes

$$\begin{aligned}\Lambda_V(\hat{\theta}_v, \sigma_p) &= \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v dG_V + \int_{\hat{\theta}_v}^{\bar{\theta}} \hat{\theta}_v dG_V \\ &= \hat{\theta}_v - \int_{\underline{\theta}}^{\hat{\theta}_v} G_V(\theta_v, \sigma_p) d\theta_v, \end{aligned} \tag{2.63}$$

and the monitoring cost is

$$\begin{aligned}\gamma\Omega_V(\hat{\theta}_v\sigma_p) &= \gamma \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v dG_V \\ &= \gamma \left[\hat{\theta}_v G_V(\hat{\theta}_v, \sigma_p) - \int_{\underline{\theta}}^{\hat{\theta}_v} G_V(\theta_v, \sigma_p) d\theta_v\right]. \end{aligned} \tag{2.64}$$

Then (2.62) can be written as

$$Z(.) = \hat{\theta}_v \gamma \left[G_V(\hat{\theta}_v, \sigma_p^A) - G_V(\hat{\theta}_v, \sigma_p^B) \right] + (1-\gamma) \int_{\underline{\theta}}^{\hat{\theta}_v} \left[G_V(\theta_v, \sigma_p^A) - G_V(\theta_v, \sigma_p^B) \right] d\theta_v. \tag{2.65}$$

Since $G_V(\theta_v, \sigma_p^A)$ and $G_V(\theta_v, \sigma_p^B)$ are MPS of $G(\hat{\theta})$ with a higher shifting parameter in country B, it follows:

(i) Capital producers in country B have a higher default probability at a given cutoff point, i.e. $G_V(\hat{\theta}_v, \sigma_p^A) < G_V(\hat{\theta}_v, \sigma_p^B)$. Thus, the first term of (2.65) is negative.

(ii) From (2.43), $G_V(\theta_v, \sigma_p^A)$ SOSD $G_V(\theta_v, \sigma_p^B)$, so the second term is also negative.

In sum, the first part of the Proposition is proved as $Z(\hat{\theta}_v, \sigma_p^A, \sigma_p^B) < 0$. With this result and the definition of the gross return on investment, the second part follows immediately:

$$GRI_V(\hat{\theta}_v, \sigma_p^B) = \frac{\alpha}{\Lambda_V(\hat{\theta}_v, \sigma_p^B) - \gamma \Omega_V(\hat{\theta}_v, \sigma_p^B)} > \frac{\alpha}{\Lambda_V(\hat{\theta}_v, \sigma_p^A) - \gamma \Omega_V(\hat{\theta}_v, \sigma_p^A)} = GRI_V(\hat{\theta}_v, \sigma_p^A). \tag{2.66}$$

□

As a result, when the aggregate volatility decreases, the GRI function shifts down at any level of $\hat{\theta}_v$. Before checking the risk premium function, let's establish the following properties that will be used next.

Proposition 2.3.2. *For a given $\hat{\theta}_v$, the lower is the aggregate risk (σ_p), the higher is the expected gross payment $\Lambda_V(\hat{\theta}_v, \sigma_p)$ and the lower is the expected monitoring cost $\gamma \Omega_V(\hat{\theta}_v, \sigma_p)$, both relative to the expected income in each state. Furthermore, the $\lambda_v(\hat{\theta}_v, \sigma_p)$ decreases when the aggregate uncertainty is lower.*

Proof. See Appendix D. □

With the previous results, it is possible to find the effect of reducing the output fluctuation on the risk premium function:

Proposition 2.3.3. *For a given cut off point, the lower is the aggregate risk (σ_p), the lower is the risk premium: $\varepsilon_V(\hat{\theta}_v, \sigma_p^A) < \varepsilon_V(\hat{\theta}_v, \sigma_p^B)$ if $\sigma_p^B > \sigma_p^A$.*

Proof. (The arguments of the functions are omitted for simplicity) If the Prosposition is true, then $\left[\varepsilon_V^A - \varepsilon_V^B\right] < 0$. From (2.56)

$$\begin{aligned}\varepsilon_V^A - \varepsilon_V^B &= \frac{\lambda_v^A}{(1-\Lambda_V^A) + \lambda_v^A(\Lambda_V^A - \gamma\Omega_V^A)} - \frac{\lambda_v^B}{(1-\Lambda_V^B) + \lambda_v^B(\Lambda_V^B - \gamma\Omega_V^B)} \\ &= \frac{\lambda_v^A\left[\left(1-\Lambda_V^B\right) + \lambda_v^B\left(\Lambda_V^B - \gamma\Omega_V^B\right)\right] - \lambda_v^B\left[\left(1-\Lambda_V^A\right) + \lambda_v^A\left(\Lambda_V^A - \gamma\Omega_V^A\right)\right]}{\left[(1-\Lambda_V^A) + \lambda_v^A(\Lambda_V^A - \gamma\Omega_V^A)\right]\left[(1-\Lambda_V^B) + \lambda_v^B(\Lambda_V^B - \gamma\Omega_V^B)\right]}.\end{aligned} \tag{2.67}$$

Since the denominator of (2.67) is always positive, the sign of the ratio is determined by the numerator,

$$\begin{aligned}sign\left(\varepsilon_V^A - \varepsilon_V^B\right) = sign\Big\{&\lambda_v^A\left[\left(1-\Lambda_V^B\right) + \lambda_v^B\left(\Lambda_V^B - \gamma\Omega_V^B\right)\right] \\ &- \lambda_v^B\left[\left(1-\Lambda_V^A\right) + \lambda_v^A\left(\Lambda_V^A - \gamma\Omega_V^A\right)\right]\Big\}.\end{aligned}$$

After adding and subtracting $\lambda_v^B\Lambda_V^B$ to the numerator, collect terms to show

$$\begin{aligned}sign\left(\varepsilon_V^A - \varepsilon_V^B\right) = sign\Big\{&\left(\lambda_v^A - \lambda_v^B\right)\left(1-\Lambda_V^B\right) + \left(\lambda_v^A - 1\right)\lambda_v^B\left(\Lambda_V^B - \Lambda_V^A\right) \\ &+ \gamma\lambda_v^A\lambda_v^B\left(\Omega_V^A - \Omega_V^B\right)\Big\}.\end{aligned}$$

From Proposition 2.3.2, we know that, for a given $\hat{\theta}_v$, $\left(\lambda_v^A - \lambda_v^B\right) < 0$, $\left(\Lambda_V^B - \Lambda_V^A\right) < 0$, and $\left(\Omega_V^A - \Omega_V^B\right) < 0$. Given that $\lambda_v > 1$ and $(1-\Lambda_V) > 0$ for any σ_p, the $sign\left(\varepsilon_V^A - \varepsilon_V^B\right)$ is always negative when $\sigma_p^B > \sigma_p^A$. In words, the reduction in aggregate

volatility lowers the risk premium. □

Therefore, the function $\varepsilon_V(\hat{\theta}_v, \sigma_p^B)$ will also shift down as the aggregate volatility decreases.

Although a reduction in the volatility lowers both sides of (2.61), reaching a new equilibrium at a lower risk premium, the effect on the optimal break-even point is ambiguous.[23] As is shown in Figure 2.3, the final effect on $\hat{\theta}_v^*$ depends on the size of the shift of each curve. That is, if the expected return on investment drops more than the risk premium, the producer chooses a relatively lower cutoff point that decreases the risk premium (Figure 2.3a). Conversely, when the risk premium decreases more than the return on investment, the producer can choose a higher cutoff point to close the gap (Figure 2.3b). The last case might be a possible result, since the value at which the producer's expected payment reaches a global maximum is higher when the aggregate risk is lower.[24]

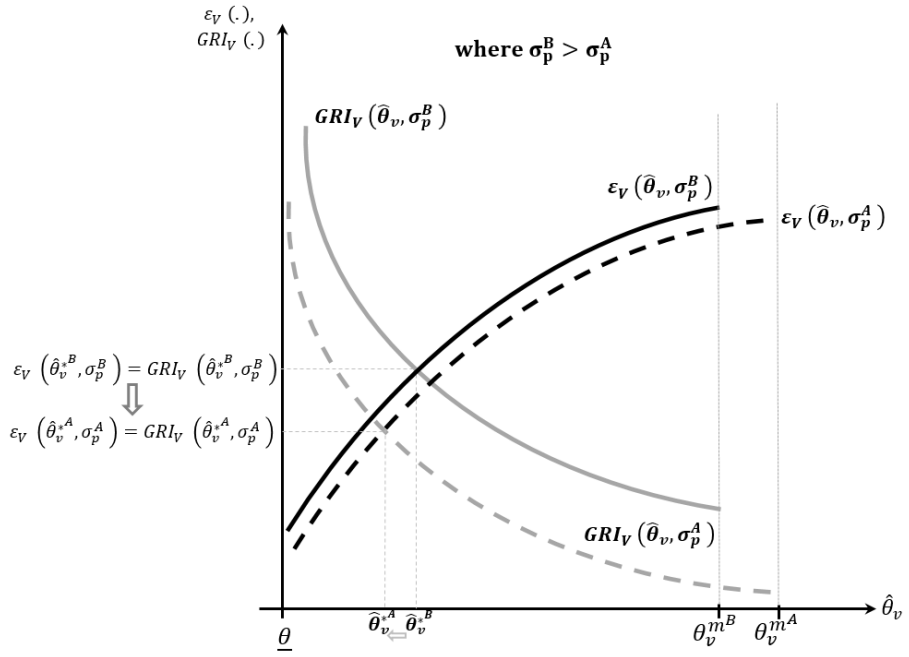
Even though one would expect that $\Delta\varepsilon_V(\hat{\theta}_v, \sigma_p) > \Delta GRI_V(\hat{\theta}_v, \sigma_p)$, nothing guarantees this outcome. If both functions change in the same proportion, the optimal cutoff point will be unaffected. In any case, the resulting optimal value of $\hat{\theta}_v^{*^A}$ will not affect the main result, because the risk premium is always lower in an economy with a lower σ_p.

[23]It is possible to show that the conditions to apply the implicit function theorem in $F(\hat{\theta}_v^*, \sigma_p) = \varepsilon_V(\hat{\theta}_v^*, \sigma_p) - GRI_V(\hat{\theta}_v^*, \sigma_p) = 0$ are satisfied, including $F'_{\hat{\theta}_v^*} \neq 0$.

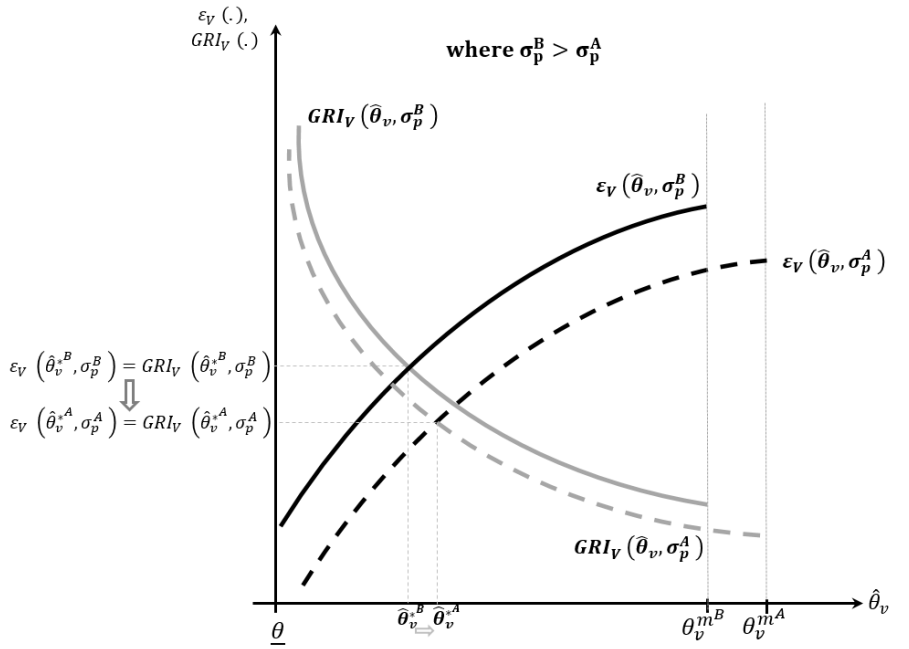
[24]The global maximum of $[\Lambda_V(\hat{\theta}_v, \sigma_p^B) - \gamma\Omega_V(\hat{\theta}_v, \sigma_p^B)]$ is reached when $\Lambda'_V(\theta_v^{m^B}, \sigma_p^B) = \gamma\Omega'_V(\theta_v^{m^B}, \sigma_p^B)$. Given that $\partial\Lambda'_V/\partial\hat{\theta}_v < 0$ and $\partial\Omega'_V/\partial\hat{\theta}_v > 0$, and that Λ'_V increases and Ω'_V decreases when the aggregate risk declines (see Appendix D, Claim D.3), then, the new maximizer must satisfy $\theta_v^{m^A} > \theta_v^{m^B}$.

Figure 2.3: Aggregate Risk Reduction: Relative Changes in ε_V and GRI_V

(a) If $\Delta\varepsilon_V(\hat{\theta}_v, \sigma_p) < \Delta GRI_V(\hat{\theta}_v, \sigma_p)$



(b) If $\Delta\varepsilon_V(\hat{\theta}_v, \sigma_p) > \Delta GRI_V(\hat{\theta}_v, \sigma_p)$



Finally, if the steady state level of investment before and after the reduction in

aggregate volatility are compared for the same R^f and k_0, we get

$$l_v^{*^B} = \left[\frac{\alpha\delta E\left[A(S)\right]}{\varepsilon_V(\hat{\theta}_v^{*^B}, \sigma_p^B)R^f}\right]^{\frac{1}{1-\alpha\delta}} < \left[\frac{\alpha\delta E\left[A(S)\right]}{\varepsilon_V(\hat{\theta}_v^{*^A}, \sigma_p^A)R^f}\right]^{\frac{1}{1-\alpha\delta}} = l_v^{*^A}, \tag{2.68}$$

as $\varepsilon_V(\hat{\theta}_v^{*^A}, \sigma_p^A) < \varepsilon_V(\hat{\theta}_v^{*^B}, \sigma_p^B)$, regardless of $\hat{\theta}_v^{*^A} \lessgtr \hat{\theta}_v^{*^B}$.

The previous finding is summarized in the following statement for an interior solution:

Result 3. *Everything else equal, a decrease (increase) in aggregate risk (σ_p) will reduce (increase) the risk premium (ε_V), increasing (decreasing) the equilibrium level of loans to the private sector (l_v^*), which is equivalent to saying that investment will be higher (lower).*

Since $B_v^{d*} = d_v^* - l_v^*$, the following Corollary holds:

Corollary 1. *Given the assumptions in this section, the demand for government bonds in steady state (B_v^{d*}) goes up as the aggregate volatility increases and the current state is high ($S_t = H$). However, if $S_t = L$, the change in B_v^{d*} is ambiguous, and it depends on the relative drop of deposits vis-à-vis the change of loans to the private sector.*

In this model, the government bond is a risk-free asset that does not require monitoring, thus it is a natural refuge in a riskier environment as the aggregate volatility increases. However, government bonds in emerging economies are subject to aggregate risk. If the return of government bonds were affected by σ_p or σ_y, the financial intermediary would have to allocate funds between positively correlated risky assets without any safety choice. This situation might be solved by adding an external risk-free asset, however, this requires opening the economy at a cost of increasing the complexity of the model. As will be mentioned in the conclusion, opening the economy is a plausible extension for this framework.

Combining Results 1 and 3, and substitution (2.58) in equation (2.59), it is possible to show that output volatility (σ_y) is a nonmonotonic function with respect to the aggregate risk parameter. For instance, a reduction in σ_p tends to decrease the output volatility, but it simultaneously boosts private investment, which tends to increase the output fluctuations. Formally,

$$\frac{\partial \sigma_y}{\partial \sigma_p} = \Pi \left[\frac{\alpha\delta E\left[A(S)\right]}{\varepsilon_V(\hat{\theta}_v^*, \sigma_p) R^f} \right]^{\frac{\alpha\delta}{1-\alpha\delta}} \lessgtr 0, \tag{2.69}$$

where $\Pi = 1 - \sigma_p \left(\frac{\alpha\delta}{1-\alpha\delta}\right) \left[\frac{\partial \varepsilon_V(\cdot)/\partial \sigma_p}{\varepsilon_V(\cdot)}\right]$. Since the second factor of (2.69) is always positive, the effect of a change in aggregate risk will depend on the $sign(\Pi)$. Let's define the elasticity of the risk premium with respect to the aggregate risk:

$$\xi_{\varepsilon_V, \sigma_p} = \frac{\sigma_p}{\varepsilon_V(\cdot)} \left[\frac{\partial \varepsilon_V(\cdot)}{\partial \sigma_p} \right]. \tag{2.70}$$

Therefore, Π can be written as

$$\Pi = 1 - \xi_{\varepsilon_V, \sigma_p} \left[\frac{\alpha\delta}{1-\alpha\delta} \right]. \tag{2.71}$$

The sign of (2.71) depends of the elasticity of the risk premium and the value of ($\alpha\delta$). The latter is the investment rate (l/y) weighted by the marginal product of investment (MPl), which is usually lower or equal than 1/2 for reasonable values of the capital share in total income (δ) and the curvature of the production function of capital goods (α). Then the expression $\left[\frac{\alpha\delta}{1-\alpha\delta}\right]$ is usually less or equal to one. From Result 3, $\partial \varepsilon_V(\cdot)/\partial \sigma_p > 0$, thus, $\xi_{\varepsilon_V, \sigma_p}$ is always positive. Given these results, three cases can be analyzed:

(i) If $\xi_{\varepsilon_V, \sigma_p} \left[\frac{\alpha\delta}{1-\alpha\delta}\right] < 1$, then (2.69) is always positive, $\frac{\partial \sigma_y}{\partial \sigma_p} > 0$. That is, an increase (decrease) in the aggregate risk, increases (reduces) the output volatility, but

at a lower amount due to the reduction (increase) in the aggregate investment level.

(ii) If $\xi_{\varepsilon_V,\sigma_p}\left[\frac{\alpha\delta}{1-\alpha\delta}\right] = 1$, then $\frac{\partial\sigma_y}{\partial\sigma_p} = 0$, which means that the effect of the aggregate risk on output is perfectly compensated by the change in investment.

(iii) Finally, if $\xi_{\varepsilon_V,\sigma_p}\left[\frac{\alpha\delta}{1-\alpha\delta}\right] > 1$, then (2.69) is always negative, $\frac{\partial\sigma_y}{\partial\sigma_p} < 0$: the effect of the aggregate risk on output volatility is more than compensated by the change in investment.

In general $\frac{\alpha\delta}{1-\alpha\delta} \leq 1$, so that cases (ii) and (iii) are only possible for high values of $\xi_{\varepsilon_V,\sigma_p}$. In other words, the more sensitive is the risk premium to changes in the aggregate risk, the lower is the direct effect of the latter on the output volatility. For relatively high values of elasticity, significant adjustments of the risk premium would lead to changes in the investment levels that more than compensate for the effects of aggregate uncertainty on output volatility, resulting in counterintuitive results. Appendix E shows different combinations of elasticities ($\xi_{\varepsilon_V,\sigma_p}$), α, and δ to check for the range of values for these variables where cases (ii) and (iii) are possible.

Since the possible effects of aggregate risk on optimal investment and output volatility are defined, Result 1 can be replaced by

Result 4. *In this framework, changes in the aggregate risk (σ_p) do not translate one-to-one into the output volatility (σ_y), since they are partially compensated by their effect on the investment level (l_v^*). For significantly high values of the risk-premium elasticity with respect to the aggregate risk, the change in investment more than compensates the direct impact of the change in aggregate uncertainty on output volatility, resulting in a null effect or in an opposite-direction variation of σ_y.*

2.3.2 Financial Depth after a Change in Aggregate Volatility

When the aggregate risk or TFP volatility decreases, keeping constant the $E[A(S)]$, the financial depth indicator (2.60) is affected by two forces. From Result 3, as the risk premium decreases, the amount of loans to the private sector increases, which tends to raise the FD indicator. On the other hand, as is stated in Result 2, the relative change in the current productivity shock generates an ambiguous effect on FD. Continuing with the previous exercise, the financial depth indicator in steady state (for a given R^f) in the economies B and A are

$$FD_v^{*^B} = \left[\frac{\alpha\delta}{\varepsilon_V(\hat{\theta}_v^{*^B}, \sigma_p^B)R^f}\right]\left[\frac{E\left[A(S)\right]}{A(S^b)}\right] \tag{2.72}$$

and

$$FD_v^{*^A} = \left[\frac{\alpha\delta}{\varepsilon_V(\hat{\theta}_v^{*^A}, \sigma_p^A)R^f}\right]\left[\frac{E\left[A(S)\right]}{A(S^a)}\right]. \tag{2.73}$$

By combining (2.72) and (2.73), a reduction in the aggregate risk will increase the financial depth if and only if:

$$\frac{\varepsilon_V(\hat{\theta}_v^{*^B}, \sigma_p^B)}{\varepsilon_V(\hat{\theta}_v^{*^A}, \sigma_p^A)} > \frac{A(S^a)}{A(S^b)}. \tag{2.74}$$

However, when volatility goes down keeping constant the mean, it is like a mean-preserving shift of the two possible productivity shocks. If $\sigma_p^B > \sigma_p^A$ with $E[A(S^b)] = E[A(S^a)]$, it must be the case that with equal probabilities

$$A(H^b) > A(H^a) > A(L^a) > A(L^b) > 0. \tag{2.75}$$

If $\sigma_p^B > \sigma_p^A$, then $\frac{\varepsilon_V(\hat{\theta}_v^{*^B}, \sigma_p^B)}{\varepsilon_V(\hat{\theta}_v^{*^A}, \sigma_p^A)} > 1$ (Result 3). Therefore, condition (2.74) is automatically satisfied in the high state ($S = H$), because $\frac{A(H^a)}{A(H^b)} < 1$. But when the aggregate

state is low ($S = L$), condition (2.74) does not automatically hold, since $\frac{A(L^a)}{A(L^b)} > 1$. In sum, one can summarize the previous findings as follows

Result 5. *Everything else equal, and under the assumption of this model, a decrease in aggregate risk (σ_p) will "automatically" increase the financial depth indicator (FD) in the high state ($S = H$). However, in the low state ($S = L$), FD increases if condition (2.74) holds. That is, when the relative reduction in the risk premium (ε_V) more than compensate the relative increase in aggregate productivity during bad times.*

Notice that if the financial depth indicator is calculated with respect to the expected output, from simplifying (2.72) and (2.73), it is straightforward to show that

Result 6. *Everything else equal, a decrease in the aggregate risk (σ_p) will always increase the financial depth indicator (FD) when is calculated with respect to the expected output, given the reduction in the risk premium (Result 3).*

2.4 Conclusions and Possible Extensions

This chapter has been devoted to analyze different effects of aggregate risk (TFP shocks) on output volatility, measured by the standard deviation of output, and the financial depth, as poxied by the credits to private sector relative to output. Several empirical works study how the increase in financial development can reduce the output volatility by using different estimation techniques. However, I have followed the approach of a different strand of theoretical and empirical literature, where high volatility is the result of a concentrated production patterns, and is ex-ante independent of the financial system.

Through a simplified theoretical model, I find that a mean preserving change in the aggregate productivity shocks can affect financial depth. That is, a reduction (increase) in the aggregate risk not only reduces (raises) the output volatility for reasonable values of the risk-premium elasticity with respect to aggregate uncertainty

(see Result 4), but it may increase (reduce) the financial depth indicator (Results 3, 5, and 6). These findings do not rule out the possibility that, ex-post, a deeper financial system can help in reducing the macro fluctuations, but they show that a shallow financial system is a by-product of the aggregate volatility. Therefore, it must be the case that for some emerging countries with high and persistent aggregate volatility, the development of a deeper financial system is restricted by the real volatility. Since the change in production patterns and technologies require time and resources, policy makers should implement more countercyclical macroeconomic policies in the meantime to lessen the aggregate fluctuations and trigger the beneficial effects on the financial system.

From a theoretical perspective, the model is flexible enough to introduce additional features of developing economies. For example, one might explore different policy interventions, such as an exogenous policy for government spending. Following with the public sector, it would be interesting to relax the assumption of a perfectly elastic supply of bonds, endogenizing the risk-free rate. Another possible extension is to open the economy by allowing domestic agents to borrow from the global financial system. If there is an external risk-free asset, the domestic government bonds might be affected by the aggregate volatility. Given the difficulties to get credit in some emerging countries, capital-producers could also use a mix of self-funding and loans to finance investment opportunities.

From an empirical point of view, one should find evidence that supports Corollary 1, analyzing the exposure of the domestic financial system to government debt as the macroeconomic volatility increases/decreases. The relationship between aggregate volatlity and financial depth can be estimated through a panel VAR to account for possible interactions. Finally, the level of concentration of domestic production could be introduced in the analysis by using a Herfindahl-Hirschman index to test theoretical predictions.

Further analysis is required to answer the question of how high and persistent macroeconomic fluctuations may hinder the financial deepening in economies affected by intrinsic volatility volatility originating in the real sectors. This study contributes by uncovering some economic channels to address this important question.

Chapter 3

Cyclicality of Fiscal Policy in Developing Countries: Magnitude and Evolution

In the first chapter, the welfare analysis of macroeconomic volatility was motivated by the original work of Lucas (1987). Recall the corollary of his finding: Since the gains of eliminating the real volatility in the US economy (after WWII) are negligible, more countercyclical policies than those already applied, would be unnecessary. However, as was discussed in Chapter 1, the welfare effects might be non-negligible under different frameworks and assumptions. Therefore, the question of whether countercyclical macroeconomic policies have been actually applied or not in developing countries in last decades is a subject of debate. In this chapter, I study some general cyclical properties of the fiscal policy from the expenditure side in different developing regions and industrialized countries, following Calderón, Chuhan-Pole, and López-Monti (2017). Although our original study focuses in Sub-Saharan African countries, the region with the highest volatility of per-capita GDP growth (see Figure 1.1), a comparative analysis of the fiscal stance in different regions is useful to answer the question of whether more countercyclical policies are needed. To check the robustness of main findings, in section 3.4, I extend the analysis to account for possible global factors.

There is evidence that industrialized countries have the ability to conduct countercyclical fiscal policies (Melitz 2000; Galí and Perotti 2003). Automatic stabilizers play an important role in these economies. Countercyclical fiscal policies in advanced countries are triggered by automatic increases in social security and welfare spending as the economy moves into a recession. Indeed, Darby and Melitz (2008) find that, on average, a reduction of one percentage point in the rate of growth is compensated by a 0.36 percentage point increase in social expenditures among OECD economies.

Furthermore, McKay and Reis (2016) show that well-targeted transfers in the US (without distorting incentives to work) help to stabilize the economy. On the other hand, earlier research suggests that fiscal policy in developing countries, and particularly in Latin America, has been procyclical (Gavin and Perotti 1997; Talvi and Végh 2005; Kaminsky et al. 2004; Ilzetzki and Végh 2008). Historically, developing country governments have cut taxes and increased spending during booms and they have retrenched outlays and raised taxes during busts. According to the literature, this destabilizing behavior is attributed to either: (a) limited access to (external) funding or (b) political economy distortions.

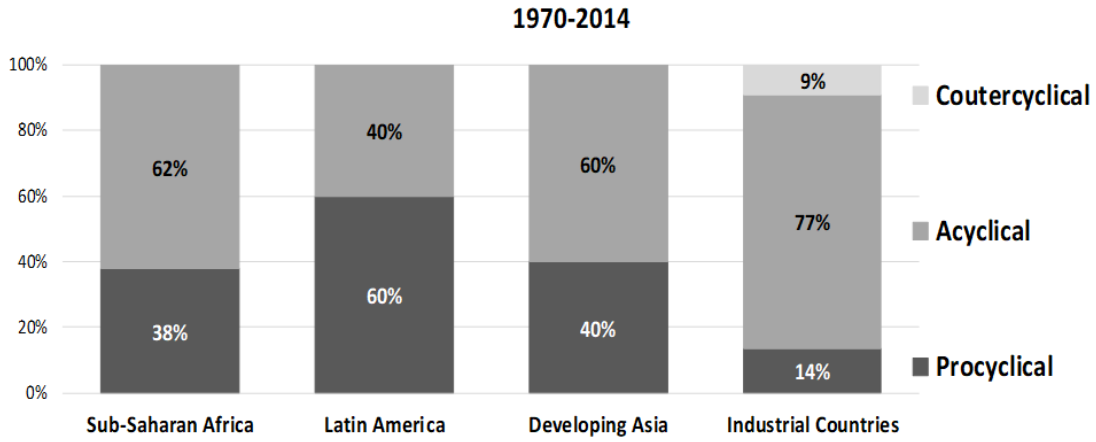
3.1 Model and Empirical Strategy

In order to examine the cyclical stance of fiscal policy, Calderón, Chuhan-Pole, and López-Monti (2017) focus on the real government consumption. The public investment (or development expenditure) is not included to analyze the stance of fiscal policy, as it is in Blanchard and Giavazzi (2004), Buiter and Grafe (2004), and Fatás (2005). In general, government investment is perceived as being quite different from consumption expenditures. First, investment decisions generally have a multi-year horizon (with very few shovel ready projects), so the reaction to the business cycle might not be contemporaneous and might potentially affect the government budget in several periods. Second, the benefits resulting from public investment (for example, services) are spread over many years, with the possibility to increase private output and tax revenues at later days. Furthermore, discretionary and non-discretionary (automatic stabilizers) components of government expenditure are not distinguished, because what really matters is the aggregate response of government consumption to the cycle, as opposed to identifying the source. Unlike most papers in the literature, the government consumption expenditure is not expressed as a percentage of GDP,

given that this ratio might be directly affected by the cyclical behavior of the GDP.[1]

Before defining the policy reaction function, Figure 3.1 shows the simple correlation coefficient between the cyclical components of real government consumption and real GDP from 1970 to 2014. The stance of government consumption along the cycle is defined as follows: government consumption expenditure is countercyclical (procyclical) if the correlation coefficient between the cyclical components of real government consumption expenditure and real GDP is negative (positive) and statistically significant at 5 percent. It is acyclical if that correlation is not statistically different from zero. Any developing regions exhibits countercyclical fiscal policies from the expenditure side. Latin American countries were more procyclical than in other developing regions. By contrast, 77 percent of industrial countries exhibited an acyclical stance and 9 percent conducted countercyclical policies.

Figure 3.1: Cyclical Properties of Government Consumption Expenditure: Proportion of Countries by Groups



Source: Calderón, Chuhan-Pole, and López-Monti (2017).
Note: The government final consumption expenditure and GDP are expressed in local currency at constant prices (source: UN-Statistics database). Their cyclical components are obtained using the Hodrick-Prescott filter. Countercyclical government consumption means that the correlation coefficient is negative and statistically significant at 5 percent level. It is procyclical when it is positive and significant, and acyclical when the coefficient is not statistically different from zero.

After looking at the correlation coefficients, the following policy reaction func-

[1]See discussion in Kaminsky et al. (2004), and Ilzetzki and Végh (2008)

tion is proposed to capture the cyclical behavior of fiscal policy from the (current) expenditure side:

$$g_{it}^{c} = \alpha_{0G} + \alpha_{1G} y_{it}^{c} + \alpha_{2G} g_{it-1}^{c} + \alpha_{3G} D_{it-1} + \varepsilon_{it}, \tag{3.1}$$

where g_{it}^{c} is the cyclical component of the log of government consumption expenditure (constant national currency) in country i at time t, y_{it}^{c} is the cyclical component of the real GDP in logs (constant national currency) in country i at time t. Additionally, the lagged stock of the general government gross debt as percentage of GDP (D_{it-1}) in each country and the lagged level of the cyclical component of the government consumption are added as control variables. Note that the regression coefficients of the policy reaction function (3.1) are initially assumed to vary across groups, country, and over time: i.e. α_{iG} for $i = 0, 1, 2, 3$ and G being a particular group of countries, such as Sub-Saharan Africa, Latin America, developing Asia, and industrial countries. The coefficient α_{1G} captures how the cyclical component of government consumption expenditure responds to the output cycles. This coefficient can be positive (procyclical), negative (countercyclical), or not statistically different from zero (acyclical).

The policy reaction function (3.1) is estimated through the pooled OLS and fixed-effect regressions. Since the deviations from a potential trend is used, the means of the cyclical components, g_{it}^{c} and y_{it}^{c}, tend to zero. Therefore, the pooled OLS and fixed-effect techniques will give similar coefficients, because the "within" estimation uses deviations from the country means. In order to account for possible endogeneity between the fiscal policy and GDP, an IV/2SLS method is used to estimate the cyclicality coefficient (α_1). Following the approach used in Galí and Perotti (2003), Lane (2003), Jaimovich and Panizza (2007), and Calderón and Nguyen (2016), the cyclical component of real GDP (y_{it}^{c}) is instrumented using its own lags and the weighted av-

erage of the GDP growth of each country's export partners. The idea is that external trade shocks do not affect government spending except through the business cycle channel. Similarly, there is no reason to think that government spending in small economies directly affects the growth rates of their trading partners. In the case of large economies, however, fiscal policy might affect the growth rates of trading partners.

Next, some specifications are considered to analyzed the properties of the cyclical stance of government consumption in different regions [2]:

(i) *Baseline Specification (Case 1)*: direct estimation of the policy reaction function (3.1).

(ii) *Asymmetric behavior along the business cycle (Case 2)*: the behavior of government consumption expenditure might be different along the cycle. The original policy reaction function is modified to account for possible asymmetries during expansions (above the trend) and recessions (below the trend):

$$g_{it}^{c} = \alpha_{0G} + \alpha_{1G} y_{it}^{c} + \alpha_{2G} DC_{it}^{+} y_{it}^{c} + \alpha_{3G} g_{it-1}^{c} + \alpha_{4G} D_{it-1} + \varepsilon_{it}, \tag{3.2}$$

where the dummy variable is defined as follows

$$DC_{it}^{+} = \begin{cases} 1 & \text{if } y_{it}^{c} > 0, \quad \text{above the trend} \\ 0 & \text{otherwise}, \quad \text{below the trend.} \end{cases} \tag{3.3}$$

In this context, the coefficient α_{1G} is the cyclical behavior during recessions while $(\alpha_{1G} + \alpha_{2G})$ is the behavior during expansions. However, to estimate IV/2SLS, the sample is split such that during the expansion phase of the cycle

[2] In Calderón, Chuhan-Pole, and López-Monti (2017), we analyze more cases to check how the cyclical stance of fiscal policy can be affected by the rent from natural resources, the foreign aid, and a fragile political environment.

the specification is

$$g_{it}^{c} = \alpha_{0G}^{+} + \alpha_{1G}^{+} y_{it}^{c} + \alpha_{2G}^{+} g_{it-1}^{c} + \alpha_{3G}^{+} D_{it-1} + \varepsilon_{it}, \quad \text{if } y_{it}^{c} > 0, \tag{3.4}$$

and during recessions

$$g_{it}^{c} = \alpha_{0G}^{-} + \alpha_{1G}^{-} y_{it}^{c} + \alpha_{2G}^{-} g_{it-1}^{c} + \alpha_{3G}^{-} D_{it-1} + \varepsilon_{it}, \quad \text{if } y_{it}^{c} < 0. \tag{3.5}$$

Specification (3.2) assumes heterogeneity in the coefficient associated to the business cycles, whereas in (3.4) and (3.5) a full heterogeneity of the fiscal policy reaction function parameters are assumed.

(iii) *Access to external borrowing (Case 3)*: It is argued that the procyclical bias of fiscal policy, and in particular of government spending, among developing countries might be attributed to their procyclical access to global capital markets. At the same time, access to global capital markets might be directly related to the extent of international financial integration of the country. Hence, we define countries with low (high) degree of financial openness as those countries where the average value of the Chinn-Ito index of financial openness is below (above) the world median by decade.[3] Similar treatment as before is performed, that is, for the least squares and fixed effect estimations, a dummy variable is added ($Fina.Open = 1$ for a high degree of financial openness), while the sample is split between high and low financial openness.

(iv) *Time-varying parameters (Case 4)*: Instead of assuming a particular year as the turning point, in Calderón, Chuhan-Pole, and López-Monti (2017) we conduct a test to detect endogenously whether there is a time break in the fiscal policy reaction function. We use a Wald-type test statistic for changes in the vector

[3] After 2000, an average and median for 2000-2014 are taken.

of parameters at unknown break points in the spirit of the test developed by Andrews (1993). Particularly, we recursively test the joint equality of the vector of coefficient estimates in the periods pre- and post-break. After running all regressions, the turning point (year) corresponds to the regression with the lowest root-mean-square error (RMSE). As can be seen in Figure 3.2, different breaking points (TP_G) are found in each developing region, providing evidence in favor of a structural break in the cyclical stance for different developing countries. Since the focues is placed on developing regions, possible breaking point (year) in industrial countries are not considered.[4] Hence, for the least squares estimates, the following specification is analyzed

$$g_{it}^c = \alpha_{0G} + \alpha_{1G} y_{it}^c + \alpha_{2G} Break_{it}^{TP} y_{it}^c + \alpha_{3G} g_{it-1}^c + \alpha_{4G} D_{it-1} + \varepsilon_{it}, \tag{3.6}$$

where the dummy variable is defined as follows

$$Break_{it}^{TP} = \begin{cases} 1 & \text{if } t \geq TP_G, \quad \text{with } TP_G \in 1985, 2014 \\ 0 & \text{otherwise.} \end{cases} \tag{3.7}$$

Again, the coefficient α_{1G} represents the cyclical behavior BEFORE the turning point and $(\alpha_{1G} + \alpha_{2G})$ the behavior AFTER that year.

Analogous analysis to (3.4) and (3.5) is performed for the IV/2SLS estimate before the breaking point found in each developing region:

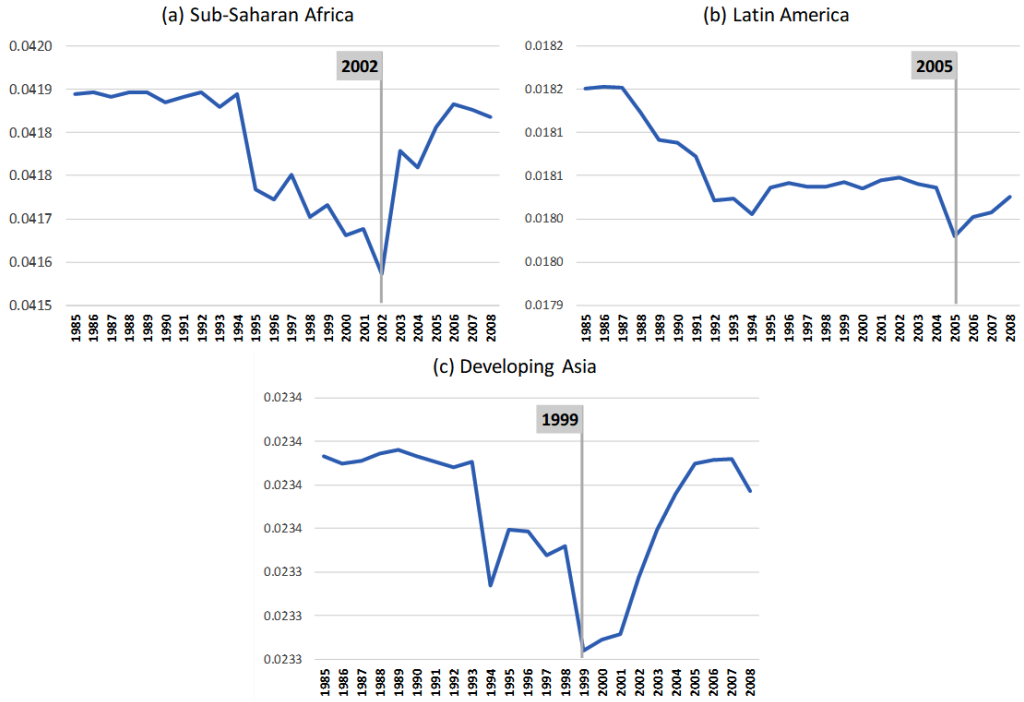
$$g_{it}^c = \alpha_{0G}^{P1} + \alpha_{1G}^{P1} y_{it}^c + \alpha_{2G}^{P1} g_{it-1}^c + \alpha_{3G}^{+P1} D_{it-1} + \varepsilon_{it}, \quad \text{if } t < TP_G, \tag{3.8}$$

[4] If a similar test is performed for industrialized countries, the breaking point (year) would be in the late seventies. However, this would leave a small sample period for regressions before the turning point from 1970.

and after the breaking point

$$g_{it}^{c} = \alpha_{0G}^{P2} + \alpha_{1G}^{P2} y_{it}^{c} + \alpha_{2G}^{P2} g_{it-1}^{c} + \alpha_{3G}^{P2} D_{it-1} + \varepsilon_{it}, \quad \text{if } t \geq TP_G. \tag{3.9}$$

Figure 3.2: Root Mean Square Errors from Multiple Panel Regressions with Different Breaking Points (years)



Source: Calderón, Chuhan-Pole, and López-Monti (2017)
Note: The unknown turning point (year) of fiscal cyclicality in each region is found through running multiple panel regressions with country fixed effect, testing the possible breaking points (dummies) from 1985 to 2008. To assess the performance of each regression (year), the Root-Mean-Square Error (RMSE) is stored. After running all regressions, the turning point (year) corresponds to the regression with the lowest RMSE.

3.2 Data

The sample includes 37 Sub-Saharan African countries (SSA), 20 countries from Latin America (LAC), 20 from developing Asia, and 22 industrial economies (see Appendix F).[5] Annual data from 1970 to 2014 are used in the analysis. The government

[5]Given data availability from 1970, countries included in each group may differ from those analyzed in Chapter 1.

consumption expenditure and real GDP in constant national currency are from the United Nations' National Accounts Database. Although most papers on fiscal cyclicality use the IMF data, the UN statistics provide a more complete and consistent national accounts data for the time period covered. To isolate the cyclical component of the government consumption expenditure and real GDP (both in logs), the HP filter with $\lambda = 6.25$ is used. General government gross debt as a percentage of GDP is from Abbas et al. (2010), updated with the IMF's World Economic Outlook.The instrumental variable is calculated with bilateral export values from the IMF's Direction of Trade Statistics and UN-COMTRADE, while the growth rates of trading partners are from the United Nations's National Accounts Database updated with the World Bank's WDI.

3.3 Summary of Results

3.3.1 Baseline Estimate: Cyclicality of Fiscal Policy

The relationship between the cyclical components of the real government consumption expenditure and real GDP is positive and statistically significant in all developing regions, with the coefficient of cyclicality being larger in Sub-Saharan Africa, but acyclical in industrial countries (Table 3.1). The results are qualitatively similar when the likely endogeneity is considered. Our instrumental variables (IV) estimation shows that government consumption is still procyclical in Sub-Saharan Africa and Latin America, with the former group having the larger coefficient. For developing Asia, the coefficient of GDP cycle is still positive but fails to be statistically significant. When we account for endogeneity, the extent of procyclicality of government consumption in Sub-Saharan Africa is greater and twice as large as that in Latin America.

The lagged debt-to-GDP indicator has a negative coefficient across all regions, implying that higher debt levels tend to constrain the government current expenditures. However, the coefficient is only significant for Latin American and industrial

countries. The coefficient on the lagged cyclical component of government consumption has a positive sign for all country groups, except Sub-Saharan Africa. But after correcting for the potential endogeneity problem, this coefficient is only significant in the group of industrial countries. Since the cyclical component of government consumption expenditure is considered, a negative coefficient for its lagged variable would suggest an oscillating behavior of fiscal policy. That is, when the cyclical component of government consumption is below its trend in the previous period, it is more likely to be above the trend in the current period, suggesting high variability in government spending.

3.3.2 Cyclical Properties of Fiscal Policy during Expansions and Recessions

The least squares estimates show that in developing countries, government consumption tends to be procyclical in both contractions and expansions, while industrial countries are slightly procyclical only in good times (Table 3.2). When accounting for possible endogeneity, the fiscal policy in developing countries tends to be procyclical in expansions, particularly in SSA that accounts for the largest and greater than one coefficient, which means that the fiscal policy in this region overreacts in good times. However, during recessions the SSA is the only region with a positive (but not statistically significant) coefficient. In other regions, the coefficient during bad times is consistent with more countercyclical policies, but only statistically significant for industrial countries. However, these results may suffer from the weak instrument problem in some developing regions for certain periods.[6]

[6]The Kleibergen-Paap-F statistic is relatively low in some developing regions for certain periods.

Table 3.1: Cyclicality of Fiscal Policy by Region. Sample Period 1970-2014 (annual data)

	Dependent Variable: *Cyclical Components of Real Government Consumption Expenditure (GovCons_Cycle)*											
	Sub-Saharan Africa			Latin America			Developing Asia			Industrial Countries		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
VARIABLES	Pooled OLS	Fixed Effects	IV/2SLS	Pooled OLS	Fixed Effects	IV/2SLS	Pooled OLS	Fixed Effects	IV/2SLS	Pooled OLS	Fixed Effects	IV/2SLS
RGDP_Cycle	**0.580*** **	**0.582*** **	**0.769** **	**0.468*** **	**0.467*** **	**0.368*** **	**0.533*** **	**0.532*** **	**0.0378**	**0.0817**	**0.0807**	**-0.00511**
	(0.139)	(0.139)	(0.331)	(0.0931)	(0.0924)	(0.114)	(0.112)	(0.113)	(0.251)	(0.0508)	(0.0503)	(0.0457)
GovCons_Cycle (t-1)	**-0.0305**	**-0.0319**	**-0.0211**	**0.0624***	**0.0636***	**0.0695**	**0.118***	**0.116***	**0.127**	**0.226*** **	**0.225*** **	**0.218*** **
	(0.0488)	(0.0485)	(0.0685)	(0.0371)	(0.0378)	(0.0716)	(0.0647)	(0.0646)	(0.118)	(0.0559)	(0.0560)	(0.0419)
PublicDebt/GDP (t-1)	**-0.00133**	**-0.00154**	**-0.00144**	**-0.00157** **	**-0.00189*** **	**-0.00189***	**-0.00167**	**-0.00231**	**-0.00266**	**-0.000648**	**-0.00106***	**-0.00123***
	(0.00287)	(0.00303)	(0.00406)	(0.000621)	(0.000559)	(0.000996)	(0.00382)	(0.00389)	(0.00433)	(0.000430)	(0.000561)	(0.000746)
Observations	1,450	1,450	1,403	854	854	823	708	708	690	957	957	915
R-squared	0.043	0.0434	0.043	0.088	0.0889	0.084	0.063	0.0625	0.024	0.060	0.0615	0.052
Number of Countries	37	37	37	20	20	20	20	20	20	22	22	22
Kleibergen-Paap rk Wald F statistic			**32.6**			**60.3**			**36.5**			**149.9**
Country FE		YES	YES		YES	YES		YES	YES		YES	YES

Source: Adapted from Calderón, Chuhan-Pole, and López-Monti (2017)

Note: Government Final Consumption Expenditures and GDP in real local currencies filtered using the Hodrick-Prescott Filter. The cyclical component of GDP is instrumented with its own lags and the weighted average of the GDP growth of each country's export partners. Trade-partner growth is weighted by the share of the country's total exports to each of its trading partners, each country's weighted-trade-partner growth is multiplied by the country's average exports/GDP. Constants are not reported. Standard errors in parentheses. Pooled and Fixed-Effect regressions are performed with Driscoll-Kraay standard errors, while IV/2SLS with HAC standard errors.

***significant at 1%, ** significant at 5%, * significant at 10%.

Table 3.2: Cyclical Properties of Fiscal Policy During Expansions and Recessions. Sample Period 1970-2014 (annual data)

Dependent Variable: *Cyclical Components of Real Government Consumption Expenditure (GovCons_Cycle)*

	Sub-Saharan Africa				Latin America				Developing Asia				Industrial Countries			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
	Pooled OLS	Fixed	IV/2SLS		Pooled OLS	Fixed	IV/2SLS		Pooled OLS	Fixed	IV/2SLS		Pooled OLS	Fixed	IV/2SLS	
VARIABLES		Effects	Bad Times	God Times		Effects	Bad Times	God Times		Effects	Bad Times	God Times		Effects	Bad Times	God Times
RGDP_Cycle	**0.699*** **	**0.697*** **	**0.918**	**1.593***	**0.404*** **	**0.409*** **	**-0.0434**	**0.835** **	**0.517** **	**0.489** **	**-1.156**	**0.712***	**0.0190**	**0.0122**	**-0.608*** **	**0.145**
	(0.180)	(0.187)	(0.628)	(0.821)	(0.0898)	(0.0954)	(0.314)	(0.381)	(0.208)	(0.211)	(1.081)	(0.414)	(0.0784)	(0.0734)	(0.122)	(0.119)
DC^{+}x (RGDP_Cycle)	**-0.247**	**-0.240**			**0.138**	**0.123**			**0.0314**	**0.0839**			**0.122**	**0.133**		
	(0.334)	(0.343)			(0.192)	(0.206)			(0.255)	(0.271)			(0.125)	(0.121)		
PublicDebt/GDP (t-1)	**-0.00102**	**-0.00113**	**0.00113**	**-0.00747**	**-0.00155** **	**-0.00186*** **	**-0.00450**	**-0.00123**	**-0.00162**	**-0.00225**	**-0.00109**	**-0.00136**	**-0.000577**	**-0.000939***	**-0.00115**	**0.000288**
	(0.00259)	(0.00272)	(0.00441)	(0.00774)	(0.000620)	(0.000542)	(0.00497)	(0.000996)	(0.00385)	(0.00386)	(0.00790)	(0.00511)	(0.000403)	(0.000530)	(0.00131)	(0.000841)
GovCons_Cylce (t-1)	**-0.0284**	**-0.0296**	**-0.0400**	**-0.0317**	**0.0616**	**0.0627**	**0.00158**	**0.0872**	**0.118***	**0.116***	**0.294** **	**-0.0612**	**0.225*** **	**0.224*** **	**0.220*** **	**0.204*** **
	(0.0494)	(0.0492)	(0.0886)	(0.108)	(0.0371)	(0.0377)	(0.102)	(0.0950)	(0.0648)	(0.0646)	(0.132)	(0.164)	(0.0552)	(0.0554)	(0.0619)	(0.0666)
Memo:																
Cyclical Pattern																
RECESSIONS	**0.699*** **	**0.697*** **	**0.918**		**0.404*** **	**0.409*** **	**-0.0434**		**0.517** **	**0.489** **	**-1.156**		**0.0190**	**0.0122**	**-0.608*** **	
EXPANSIONS	**0.452***	**0.457***		**1.593***	**0.542*** **	**0.532*** **		**0.835** **	**0.548*** **	**0.572*** **		**0.712***	**0.141**	**0.145***		**0.145**
Observations	1,450	1,450	702	701	854	854	399	424	708	708	335	355	957	957	467	448
Number of Countries	37	37	37	37	20	20	20	20	20	20	20	20	22	22	22	22
Kleibergen-Paap rk Wald F statistic			**12.1**	**9.6**			**11.5**	**11.4**			**4.8**	**10.4**			**36.1**	**36.9**
Country FE		YES	YES	YES		YES	YES	YES		YES	YES	YES		YES	YES	YES

Source: Adapted from Calderón, Chuhan-Pole, and López-Monti (2017)

Note: Government Final Consumption Expenditures and GDP in real local currencies filtered using the Hodrick-Prescott Filter. The cyclical component of GDP is instrumented with its own lags and the weighted average of the GDP growth of each country's export partners. Trade-partner growth is weighted by the share of the country's total exports to each of its trading partners, each country's weighted-trade-partner growth is multiplied by the country's average exports/GDP. DC^{+} is the dummy for positive cycles. Constants are not reported. Standard errors in parentheses. Pooled and Fixed-Effect regressions are performed with Driscoll-Kraay standard errors, while IV/2SLS with HAC standard errors.

***significant at 1%, ** significant at 5%, * significant at 10%.

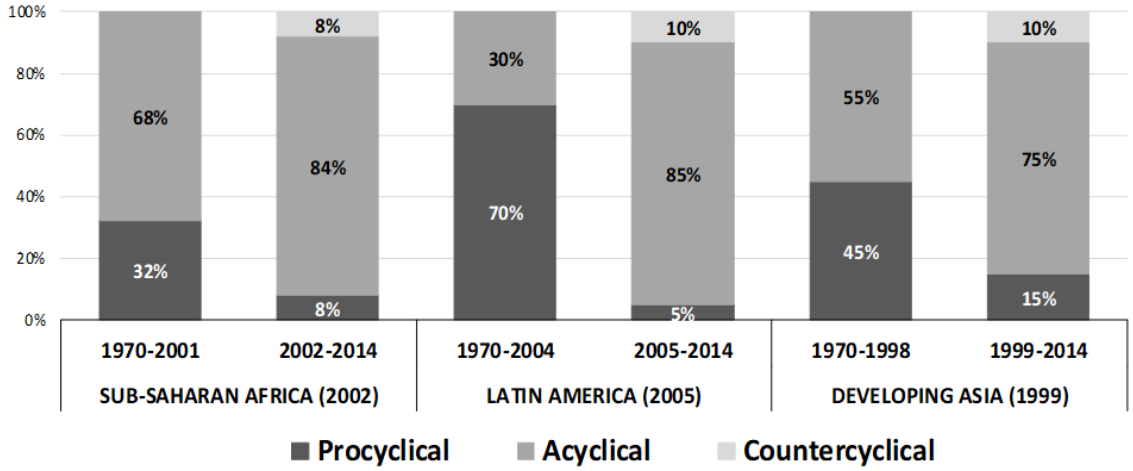
3.3.3 Cyclicality of Fiscal Policy and Financial Openness

Table 3.3 explores the effect of the country's degree of financial openness on the cyclical stance of its fiscal policy. The IV/2SLS estimates show that government consumption expenditure is procyclical in SSA and LAC with low financial openness and acyclical among countries with high financial openness. This finding suggests that the likely access to foreign borrowing during downturns may lower the degree of procyclicality of fiscal policy. For developing Asia and industrial countries, on the other hand, government consumption expenditure is acyclical in countries with both low and high financial openness.

3.3.4 Fiscal Cyclicality Before and After the Turning Point (year) by Region

All Developing regions have made some progress, at least from the expenditure side, towards more acyclical (and countercyclical in some cases) fiscal policies after the breaking points (years) identified in previous section. As is shown in Figure 3.3, there were no countries with negative and statistically significant correlation coefficients (countercyclical fiscal policies) in the pre-break period. This situation has started changing by the end of the nineties. After the turning year, the proportion of countries with countercyclical fiscal policies have increased to 8 percent in Sub-Saharan Africa and 10 percent in both Latin America and developing Asia. At the same time, the proportion of countries with positive and statistically significant correlation coefficient (procyclical fiscal policies) have decreased considerably.

Figure 3.3: Cyclical Properties of Government Consumption Expenditure Before and After the Turning Point (year): Proportion of Countries by Groups



Source: Calderón, Chuhan-Pole, and López-Monti (2017).
Note: The government final consumption expenditure and GDP are expressed in local currency at constant prices (source: UN-Statistics database). Their cyclical components are obtained using the Hodrick-Prescott filter. Countercyclical government consumption means that the correlation coefficient is negative and statistically significant at 5 percent level. It is procyclical when it is positive and significant, and acyclical when the coefficient is not statistically different from zero.

In terms of the policy reaction functions, Table 3.4 shows that all developing regions have moved from procyclical fiscal policies (at least from the expenditure side) to acyclical or countercyclical policies after their turning point (year). Latin America is the only region that has moved to countercyclical fiscal policies after 2005. Sub-Saharan Africa has made important progress since 2002 toward acyclical policies. The reduction in the degree of procyclicality across developing regions has been the result of better fiscal management, particularly during the strong global growth experienced in 2000-08. This period was characterized by high commodity prices and increasing domestic savings, which better prepared many developing countries to face the international financial crisis of 2008.[7]

[7]See Kacef and López-Monti (2010) for a macroeconomic assessment of Latin American countries before the international crisis.

Table 3.3: Cyclicality of Fiscal Policy and Financial Openness. Sample Period 1970-2014 (annual data)

Dependent Variable: *Cyclical Components of Real Government Consumption Expenditure (GovCons_Cycle)*

	Sub-Saharan Africa				Latin America				Developing Asia				Industrial Countries			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
			IV/2SLS				IV/2SLS				IV/2SLS				IV/2SLS	
VARIABLES	Pooled OLS	Fixed Effects	Low Financial Openness	High Financial Openness	Pooled OLS	Fixed Effects	Low Financial Openness	High Financial Openness	Pooled OLS	Fixed Effects	Low Financial Openness	High Financial Openness	Pooled OLS	Fixed Effects	Low Financial Openness	High Financial Openness
RGDP_Cycle	**0.601*** **	**0.604*** **	**0.894** **	**-0.0898**	**0.620*** **	**0.624*** **	**0.524*** **	**0.190**	**0.675*** **	**0.674*** **	**-0.121**	**0.229**	**0.137**	**0.137**	**0.0939**	**-0.0428**
	(0.136)	(0.136)	(0.383)	(0.711)	(0.0827)	(0.0835)	(0.177)	(0.138)	(0.123)	(0.121)	(0.492)	(0.234)	(0.138)	(0.137)	(0.122)	(0.0498)
RGDP_Cycle x Finan.Open	**-0.0719**	**-0.0765**			**-0.298*** **	**-0.309*** **			**-0.338**	**-0.338**			**-0.0783**	**-0.0792**		
	(0.339)	(0.335)			(0.110)	(0.108)			(0.218)	(0.221)			(0.133)	(0.132)		
GovCons_Cycle (t-1)	**-0.0299**	**-0.0313**	**-0.0534**	**0.184**	**0.0576**	**0.0588**	**0.136**	**-0.0271**	**0.116***	**0.114***	**0.0706**	**0.191*** **	**0.224*** **	**0.223*** **	**0.0866**	**0.258*** **
	(0.0490)	(0.0485)	(0.0755)	(0.137)	(0.0380)	(0.0387)	(0.112)	(0.0773)	(0.0640)	(0.0639)	(0.162)	(0.0733)	(0.0537)	(0.0538)	(0.0881)	(0.0493)
PublicDebt/GDP (t-1)	**-0.00136**	**-0.00157**	**-0.00123**	**-0.000890**	**-0.00154** **	**-0.00189*** **	**-0.00279*** **	**-0.00347**	**-0.00172**	**-0.00247**	**-0.00280**	**-0.00527**	**-0.000648**	**-0.00106***	**-0.00312**	**-0.00121**
	(0.00287)	(0.00302)	(0.00356)	(0.00583)	(0.000622)	(0.000568)	(0.00107)	(0.00328)	(0.00384)	(0.00390)	(0.00498)	(0.00859)	(0.000436)	(0.000574)	(0.00371)	(0.000771)
Memo:																
Cyclical Pattern																
Low Financial Openness	**0.601*** **	**0.604*** **	**0.894** **		**0.620*** **	**0.624*** **	**0.524*** **		**0.675*** **	**0.674*** **	**-0.121**		**0.137**	**0.137**	**0.0939**	
High Financial Openness	**0.529**	**0.528**		**-0.0898**	**0.322*** **	**0.315*** **		**0.190**	**0.337***	**0.336***		**0.229**	**0.0587**	**0.0578**		**-0.0428**
Observations	1,450	1,450	1,123	280	854	854	335	487	708	708	449	241	957	957	130	785
Number of Countries	37	37	36	23	20	20	19	19	20	20	20	12	22	22	12	22
Kleibergen-Paap rk Wald F statistic			**28.8**	**7.5**			**24.2**	**37.4**			**17.2**	**18.2**			**27.5**	**124.1**
Country FE		YES	YES	YES		YES	YES	YES		YES	YES	YES		YES	YES	YES

Source: Adapted from Calderón, Chuhan-Pole, and López-Monti (2017)

Note: Government Final Consumption Expenditures and GDP in real local currencies filtered using the Hodrick-Prescott Filter. The cyclical component of GDP is instrumented with its own lags and the weighted average of the GDP growth of each country's export partners. Trade-partner growth is weighted by the share of the country's total exports to each of its trading partners, each country's weighted-trade-partner growth is multiplied by the country's average exports/GDP. Low (high) financial openness is defined when the average value of the Chinn-Ito index of financial openness is below (above) the world median by decade. Constants are not reported. Standard errors in parentheses. Pooled and Fixed-Effect regressions are performed with Driscoll-Kraay standard errors, while IV/2SLS with HAC standard errors.

***significant at 1%, ** significant at 5%, * significant at 10%.

Table 3.4: Fiscal Cyclicality Before and After the Turning Point (year) by Region. Sample Period 1970-2014 (annual data)

	Dependent Variable: *Cyclical Components of Real Government Consumption Expenditure (GovCons_Cycle)*											
	Sub-Saharan Africa (Turning Point = 2002)				**Latin America (Turning Point = 2005)**				**Developing Asia (Turning Point = 1999)**			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
			IV/2SLS				IV/2SLS				IV/2SLS	
VARIABLES	Pooled OLS	Fixed Effects	Before Turning Point	After Turning Point	Pooled OLS	Fixed Effects	Before Turning Point	After Turning Point	Pooled OLS	Fixed Effects	Before Turning Point	After Turning Point
RGDP_Cycle	**0.814*** **	**0.818*** **	**0.922*** **	**0.395**	**0.550*** **	**0.549*** **	**0.546*** **	**-0.447** **	**0.655*** **	**0.655*** **	**0.108**	**-0.127**
	(0.0823)	(0.0836)	(0.292)	(0.849)	(0.0860)	(0.0854)	(0.132)	(0.205)	(0.122)	(0.124)	(0.363)	(0.312)
Break^{TurningPoint}x (RGDP_Cycle)	**-0.866*** **	**-0.876*** **			**-0.692*** **	**-0.696*** **			**-0.501** **	**-0.504***		
	(0.176)	(0.179)			(0.119)	(0.120)			(0.242)	(0.255)		
PublicDebt/GDP (t-1)	**-0.00188**	**-0.00237**	**-0.00216**	**-0.000240**	**-0.00154** **	**-0.00183*** **	**-0.00189***	**0.00444**	**-0.00173**	**-0.00246**	**-0.00240**	**-0.00914**
	(0.00256)	(0.00260)	(0.00348)	(0.00592)	(0.000621)	(0.000570)	(0.00102)	(0.00959)	(0.00382)	(0.00393)	(0.00440)	(0.0127)
GovCons_Cylce (t-1)	**-0.0280**	**-0.0296**	**-0.00946**	**-0.0371**	**0.0622***	**0.0635***	**0.103**	**-0.142**	**0.115***	**0.113***	**0.0563**	**0.199** **
	(0.0487)	(0.0483)	(0.0769)	(0.141)	(0.0366)	(0.0371)	(0.0774)	(0.146)	(0.0633)	(0.0631)	(0.187)	(0.0949)
Memo:												
Cyclical Pattern												
BEFORE Turning Point (year)	**0.814*** **	**0.818*** **	**0.922*** **		**0.573*** **	**0.571*** **	**0.546*** **		**0.643*** **	**0.644*** **	**0.108**	
AFTER Turning Point (year)	**-0.052**	**-0.058**		**0.395**	**-0.142***	**-0.147***		**-0.447** **	**0.154**	**0.151**		**-0.127**
Observations	1,450	1,450	923	480	854	854	623	200	708	708	372	318
Number of Countries	37	37	37	37	20	20	20	20	20	20	20	20
Kleibergen-Paap rk Wald F statistic			**27.4**	**7.5**			**41.3**	**38.6**			**20.4**	**24.9**
Country FE		YES	YES	YES		YES	YES	YES		YES	YES	YES

Source: Adapted from Calderón, Chuhan-Pole, and López-Monti (2017)

Note: Government Final Consumption Expenditures and GDP in real local currencies filtered using the Hodrick-Prescott Filter. The cyclical component of GDP is instrumented with its own lags and the weighted average of the GDP growth of each country's export partners. Trade-partner growth is weighted by the share of the country's total exports to each of its trading partners, each country's weighted-trade-partner growth is multiplied by the country's average exports/GDP. $Break^{TurningPoint}$ is the dummy variable for observations after turning point (year). Constants are not reported. Standard errors in parentheses. Pooled and Fixed-Effect regressions are performed with Driscoll-Kraay standard errors, while IV/2SLS with HAC standard errors.

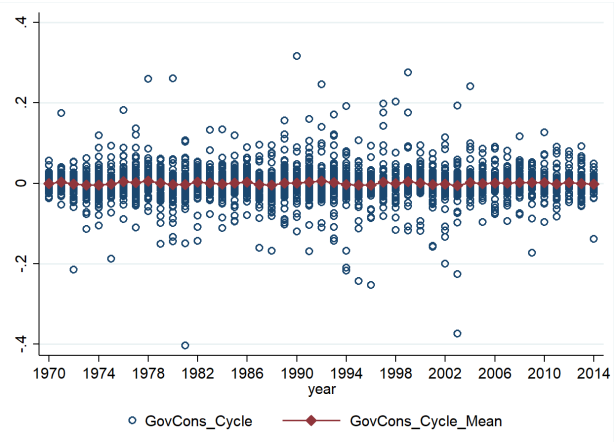
***significant at 1%, ** significant at 5%, * significant at 10% .

3.4 Controlling for Global Factors

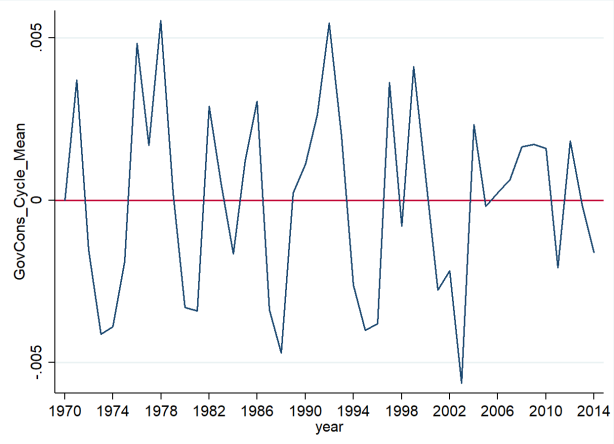
In Calderón, Chuhan-Pole, and López-Monti (2017), we do not consider possible global factors that may affect our estimates and the relevance of instruments used. Global shocks can trigger correlated business cycles across countries, which affect government expenditures and exports. Figure 3.4a shows a heterogeneous distribution of the cyclical component of government consumption (`GovCons_Cycle`) across time, resulting in small variations of its mean (`GovCons_Cycle_Mean`) around zero (Figure 3.4b). In addition to the country fixed effects, in this section, I include year fixed effects to previous specifications to account for possible global factors.

Figure 3.4: Cyclical Component of Government Consumption Across Time.

(a) Heterogeneity Across Years



(b) Annual Average



I summarize in Table 3.5 the least squares and IV/2SLS estimates when global factors are added to check the cyclicality of fiscal policy in each region, its possible asymmetric response during expansions and recessions, the effect of financial openness, and the turning points (years) found in each developing region. The cyclical behavior of government consumption expenditure is reported after controlling for global shocks and the same set of control variables used before, i.e. the lagged stock of general government gross debt over GDP and a lagged level of the cyclical component of government consumption.

The baseline specification (Case 1) for developing countries show similar pattern than the previous results without year fixed effects, that is, SSA and LAC regions registering procyclical fiscal policies, while developing Asia an acyclical behavior when endogeneity is considered. However, the fiscal policy in industrial countries that was classified as acyclical before (see Table 3.1) is now procyclical. Despite this contradicting result, least squares estimates suggest that industrial countries register the lowest level of procyclical fiscal policy.

Without accounting for possible endogeneity, the cyclical component of government consumption expenditures is procyclical in recessions and expansions in all regions (Case 2). Nevertheless, IV estimates show acyclical fiscal policies during recessions and procyclical ones during expansion in LAC and industrial economies, but acyclical in developing Asia along the cycles. In SSA, fiscal policy is procyclical in both recessions and expansions with coefficients greater than one, like LAC in good times, suggesting that government consumption expenditure tends to overreact to business cycles. In any case, IV estimates should be taken with caution when global factors are considered, since these results may suffer from weak instrument problems.

Table 3.5: Cyclical Properties of Fiscal Policy: Controlling for Global Factors. Sample Period 1970-2014 (annual data)

	Dependent Variable: *Cyclical Components of Real Government Consumption Expenditure (GovCons_Cycle)*							
	Sub-Saharan Africa		**Latin America**		**Developing Asia**		**Industrial Countries**	
	(1) Least Squares	(2) IV/2SLS	(3) Least Squares	(4) IV/2SLS	(5) Least Squares	(6) IV/2SLS	(7) Least Squares	(8) IV/2SLS
	Baseline Specification (Case 1): Cyclicality of Fiscal Policy							
RGDP_Cycle	**0.584*** **	**0.825*** **	**0.496*** **	**0.495*** **	**0.583*** **	**-0.0330**	**0.195*** **	**0.222*** **
Kleibergen-Paap rk Wald F statistic		19.8		25.6		15.1		36.4
	Cyclical Properties of Fiscal Policy during Expansions and Recessions (Case 2)							
RECESSIONS	**0.724*** **	**1.078***	**0.455*** **	**0.0483**	**0.502*** **	**-1.731**	**0.220*** **	**-0.0511**
Kleibergen-Paap rk Wald F statistic		6.9		7.2		2.0		15.6
EXPANSIONS	**0.434***	**1.523***	**0.547*** **	**1.048** **	**0.661*** **	**0.541**	**0.1691** **	**0.425** **
Kleibergen-Paap rk Wald F statistic		6.5		6.2		5.6		9.3
	Cyclicality of Fiscal Policy and Financial Openness (Case 3)							
Low Financial Openness	**0.613*** **	**0.961*** **	**0.652*** **	**0.542*** **	**0.706*** **	**-0.143**	**0.232** **	**0.240**
Kleibergen-Paap rk Wald F statistic		15.9		15.9		9.9		2.4
High Financial Openness	**0.5133***	**0.204**	**0.322*** **	**0.549** **	**0.405** **	**0.0903**	**0.174*** **	**0.186** **
Kleibergen-Paap rk Wald F statistic		4.9		12.9		7.0		34.9
	The Cyclical Behaviour of Fiscal Policy before and after the Turning Point (Case 4)							
	(2002)		*(2005)*		*(1999)*		-	
BEFORE Turning Point (year)	**0.828*** **	**0.968*** **	**0.523*** **	**0.584*** **	**0.689*** **	**0.0315**	-	-
Kleibergen-Paap rk Wald F statistic		15.0		26.3		11.1		
AFTER Turning Point (year)	**-0.058**	**0.396**	**0.158**	**-0.286**	**0.233**	**-0.152**	-	-
Kleibergen-Paap rk Wald F statistic		5.6		4.8		6.7		
Country FE	YES	YES	YES	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES	YES	YES	YES

Source: Author's calculations.
Note: Government Final Consumption Expenditures and GDP in real local currencies filtered using the Hodrick-Prescott Filter. The cyclical component of GDP is instrumented with its own lags and the weighted average of the GDP growth of each country's export partners. Trade-partner growth is weighted by the share of the country's total exports to each of its trading partners, each country's weighted-trade-partner growth is multiplied by the country's average exports/GDP. Least Squares estimates with country and year fixed effects are performed with Driscoll-Kraay standard errors, while IV/2SLS with HAC standard errors.
***significant at 1%, ** significant at 5%, * significant at 10%.

Regarding the effect of financial openness on fiscal cyclicality (Case 3), the least squares regressions suggest that the higher is the financial openness with respect to the world median, the lower is the degree of procyclicality in all regions, including advanced economies. When accounting for possible endogeneity, only LAC register

an unchanged procyclical behavior regardless the level of access to foreign borrowing. However, in line with previous findings (Table 3.3), IV estimates suggest that fiscal policy in SSA tends to be acyclical with high financial openness and procyclical with low access to external resources, while in developing Asia is acyclical in both cases. The major change with respect to previous findings is registered in industrial countries, where the likely access to external borrowing switches the policy from acyclical to procyclical. Once more, in some cases, these IV estimates may suffer from the weak instrument problem. Finally, adding year fixed effects does not change the original estimates of fiscal cyclicality before and after the turning point (year) in developing regions (Case 4).

In sum, the main results from previous section do not qualitative change for developing countries after controlling for global shocks in least-squares estimates, but they do for industrial countries. On the other hand, original findings from IV estimates may change in some regions and specifications when year fixed effects are added, but these results may be affected by weak instrument problems.

3.5 Conclusions

The government consumption expenditure has been mainly procyclical in developing countries during the entire period 1970-2014, while acyclical in industrialized economies only when possible global factors are not considered. In terms of the asymmetric behavior during good and bad times, least squares estimates show that in developing countries government consumption tends to be procyclical in both contractions and expansions, while industrial countries are slightly procyclical only in good times. When controlling for global shocks, the latter group becomes procyclical along the business cycle.

However, when accounting for possible endogeneity and without year fixed effects, Sub-Saharan Africa is the only region that during recessions has a positive (but not

statistically significant) coefficient. In other developing regions, the coefficient during bad times is consistent with more countercyclical policies, but only statistically significant for industrial countries. After including year fixed effects, SSA and LAC register procyclical fiscal policy during booms, but only the former region has a procyclical behavior in busts.

There is evidence that the likely access to foreign borrowing may lower the degree of procyclicality in all regions. In fact, estimates without global factor adjustments show that the procyclical expenditure policies are lower or turns to an acyclical behavior as countries are financially more open. Similar pattern is found when year fixed effects are introduced in least square regressions. Although government consumption expenditures in developing countries were mainly procyclical for the period 1970-2014, they have been moving to acyclical or countercyclical policies (at least from the expenditure side) since the late nineties.

Appendix A

Benchmark Model: Some Comparative Statics

Figure A1: An Increase in the Risk-Free Rate of Return

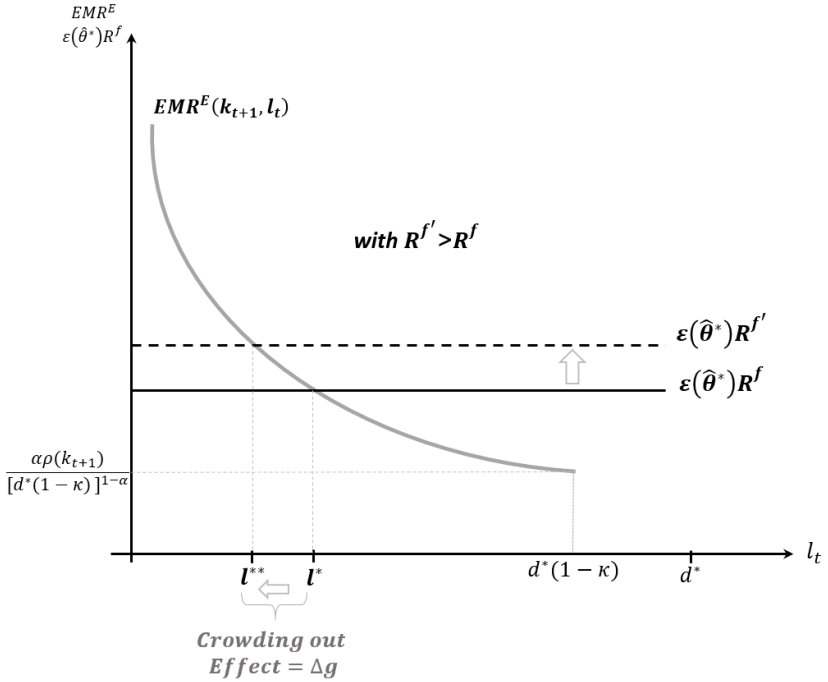


Figure A2: An Increase in the Excess Return

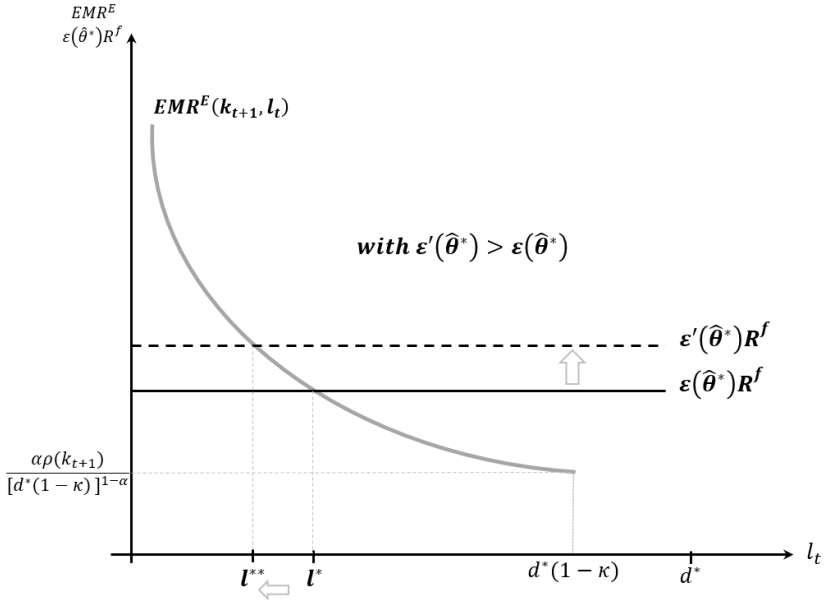
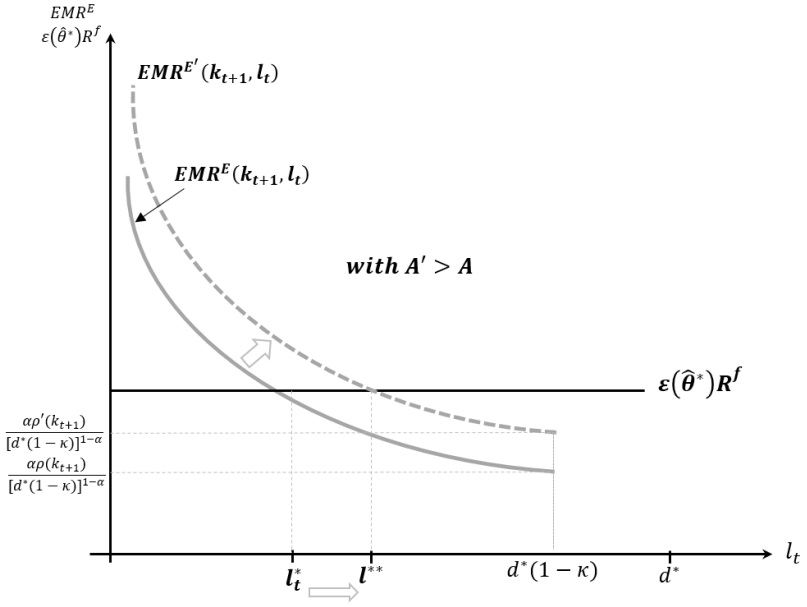


Figure A3: An Increase in the Total Factor Productivity



Appendix B

Benchmark Model: Second-Order Condition

To satisfy the sufficient condition for a local maximum, the corresponding Bordered Hessian matrix (B) must be the negative definite. Since the sign of the first leading principal minor is always negative, it is sufficient to check the sign of the determinant of the entire Bordered Hessian defined as follows:

$$\det B = \begin{vmatrix} 0 & \mathcal{L}_{\lambda,\hat{\theta}} & \mathcal{L}_{\lambda,l_t} \\ \mathcal{L}_{\lambda,\hat{\theta}} & \mathcal{L}_{\hat{\theta},\hat{\theta}} & \mathcal{L}_{\hat{\theta},l_t} \\ \mathcal{L}_{\lambda,l_t} & \mathcal{L}_{\hat{\theta},l_t} & \mathcal{L}_{l_t,l_t} \end{vmatrix}_{(\hat{\boldsymbol{\theta}}^*,\boldsymbol{l}^*)}$$

All partial derivatives in this matrix are evaluated at the critical values $(\hat{\theta}^*, l^*)$.

Claim B.1. *If* $\left[\Lambda''(\hat{\theta}) - \gamma\Omega''(\hat{\theta})\right]\lambda - \Lambda''(\hat{\theta}) < 0$ *then the Bordered Hessian matrix (B) associated with the maximization of (2.20) is negative definite. Therefore, a (local) maximum is achieved at* $(\hat{\boldsymbol{\theta}}^*, \boldsymbol{l}^*)$.

Proof. Recall the determinant is

$$\det B = \mathcal{L}_{\lambda,\hat{\theta}}\mathcal{L}_{l_t,\hat{\theta}}\mathcal{L}_{\lambda,l_t} + \mathcal{L}_{\lambda,l_t}\mathcal{L}_{\lambda,\hat{\theta}}\mathcal{L}_{\hat{\theta},l_t} - \left[(\mathcal{L}_{\lambda,l_t})^2\mathcal{L}_{\hat{\theta},\hat{\theta}} + \mathcal{L}_{l_t,l_t}(\mathcal{L}_{\lambda,\hat{\theta}})^2\right], \tag{B.1}$$

where:

$$\mathcal{L}_{\lambda,\hat{\theta}} = \rho(k_{t+1})l_t^{\alpha}\left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right], \tag{B.2}$$

$$\mathcal{L}_{\lambda,l_t} = \alpha\rho(k_{t+1})l_t^{\alpha-1}\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right] - R_{t+1}^f, \tag{B.3}$$

$$\mathcal{L}_{\hat{\theta},\hat{\theta}} = \rho(k_{t+1})l_t^{\alpha}\left\{\left[\Lambda''(\hat{\theta}) - \gamma\Omega''(\hat{\theta})\right]\lambda - \Lambda''(\hat{\theta})\right\}, \tag{B.4}$$

$$\mathcal{L}_{\hat{\theta},l_t} = \alpha\rho(k_{t+1})l_t^{\alpha-1}\left\{\left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right]\lambda - \Lambda'(\hat{\theta})\right\}, \tag{B.5}$$

$$\mathcal{L}_{l_t,\hat{\theta}} = \alpha\rho(k_{t+1})l_t^{\alpha-1}\left\{-\Lambda'(\hat{\theta}) + \lambda\left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right]\right\}, \tag{B.6}$$

$$\mathcal{L}_{l_t,l_t} = \alpha(\alpha-1)\rho(k_{t+1})l_t^{\alpha-2}\left\{\left[1 - \Lambda(\hat{\theta})\right] + \lambda\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]\right\}. \tag{B.7}$$

From the FOC (2.21a) $\left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right]\lambda - \Lambda'(\hat{\theta}) = 0$, thus, $\mathcal{L}_{\hat{\theta},l_t} = \mathcal{L}_{l_t,\hat{\theta}} = 0$ and (B.1) can be written as

$$\det B = -(\mathcal{L}_{\lambda,l_t})^2\mathcal{L}_{\hat{\theta},\hat{\theta}} - \mathcal{L}_{l_t,l_t}(\mathcal{L}_{\lambda,\hat{\theta}})^2. \tag{B.8}$$

To be negative definite the $\det B$ must be positive. Since $\mathcal{L}_{l_t,l_t} < 0$, the sign of (B.8) depends on the sign of $\mathcal{L}_{\hat{\theta},\hat{\theta}}$, which is negative iff $\left[\Lambda''(\hat{\theta}) - \gamma\Omega''(\hat{\theta})\right]\lambda - \Lambda''(\hat{\theta}) < 0$. □

Appendix C

Properties of the Excess Return at the Cutoff Point

Since $0 < \Lambda(\hat{\theta}) < 1$ and $\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta}) > 0$ given that $\hat{\theta}^* \in (\underline{\theta}, \theta^m)$, it must be that $0 < \Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta}) < 1$. As $\Lambda'(\hat{\theta}) > 0$ and $(1 - \Lambda'(\hat{\theta})) > 0$, then $\varepsilon(\hat{\theta}) > 0$ and by contradiction is straightforward to prove that [1]

$$\varepsilon(\hat{\theta}) = \frac{\Lambda'(\hat{\theta})}{\left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right]\left[1 - \Lambda(\hat{\theta})\right] + \Lambda'(\hat{\theta})\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]} > 1. \tag{C.1}$$

Taking derivative with respect to the break-even point:

$$\varepsilon'(\hat{\theta}) = \frac{\Lambda''(\hat{\theta})D(\hat{\theta}) - D'(\hat{\theta})\Lambda'(\hat{\theta})}{\left[D(\hat{\theta})\right]^2}, \tag{C.2}$$

where

$$D(\hat{\theta}) = \left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right]\left[1 - \Lambda(\hat{\theta})\right] + \Lambda'(\hat{\theta})\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right], \tag{C.3}$$

$$D'(\hat{\theta}) = \left[\Lambda''(\hat{\theta}) - \gamma\Omega''(\hat{\theta})\right]\left[1 - \Lambda(\hat{\theta})\right] + \Lambda''(\hat{\theta})\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]. \tag{C.4}$$

The sign of $\varepsilon'(\hat{\theta})$ is defined by the sign of the numerator of (C.2). Substituting (C.3) and (C.4) in the numerator and simplifying terms I get

$$\begin{aligned}\Lambda''(\hat{\theta})D(\hat{\theta}) - D'(\hat{\theta}) &= \left[1 - \Lambda(\hat{\theta})\right]\left\{\Lambda''(\hat{\theta})\left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right] - \Lambda'(\hat{\theta})\left[\Lambda''(\hat{\theta}) - \gamma\Omega''(\hat{\theta})\right]\right\} \\ &= \gamma\left[1 - \Lambda(\hat{\theta})\right]\left[\Lambda'(\hat{\theta})\Omega''(\hat{\theta}) - \Lambda''(\hat{\theta})\Omega'(\hat{\theta})\right].\end{aligned}$$

Since $\Lambda'(\hat{\theta}) > 0$ and $\Lambda''(\hat{\theta}) < 0$, while both $\Omega'(\hat{\theta})$ and $\Omega''(\hat{\theta})$ are positive, the numerator of (C.2) is positive. Therefore, $\varepsilon(\hat{\theta})$ is increasing in the cutoff point.

[1] If $0 < \varepsilon(\hat{\theta}) < 1$ then $\left[-\gamma\Omega'(\hat{\theta})\left[1 - \Lambda(\hat{\theta})\right] - \gamma\Lambda'(\hat{\theta})\Omega(\hat{\theta})\right] > 0$, which is not possible given the properties of $\Lambda(\cdot)$ and $\Omega(\cdot)$. Thus, the risk premium must be greater than 1.

Appendix D

Proof of Preposition 2.3.2

To prove the Preposition, let's consider a change in the aggregate risk such as $\sigma_p^B > \sigma_p^A$. Then, the following properties hold:

Claim D.1. *For a given $\hat{\theta}_v$, the expected gross payment, relative to the expected income, is higher in economy A, then $\left[\Lambda_V(\hat{\theta}_v, \sigma_p^B) - \Lambda_V(\hat{\theta}_v, \sigma_p^A)\right] < 0$.*

Proof. In general, $\Lambda_V(\hat{\theta}_v, \sigma_p) = \hat{\theta}_v - \int_{\underline{\theta}}^{\hat{\theta}_v} G_V(\theta_v, \sigma_p) d\theta_v$ for any σ_p, then

$$\left[\Lambda_V(\hat{\theta}_v, \sigma_p^B) - \Lambda_V(\hat{\theta}_v, \sigma_p^A)\right] = \int_{\underline{\theta}}^{\hat{\theta}_v} \left[G_V(\theta_v, \sigma_p^A) - G_V(\theta_v, \sigma_p^B)\right] d\theta_v, \tag{D.1}$$

which is negative since $G_V(\theta_v, \sigma_p^A)$ SOSD $G_V(\theta_v, \sigma_p^B)$. □

Claim D.2. *For a given $\hat{\theta}_v$, the expected monitoring cost, relative to the expected income, is higher in economy B, then $\gamma\left[\Omega_V(\hat{\theta}_v, \sigma_p^A) - \Omega_V(\hat{\theta}_v, \sigma_p^B)\right] < 0$.*

Proof. This result comes directly from (Assumption 4):
From equation (2.53), $\Omega_V(\hat{\theta}_v, \sigma_p) = \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v g_V(\theta_v, \sigma_p) d\theta_v$, thus

$$\begin{aligned}\left[\Omega_V(\hat{\theta}_v, \sigma_p^A) - \Omega_V(\hat{\theta}_v, \sigma_p^B)\right] &= \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v g_V(\theta_v, \sigma_p^A) d\theta_v - \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v g_V(\theta_v, \sigma_p^B) d\theta_v \\ &= \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v \left[g_V(\theta_v, \sigma_p^A) - g_V(\theta_v, \sigma_p^B)\right] d\theta_v < 0, \end{aligned} \tag{D.2}$$

since the mean-preserving increase in risk will shift up the density function over the interval $[\underline{\theta}, \hat{\theta}_v]$ □

Claim D.3. *For a given* $\hat{\theta}_v$, $\left[\lambda_v(\hat{\theta}_v, \sigma_p^A) - \lambda_v(\hat{\theta}_v, \sigma_p^B)\right] < 0$.

Proof. To simplify notation, the arguments of the functions are omitted. From the FOC (2.55a)

$$\begin{aligned}\left[\lambda_v^A - \lambda_v^B\right] &= \frac{\Lambda_V'^A}{\Lambda_V'^A - \gamma\Omega_V'^A} - \frac{\Lambda_V'^B}{\Lambda_V'^B - \gamma\Omega_V'^B} \\ &= \frac{\Lambda_V'^A\left[\Lambda_V'^B - \gamma\Omega_V'^B\right] - \Lambda_V'^B\left[\Lambda_V'^A - \gamma\Omega_V'^A\right]}{\left[\Lambda_V'^A - \gamma\Omega_V'^A\right]\left[\Lambda_V'^B - \gamma\Omega_V'^B\right]}. \end{aligned} \tag{D.3}$$

The denominator of (D.3) is positive, so the sign of the ratio is defined by the numerator, which can be written as $\gamma\left[\Lambda'^B\Omega_V'^A - \Lambda'^A\Omega_V'^B\right]$. By adding and subtracting $\gamma\Lambda'^B\Omega_V'^B$ to the numerator, the sign is determined by

$$sign\left(\lambda_v^A - \lambda_v^B\right) = sign\left\{\Lambda_V'^B\left[\Omega_V'^A - \Omega_V'^B\right] + \Omega_V'^B\left[\Lambda_V'^B - \Lambda_V'^A\right]\right\}. \tag{D.4}$$

Given that $\Lambda_V' > 0$ and $\Omega_V' > 0$, we just need to find the signs of the changes in marginal (expected) gross payment and monitoring cost. Since $\Lambda_V' = 1 - G_V(\hat{\theta}_v, \sigma_p)$, then

$$\left[\Lambda_V'^B - \Lambda_V'^A\right] = G_V(\hat{\theta}_v, \sigma_p^A) - G_V(\hat{\theta}_v, \sigma_p^B) < 0 \quad \text{(from Assumption 4).} \tag{D.5}$$

While $\Omega_V' = \hat{\theta} g_V(\hat{\theta}_v, \sigma_p)$

$$\left[\Omega_V'^A - \Omega_V'^B\right] = \hat{\theta}\left[g_V(\hat{\theta}_v, \sigma_p^A) - g_V(\hat{\theta}_v, \sigma_p^B)\right] < 0 \quad \text{(from Assumption 4).} \tag{D.6}$$

Consequently, $\left(\lambda_v^A - \lambda_v^B\right) < 0$. □

Appendix E

Sensitivity Analysis for Different Values of Elasticities ($\xi_{\varepsilon_V,\sigma_p}$), α, and δ

Elasticity = 0.5	(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V,\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$	δ = share of capital in total income					
	0.3	0.4	0.5	0.6	0.7	0.8
0.5	0.09	0.13	0.17	0.21	0.27	0.3
α = curvature of 0.6	0.11	0.16	0.21	0.28	0.36	0.5
the investment's 0.7	0.13	0.19	0.27	0.36	0.48	0.6
production 0.8	0.16	0.24	0.33	0.46	0.64	0.9
function 0.9	0.18	0.28	0.409	0.59	0.85	1.3
0.99	0.21	0.33	0.490	0.73	1.13	1.9

Elasticity = 0.8	(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V,\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$	δ = share of capital in total income					
	0.3	0.4	0.5	0.6	0.7	0.8
0.5	0.14	0.20	0.27	0.34	0.43	0.5
α = curvature of 0.6	0.18	0.25	0.34	0.45	0.58	0.7
the investment's 0.7	0.21	0.31	0.43	0.58	0.77	1.0
production 0.8	0.25	0.38	0.53	0.74	1.02	1.4
function 0.9	0.30	0.45	0.655	0.94	1.36	2.1
0.99	0.34	0.52	0.784	1.17	1.81	3.0

Elasticity = 1	(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V,\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$	δ = share of capital in total income					
	0.3	0.4	0.5	0.6	0.7	0.8
0.5	0.18	0.25	0.33	0.43	0.54	0.7
α = curvature of 0.6	0.22	0.32	0.43	0.56	0.72	0.9
the investment's 0.7	0.27	0.39	0.54	0.72	0.96	1.3
production 0.8	0.32	0.47	0.67	0.92	1.27	1.8
function 0.9	0.37	0.56	0.818	1.17	1.70	2.6
0.99	0.42	0.66	0.980	1.46	2.26	3.8

Elasticity = 1.5	(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V,\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$	δ = share of capital in total income					
	0.3	0.4	0.5	0.6	0.7	0.8
0.5	0.26	0.38	0.50	0.64	0.81	1.0
α = curvature of 0.6	0.33	0.47	0.64	0.84	1.09	1.4
the investment's 0.7	0.40	0.58	0.81	1.09	1.44	1.9
production 0.8	0.47	0.71	1.00	1.38	1.91	2.7
function 0.9	0.55	0.84	1.227	1.76	2.55	3.9
0.99	0.63	0.98	1.470	2.19	3.39	5.7

Elasticity = 2	(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V,\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$	δ = share of capital in total income					
	0.3	0.4	0.5	0.6	0.7	0.8
0.5	0.35	0.50	0.67	0.86	1.08	1.3
α = curvature of 0.6	0.44	0.63	0.86	1.13	1.45	1.8
the investment's 0.7	0.53	0.78	1.08	1.45	1.92	2.5
production 0.8	0.63	0.94	1.33	1.85	2.55	3.6
function 0.9	0.74	1.13	1.636	2.35	3.41	5.1
0.99	0.84	1.31	1.960	2.93	4.51	7.6

Elasticity = 2.5	(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V,\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$	δ = share of capital in total income					
	0.3	0.4	0.5	0.6	0.7	0.8
0.5	0.44	0.63	0.83	1.07	1.35	1.7
α = curvature of 0.6	0.55	0.79	1.07	1.41	1.81	2.3
the investment's 0.7	0.66	0.97	1.35	1.81	2.40	3.2
production 0.8	0.79	1.18	1.67	2.31	3.18	4.4
function 0.9	0.92	1.41	2.045	2.93	4.26	6.4
0.99	1.06	1.64	2.450	3.66	5.64	9.5

Elasticity = 3	(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V,\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$	δ = share of capital in total income					
	0.3	0.4	0.5	0.6	0.7	0.8
0.5	0.53	0.75	1.00	1.29	1.62	2.0
α = curvature of 0.6	0.66	0.95	1.29	1.69	2.17	2.8
the investment's 0.7	0.80	1.17	1.62	2.17	2.88	3.8
production 0.8	0.95	1.41	2.00	2.77	3.82	5.3
function 0.9	1.11	1.69	2.455	3.52	5.11	7.7
0.99	1.27	1.97	2.941	4.39	6.77	11.4

Elasticity = 4	(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V,\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$	δ = share of capital in total income					
	0.3	0.4	0.5	0.6	0.7	0.8
0.5	0.71	1.00	1.33	1.71	2.15	2.7
α = curvature of 0.6	0.88	1.26	1.71	2.25	2.90	3.7
the investment's 0.7	1.06	1.56	2.15	2.90	3.84	5.1
production 0.8	1.26	1.88	2.67	3.69	5.09	7.1
function 0.9	1.48	2.25	3.273	4.70	6.81	10.3
0.99	1.69	2.62	3.921	5.85	9.03	15.2

Elasticity = 5	(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V,\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$	δ = share of capital in total income					
	0.3	0.4	0.5	0.6	0.7	0.8
0.5	0.88	1.25	1.67	2.14	2.69	3.3
α = curvature of 0.6	1.10	1.58	2.14	2.81	3.62	4.6
the investment's 0.7	1.33	1.94	2.69	3.62	4.80	6.4
production 0.8	1.58	2.35	3.33	4.62	6.36	8.9
function 0.9	1.85	2.81	4.091	5.87	8.51	12.9
0.99	2.11	3.28	4.901	7.32	11.29	19.0

Elasticity = 5.7	(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V,\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$	δ = share of capital in total income					
	0.3	0.4	0.5	0.6	0.7	0.8
0.5	1.01	1.43	1.90	2.44	3.07	3.8
α = curvature of 0.6	1.25	1.80	2.44	3.21	4.13	5.3
the investment's 0.7	1.52	2.22	3.07	4.13	5.48	7.3
production 0.8	1.80	2.68	3.80	5.26	7.25	10.1
function 0.9	2.11	3.21	4.664	6.69	9.71	14.7
0.99	2.41	3.74	5.587	8.34	12.87	21.7

Appendix F

List of Countries and Regions Analyzed in Chapter 3

Table F.1: List of Countries by Region

Sub-Saharan Africa	*Latin America*	*Developing Asia*	*Industrial Countries*
Angola	Argentina	Bangladesh	Australia
Benin	Bolivia	Cambodia	Austria
Botswana	Brazil	China	Belgium
Burkina Faso	Chile	Fiji	Canada
Burundi	Colombia	India	Denmark
Cameroon	Costa Rica	Indonesia	Finland
Cape Verde	Dominican Republic	Lao P.D.R.	France
Central African Republic	Ecuador	Malaysia	Germany
Chad	El Salvador	Maldives	Greece
Comoros	Guatemala	Mongolia	Iceland
Congo, Rep.	Haiti	Myanmar	Ireland
Congo, Dem. Rep.	Honduras	Nepal	Italy
Cote d'Ivoire	Mexico	Papua New Guinea	Japan
Gabon	Nicaragua	Philippines	Netherlands
Gambia, The	Panama	Samoa	New Zealand
Ghana	Paraguay	Sri Lanka	Norway
Guinea	Peru	Thailand	Portugal
Guinea-Bissau	Trinidad and Tobago	Tonga	Spain
Kenya	Uruguay	Vanuatu	Sweden
Liberia	Venezuela, RB	Vietnam	Switzerland
Madagascar			United Kingdom
Malawi			United States
Mali			
Mauritania			
Mauritius			
Mozambique			
Niger			
Nigeria			
Rwanda			
Senegal			
Sierra Leone			
South Africa			
Tanzania			
Togo			
Uganda			
Zambia			
Zimbabwe			

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